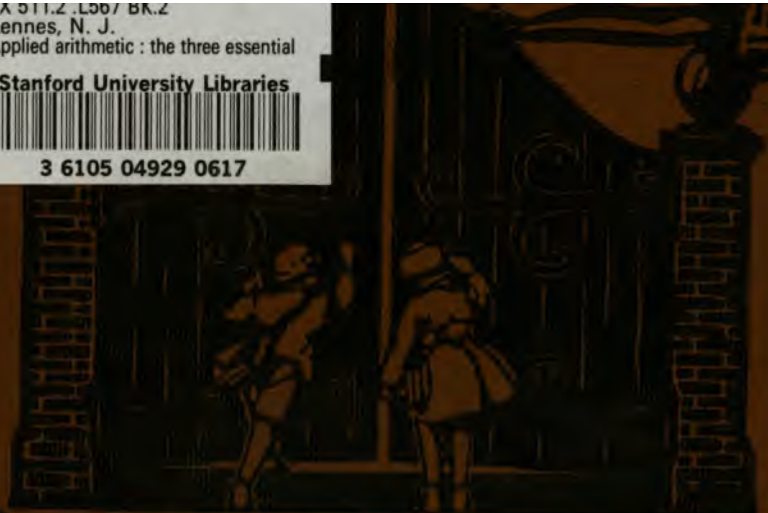


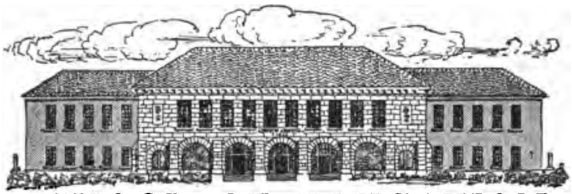
APPLIED ARITHMETIC

THE THREE ESSENTIALS

TEACHER'S EDITION

BOOK 2

LENNES AND JENKINS



LENNES AND JENKINS

LIPPINCOTT'S SCHOOL TEXT SERIES
EDITED BY WILLIAM F. RUSSELL, PH.D.
DEAN, COLLEGE OF EDUCATION, STATE UNIVERSITY OF IOWA

APPLIED ARITHMETIC
THE THREE ESSENTIALS

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BOOK II

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PREFACE

THIS book is the second of a three-book series and is intended to cover the work in arithmetic of the fifth and sixth grades. The principles which have guided the authors may be grouped under three main heads:

1. *Selection and Organization of Subject Matter.* Recent discussion and practice, as revealed in the literature and in published curricula, seem to show substantial agreement as to what topics should be included in the earlier part of a course in Arithmetic, while there is yet considerable difference of opinion as to topics to be included in the later parts. All topics whose inclusion or exclusion is now being debated are placed among the supplementary topics at the end of books two and three. The main body of each book furnishes a minimum course which may be studied without break in continuity. At convenient points in the text supplementary topics may be taken up. It is believed that in this respect these books will serve each of many different needs just as effectively as if they were prepared to meet such need exclusively. The business of the maker of text books is to furnish teachers and supervising officers effective instruments for carrying out their purposes, rather than to seek to impose rigidly his own personal predilections.

It has been the purpose of the authors to arrange the subject matter in such manner that the greatest simplicity of treatment may be attained. In the second book this is exemplified by the consistent use of the principle of product and factors in the solution of problems, as shown on pages 24–29, 88, 89, 176, 177, 206–216. The problems considered on these pages constitute the most important applications of arithmetic within the scope of this book, and their solution is effected by means of the simple ideas developed in book one in connection with such simple combinations as $3\times4=12$ and $2\times3\times4=24$.

In book three this unifying and simplifying of the subject matter is carried out consistently.

2. *Derivation and Application.* The subject matter of Arithmetic may be divided into two parts:

(*a*) The four fundamental operations on integers and common and decimal fractions.

(*b*) The application of these operations to the solution of problems.

In this book the fundamental operations are completed by a systematic study of common and decimal fractions. These are developed from simple objects well known to all children and from drawings which every child can make without trouble (see pages 34–38, 41, 42, 44, 45, 64, 78, 98, 112, 113). Few mistakes in teaching are more common or more pernicious than the subjection of the child to special and transient experiences for the sake of illustrative material. It has been the purpose of the authors to find the basis for arithmetic in the simple, obvious facts of the stream of life that flows about the child.

In the applications of arithmetic the only new element is the situation which gives rise to the problem. In percentage there is nothing new except the fact that *hundreths* or *per cents* are used very generally in practical life. In discount, the only new element is the fact that a reduction in price stated as a certain number of hundredths or per cents of the original price is often made. Similar remarks apply to commission, interest, profit and loss, taxes, insurance and other applications of percentage. A serious effort has been made to make this clear to the child and to put him in the way of obtaining solid information about the only new matter with which he is confronted as each topic is taken up. On this point see pages, 134, 135, 136, 160, 161, 178, 182, 186, 188, 220, 221, 276, 278, 280, 282, 285.

3. *Motivation.* The subject matter of arithmetic can be motivated most effectively only when the freest possible use is made of the child's many spontaneous interests. The authors believe they have not neglected any opportunity to interest the child in the

subject matter itself and its manifold applications. They have recognized, however, that it is possible so to connect the learning of arithmetic with other activities which in themselves are of compelling interest to the child, that the combination will be a source of joy and life when the arithmetic elements alone would lead to sadness and forced labor. For this reason systematic use has been made of games of group competition. On this point see the pages referred to in the index under the head, "Games used in drills."

The most effective applications of arithmetic can be made only when considerable local material is brought in. In the upper grades such material is also an important element in motivation. The child is becoming more and more interested in the activities of the grown-up. He is curious to know how things are really done, and if properly directed will gladly gather information from his own environment. For this reason many suggestions for bringing in local material are made throughout the book. See for example pages 20, 27, 69, 71, 93, 103, 104, 105, 107, 109, 136, 178, 187, 190, 194, 200.

Some other points may be mentioned. It is found on investigation that the fractions which are in common use are very simple. Denominators other than 2, 3, 4, 8, 12, 16, 32, occur so seldom as to be almost negligible. In fractions to be added, subtracted or divided they may be said not to occur at all. A problem like $\frac{3}{5}+\frac{2}{7}$ is as rare in practical life as the buffalo on our western prairies. For this reason much practice has been given on the manipulation of very simple fractions, while the more complicated fractions have been given less space. See pages 39, 40, 42, 43, 51, 52, 78, 96, 97, 98, 100, 154, 155.

As in book one, the two standard methods of subtraction have been given equal prominence. The addition method has been carried through consistently in fractions and in denominate numbers. See pages 54, 55, 247.

No effort has been spared to make the books attractive in

appearance and convenient in the arrangement of subject matter on the page.

The reason for producing this series lies, not in any *one* of the features mentioned here, but in the belief that by careful and systematic use of *all* that is best in present knowledge and usage it should be possible to produce a series of texts that would more adequately meet the requirements of the modern school than is done by any texts now in existence.

These books have been built leisurely. The first draft was made nearly ten years ago. During the intervening time the work of selecting what has proved most certainly valuable, both in what is old and what is new, and in organizing and relating the various parts, has been in constant progress. It is difficult, if not impossible to make proper acknowledgement to all who have helped in this work. The most prominent among these, however, have been Dr. Theodore Lindquist, Head of the Department of Mathematics in the State Normal School at Emporia, Kansas, Mr. H. C. Pearson, Principal of the Horace Mann School in New York City, and several of the Horace Mann teachers.

THE AUTHORS.

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BOOK II

CHAPTER I

1. The first part of this book consists of a review of what you have already learned about Arithmetic, together with many new applications which will show you more of the uses that people all about you are making of this important subject.

2. **Digits.** The numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, are called digits.

3. **Place Value.** In a number with more than five digits the digit in the 6th place represents hundreds of thousands, the digit in the 7th place represents millions, and so on, as indicated below.

billions	hundred millions, ten millions, millions	hundred thousands, ten thousands, thousands	hundreds, tens, ones
5	397	482	684
Billions Period.	Millions Period.	Thousands Period.	Ones Period.

For convenience in reading, the digits of large numbers are separated by commas into groups of 3 digits each. Thus, the above number is written 5,397,482,684. These groups are called *periods*.

In a whole number the digit in the right-hand place represents *ones*, the digit in the next place represents *tens*, and so on.

State what is represented by each digit in the number 5,397,482,684.

4. Reading Numbers. The number 8,604,896 is read *eight million, six hundred four thousand, eight hundred ninety-six.*

Notice that the zero is not read. In this number it signifies *no ten thousands.*

ORAL EXERCISES

Read the following:

1. In the year 1918 there were produced in the United States,
918,920,000 bushels of wheat,
3,159,494,000 bushels of corn,
11,816,000 bales of cotton,
634,594,000 tons of coal.

2. In the year 1910 the populations of the world's most important countries were:

United States *	91,972,267
British Empire	396,294,758
Germany	64,903,423
Russian Empire	160,095,200
France	38,961,945
Austria	49,418,596
Italy	32,475,253
Spain	19,503,008
China	439,214,000
Japan	53,875,390

WRITTEN EXERCISES

Write the following numbers, using figures:

3. One million, six hundred thirty-five thousand, three hundred ninety-eight.

4. Twelve million, one hundred twenty-eight thousand, six hundred twenty-one.

* Not including any foreign possessions.

5. One hundred eight million, thirty-seven thousand, five hundred nine.

6. Seventy-four million, eight hundred two thousand, seventy-six.

5. **Roman Numerals.** Roman numerals are formed by means of the seven letters:

Letters	I	V	X	L	C	D	M
Values	1	5	10	50	100	500	1000

1. Read the Roman numerals II, IV, VI, IX, XI, XII.

2. What is the difference between IV and VI? between IX and XI?

Explain the system according to which the Roman numerals are formed.

3. Read the Roman numerals XXV, XL, IL, LIX, LXXIV, CCXXIX, MCLXXXVII, MCMXVI.

4. Write the following dates in Roman numerals: 1492, 1776, 1861, 1865, 1907, 1917.

6. **Addition, Addends, Sum.** The process of taking two numbers together to form one number is called *addition*. The numbers added are called *addends*. The result is called the *sum*.

The sign +, is the sign of addition, and is read *plus*.

Thus $9+6=15$ is read "9 plus 6 equals 15." In this combination 9 and 6 are the addends and 15 is the sum.

Make a list of all the addition combinations (45 in all) of two numbers, each below 10.

You should know each of these instantly and without having to think of any other combination.

Thus, for example, as soon as you hear the numbers 9 and 6 or see them you should know that their sum is 15. Unless you know these combinations thoroughly you will not make a good record in your number work this year.

Add: 853	The sum of the first column is 25. Explain what you
70	do with the 5 ones and the 2 tens. The sum of the
66	second column, including the 2 tens *carried* is how
98	many tens? Explain fully what you do with this
438	number. How many hundreds are *carried* to the
	hundreds' column?

Test by adding each column both ways. In adding the first column say 3, 9, 17, 25. Do not say $3+6=9$, $9+8=17$, $17+8=25$.

In practice we do not even *think* that we carry so many *tens*, *hundreds* or *thousands*, but simply put down the digit to the right in each sum and carry the other.

WRITTEN EXERCISES

Add the following and keep a record of the time in which you do it.

1.	2.	3.	4.
938	7184	43742	99437
5873	76148	634134	549312
174	1972	9182	191563
96	30050	431876	10690
484	17436	85734	302715
8300	23456	91360	487659
15865	156246	1296028	1641376

5.	6.	7.	8.
2467	4984	28040	12134
841	617	1060	53143
84976	174836	467891	34567
87284	68475	59387	21098
29437	457893	86579	53174
59848	9748	165893	256
264853	716553	808850	174372

9.	10.	11.	12.
434	87065	97635	627
5678	126936	8976	29189
3219	90531	340698	577
46897	78964	69784	910
1071	1547	25652	8101
9140	13702	7814	25170
66439	398745	550559	64574

Below are the populations of the States of the Union as given by the census of 1920. First find the population of each group of States, and then find the total population. **105,710,620**

New England States:

Maine	768,014
New Hampshire	443,083
Vermont	352,428
Massachusetts	3,852,356
Rhode Island	604,397
Connecticut	1,380,631
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New York	10,385,227
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Kansas	1,769,257
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7. **The Decimal Number System.** The most important thing about our number system is that we count to *ten*, then to *twenty*, or two times ten, then to *thirty* or three times ten, and so on to *ten times ten* or *one hundred*, and so on. We thus see that the number *ten* is of special importance in our number system. For this reason it is called the *decimal number system*. The word decimal comes from the Latin word *decem*, meaning ten.

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8. Subtraction. *Subtraction consists in taking one number away from another.*

The sign $-$, is the sign of subtraction, and is read *minus.*

Subtraction is also defined as the process of finding the difference between two numbers; or as finding how much must be added to one number to make the sum equal to another number.

Thus, to subtract 8 from 17 consists in taking 8 from 17, or in finding the difference between 8 and 17, or in finding how much must be added to 8 to make 17. That is, $17-8=9$ *because* $9+8=17$.

9. Minuend, Subtrahend, Remainder. The number before the minus sign is called the *minuend,* the number after the minus sign is called the *subtrahend,* and the result is called the *difference* or *remainder.*

ORAL EXERCISES.

In each of the following state what process must be used and why:

1. How much more is 92 than 34?
2. John and Henry together have 27 cents. How much has John if Henry has 16 cents?
3. John has 22 cents and Henry has 14 cents. How much more has John than Henry?
4. By how much does 25 exceed 16?
5. If the sum of the populations in the cities of St. Paul and Minneapolis is given, and also the population of St. Paul, how would you find the population of Minneapolis?
6. If you know how much a man has put into a bank and also how much he has left in the bank, how would you find how much he has drawn out?
7. If you know how much older a father is than his son, and also the age of the father, how do you find the son's age?

Example. From 3204 subtract 1846.

First method (adding or Austrian method): *

3204	*Ones.*	14=6+8	Write *8* Carry 1
1846	*Tens.*	10=5+5	Write *5* Carry 1
1358	*Hundreds.*	12=9+3	Write *3* Carry 1
	Thousands.	3=2+1	Write *1*

In each column we find what must be added to the subtrahend to make the sum equal to the minuend (increased, if necessary, by a 1 in the next higher place). A fuller explanation is given in Book III.

Second method (taking away method):

Since we can not take 6 from 4 we take 1 hundred from the 2 hundred and regard it as a 90+10. The 10 is added to the 4. Then 6 from 14 leaves 8, 4 (tens) from 9 (tens) leaves 5 (tens).

Again, we can not take 8 (hundreds) from 1 (hundred). Hence, we take 1 (thousand) from the 3 (thousands) and add to the 1 (hundred). Then 8 from 11 leaves 3, and 1 from 2 leaves 1.

WRITTEN EXERCISES.

In this manner subtract the following:

1. 3198	2. 7490	3. 5514	4. 6542
1824	579	1678	1376
1374	6911	3836	5166
5. 1914	6. 4917	7. 7910	8. 9400
265	1728	2837	3782
1649	3189	5073	5618

9. How much must be added to 346 to make the sum 691? 345

10. The sum of two numbers is 8691. One of the numbers is 3649. Find the other number. 5042

11. What is the difference between 24,791 and 8,691? 16,100

* Use the method which the child has already learned and pay no attention to the other method.

This man is depositing money in the bank. The bank keeps the money for him until he needs it, when he draws it out again. This man owns a store, and deposits his money each day.

WRITTEN EXERCISES.

1. Find how much money this man put in the bank in one week if he deposited \$186.50 on Monday, \$371.40 on Tuesday, \$249.30 on Wednesday, \$517.80 on Thursday, \$398.60 on Friday, and \$492.40 on Saturday. **\$2216.00**

Find the total deposits for each of these men if they made deposits as follows:

2. C. J. Hicks.

Monday	\$481.50	Tuesday	\$468.60	
Wednesday	532.25	Thursday	425.30	
Friday	416.80	Saturday	694.40	
				\$3018.85

3. Gilbert A. Harris.

Monday	\$769.50	Tuesday	\$531.00	
Wednesday	567.50	Thursday	604.75	
Friday	512.25	Saturday	648.00	
				\$3633.00

4. James B. Ashton.

Monday	\$476.00	Tuesday	\$321.60	
Wednesday	367.90	Thursday	619.75	
Friday	390.65	Saturday	405.40	
				\$2581.30

5. John R. Walker.

Monday	\$1984.72	Tuesday	\$1580.75	
Wednesday	1267.80	Thursday	1460.50	
Friday	894.65	Saturday	2170.70	**\$9359.12**

WRITTEN EXERCISES.

At the end of each month, or when asked to do so, the bank gives each depositor a list of all the money he has withdrawn since the last list was made.

A woman who paid the running expenses of her household with money drawn from the bank received the following lists of withdrawals:

January	February	March	April
$35.00	$35.00	$21.26	$35.00
12.55	5.40	35.00	17.50
8.70	11.76	8.53	8.30
7.60	5.00	5.00	2.60
5.00	15.00	4.23	4.70
4.60	4.54	2.64	24.60
10.00	2.31	4.69	9.10
20.00	3.86	10.00	7.40
13.40	10.00	2.64	14.75
11.60	15.00	62.25	7.13
5.00	3.45	7.80	9.40
2.00	5.35	4.60	12.30
4.00	4.40	21.30	7.90
$139.45	**$121.07**	**$189.94**	**$160.68**

WRITTEN EXERCISES.

1. How much did this woman withdraw from the bank altogether? *Suggestion:* First find out how much she withdrew each month. **$611.14**

2. If this woman deposited $250 in January, did she draw out more or less than she deposited that month? how much? **Less $110.55**

3. This woman deposited $185 in February, $210 in March, and $217.50 in April. Did she draw out more or less than she deposited in each of these months? how much for each month? **$63.93 less, $20.06 less, $56.82 less.**

10. Multiplication, Multiplicand, Multiplier, Product. Multiplying a number by an integer gives the same result as adding the number to itself as often as indicated by the integer.

The number multiplied is called the *multiplicand.*

The number by which we multiply is called the *multiplier.*

The result obtained in multiplication is called the *product.*

The sign $\times$, is the sign of multiplication, and is read *times.*

Thus "$5\times15=75$" is read "5 times 15 equals 75."

In this example 15 is the *multiplicand,* 5 is the *multiplier,* and 75 is the *product.*

Example. Multiply 437 by 6.

437	(multiplicand)	Also find 6×437 by addition, and thus show that multiplication may be regarded as a short way of adding.
6	(multiplier)	
2622	(product)	

In practice the process of multiplying one number by another is based on the multiplication table. The number facts of this table must be known instantly and separately.

ORAL EXERCISES.

Read rapidly and supply the missing numbers in each of the following:

$8\times8=?$	$5\times9=?$	$10\times7=?$	$5\times7=?$
$7\times9=?$	$6\times5=?$	$2\times6=?$	$10\times4=?$
$6\times6=?$	$5\times8=?$	$10\times8=?$	$9\times2=?$
$4\times8=?$	$6\times7=?$	$3\times7=?$	$8\times9=?$
$4\times9=?$	$7\times7=?$	$9\times3=?$	$7\times8=?$
$6\times4=?$	$9\times9=?$	$10\times9=?$	$3\times6=?$
$10\times5=?$	$10\times6=?$	$7\times2=?$	$9\times6=?$
$2\times8=?$	$3\times8=?$	$4\times7=?$	$6\times8=?$

If any of the combinations of the multiplication table cause you to hesitate in the least, make a list of these and practice on them till you know them well.

11. The Unit. A single object or a group of objects taken together as one, and used for the purposes of counting or measuring is called a *unit.*

Thus, one book, one pencil, one desk, are units.

One dozen, or one herd of cattle, may also be regarded as units.

12. Concrete and Abstract Numbers. When a number refers to a definite kind of thing, the number is said to be *concrete.* Otherwise, it is said to be *abstract.*

Thus, 6 horses, 14 men, 10 dollars, 3 days, are concrete numbers, while 6, 14, 10, 3, are abstract numbers.

13. The Multiplier Abstract. We can multiply 8 dollars by 4, getting 32 dollars, but we cannot multiply 4 by 8 dollars.

That is, the multiplier must be an abstract number. If the multiplicand is a concrete number, the product will be a concrete number of the same kind as the multiplicand.

14. The Method Used in Practice. In actual practice, however, numbers are multiplied as if they were all abstract, the smaller number usually being used as the multiplier.

There is seldom any difficulty in telling what kind of number the product is.

Thus, to find the amount earned in 274 days at $4.00 a day, we multiply 274 by 4, and not $4 by 274. No practical business man would ever solve this problem by multiplying $4 by 274.

ORAL EXERCISES

1. Name several concrete and several abstract numbers.

2. A farmer bought 575 sheep at $9 a head. How do you find how much he paid for all the sheep?

3. How would you solve the following problem: If the average yield of corn in certain parts of Illinois is 58 bu. to the acre, how many bu. may be expected in a county where 149,800 acres are planted in corn?

Example 1. Multiply 4 by 6. Then multiply 4 by 3 and the product by 2. Compare the two products.

From this we see that to multiply a number by 3 and the product by 2 gives the same result as to multiply it by 2×3 or 6.

Example 2. Multiply 4 by 15. Also multiply 4 by 3 and the product by 5. Compare results.

In the same manner as in these examples we may multiply a number by 20 by multiplying it by 2 and the product by 10. To multiply a number by 700, we may multiply it by 7 and the product by 100.

ORAL EXERCISES

1. How do we multiply a number by 10? by 100? by 1000?
2. What is a convenient way of multiplying a number by 30? by 40? by 50? by 60? by 70? by 80? by 90?
3. What is a convenient way of multiplying a number by 200? by 300? by 400? by 500? by 600? by 800? by 900?
4. What is a convenient way of multiplying a number by 3000? by 8000? by 9000? by 4000?

WRITTEN EXERCISES

Find the products of the following:

1. $50 \times 165 = ?$ 8,250
2. $80 \times 642 = ?$ 51,360
3. $500 \times 42 = ?$ 21,000
4. $200 \times 76 = ?$ 15,200
5. $40 \times 308 = ?$ 12,320
6. $700 \times 531 = ?$ 371,700
7. $400 \times 59 = ?$ 23,600
8. $70 \times 197 = ?$ 13,790
9. $800 \times 396 = ?$ 316,800
10. $4000 \times 297 = ?$ 1,188,000
11. $7000 \times 834 = ?$ 5,838,000
12. $6000 \times 397 = ?$ 2,382,000
13. $500 \times 219 = ?$ 109,500
14. $300 \times 807 = ?$ 242,100
15. $600 \times 598 = ?$ 358,800
16. $700 \times 379 = ?$ 265,300
17. $8000 \times 89 = ?$ 712,000
18. $7000 \times 298 = ?$ 2,086,000
19. $900 \times 574 = ?$ 516,600
20. $600 \times 853 = ?$ 511,800
21. $3000 \times 947 = ?$ 2,841,000
22. $4000 \times 576 = ?$ 2,304,000
23. $700 \times 197 = ?$ 137,900
24. $900 \times 643 = ?$ 578,700

Example. 1. Multiply 6492 by 32.

6492	Multiply 6492 by 2, then by 30, and add the products.
32	How do we multiply by 30?
12984	Note, that when multiplying by 30, the first figure is written in tens' column and the zero is omitted. It is clear that this amounts to the same thing as to multiply by 30 and writing the first figure (zero) in ones' place.
19476	
207744	

Example. 2 Multiply 4502 by 207.

4502	Multiply 4502 by 7, then by 200, and add the products. Instead of multiplying by 2 and then by 100 by annexing 2 zeros we simply multiply by 2 and write the first figure in hundreds' place and omit the zeros.
207	
31514	
9004	
931914	

WRITTEN EXERCISES

1. Explain fully the multiplication of 489 by 648. Check your work by multiplying 648 by 489. **316,872**

Multiply the following and check the work:

2. 876	3. 498	4. 398	5. 642
154	576	680	857
134,904	**286,848**	**270,640**	**550,194**
6. 297	7. 897	8. 408	9. 5987
806	587	796	846
239,382	**526,539**	**324,768**	**5,065,002**
10. 4934	11. 1398	12. 4572	13. 3984
902	789	597	276
4,450,468	**1,103,022**	**2,729,484**	**1,099,584**

14. Multiply \$14.75 by 68.

Suggestion: Multiply as if there were no decimal point, and place a decimal point in the product directly under the decimal point in the multiplicand. **\$1003.00**

15. The area of the city of Chicago is about 190 square miles. How many acres is this? (One square mile = 640 acres.) How many farms averaging 100 acres would this make? If these farms had an average population of 8 persons to the farm, how many people would be living on this land? **121,600 acres, 1,216 farms, 9,728 persons**

John and Eleanor are helping their mother canning vegetables for the winter.

ORAL EXERCISES

1. At 4 cents a pound how much did 50 pounds of unshelled peas cost?

2. After shelling and drying, these peas filled 8 quart-jars. How much did the dried peas cost per quart?

3. At five cents a pound, how much did 120 pounds of string beans cost?

4. If these beans filled 40 quart-jars, how much did the canned beans cost per quart?

5. At 25 cents for 1 dozen bunches, how much did 8 dozen bunches of beets cost?

6. If these beets filled 8 quart-jars, how much did canned beets cost per quart?

7. At 20 cents per dozen ears, how much did 3 sacks of corn cost if each sack contained 15 dozen ears?

WRITTEN EXERCISES

1. Cherries may be bought for 55 cents a gallon when delivered to the house, and for 20 cents a gallon when bought on the trees. How much did John and Eleanor save by picking 14 gallons of cherries? **$4.90**

2. At 20 cents a gallon for cherries how much will it cost to can 30 quarts, if 3 cents' worth of sugar is used for each quart and if 35 cents' worth of gas is used for the 30 quarts? The value of the labor is not counted. **$2.75**

3. During the summer this family put down 45 dozen eggs at a total cost of 32 cents per dozen. If bought at the store during the winter, these eggs would cost on an average 48 cents a dozen. How much do the family save by putting down the eggs? **$7.20**

4. John's mother bought currants for 35 cents a gallon. If one gallon of currants made 12 glasses of jelly, how much did she pay for enough currants to make 60 glasses of jelly? **$1.75**

15. Division, Dividend, Divisor, Quotient. If the product of two numbers is given and one of the numbers, the process of finding the other number is called *division.*

The given product is called the *dividend.*

That one of the two numbers which is given is called the *divisor.* The one which is to be found is called the *quotient.*

Thus in $3\times ?=12$, the finding of the missing number is *division,* 12 is the *dividend,* 3 the *divisor,* and the missing number, 4, is the *quotient.*

It is important that this definition of division should be grasped thoroughly, as it will be a great help in solving practical problems.

ORAL EXERCISES

The finding of the missing numbers in each of the following is a problem in division. Give the dividend, the divisor and the quotient of each:

1. $3\times ?=6$	$5\times ?=10$	$3\times ?=15$	$?\times 3=27$
2. $7\times ?=21$	$2\times ?=16$	$?\times 4=20$	$5\times ?=35$

3. If the product of two numbers is known and also one of the numbers, how do you find the other?

Example. Divide 8786 by 6.

$\begin{array}{l} \text{1464, remainder 2.} \\ 6\overline{)8786} \end{array}$ Explain this process as fully as you can. See if you can understand it better now than you did in the fourth grade.

WRITTEN EXERCISES

Divide:

1. $\begin{array}{r} 214\text{-}3 \\ 4\overline{)859} \end{array}$	5. $\begin{array}{r} 237\text{-}4 \\ 9\overline{)2137} \end{array}$	9. $\begin{array}{r} 552 \\ 7\overline{)3864} \end{array}$	13. $\begin{array}{r} 656\text{-}1 \\ 7\overline{)4593} \end{array}$
2. $\begin{array}{r} 1334\text{-}2 \\ 7\overline{)9340} \end{array}$	6. $\begin{array}{r} 396\text{-}4 \\ 5\overline{)1984} \end{array}$	10. $\begin{array}{r} 698\text{-}5 \\ 7\overline{)4891} \end{array}$	14. $\begin{array}{r} 474\text{-}2 \\ 8\overline{)3794} \end{array}$
3. $\begin{array}{r} 1142 \\ 8\overline{)9136} \end{array}$	7. $\begin{array}{r} 4572\text{-}3 \\ 4\overline{)18291} \end{array}$	11. $\begin{array}{r} 475\text{-}1 \\ 5\overline{)2376} \end{array}$	15. $\begin{array}{r} 587\text{-}4 \\ 9\overline{)5287} \end{array}$
4. $\begin{array}{r} 246\text{-}3 \\ 6\overline{)1479} \end{array}$	8. $\begin{array}{r} 986\text{-}2 \\ 3\overline{)2960} \end{array}$	12. $\begin{array}{r} 865 \\ 8\overline{)6920} \end{array}$	16. $\begin{array}{r} 765\text{-}3 \\ 6\overline{)4593} \end{array}$

Example 1. Divide 71496 by 9, using short division. Then divide by using long division.

Compare these processes to see that they are alike, except that in long division more of the work is written down.

Example 2. Divide 9478 by 600.

15, remainder 478. Explain this process as fully as you can.
$6\cancel{0}\cancel{0})\overline{94\cancel{7}\cancel{8}}$

WRITTEN EXERCISES

Divide:

1. $\begin{array}{r}49\text{-}58\\ 80)\overline{3978}\end{array}$	2. $\begin{array}{r}87\text{-}12\\ 90)\overline{7842}\end{array}$	3. $\begin{array}{r}94\text{-}7\\ 70)\overline{6587}\end{array}$	4. $\begin{array}{r}83\text{-}7\\ 60)\overline{4987}\end{array}$
5. $\begin{array}{r}143\text{-}10\\ 40)\overline{5730}\end{array}$	6. $\begin{array}{r}48\text{-}786\\ 800)\overline{39186}\end{array}$	7. $\begin{array}{r}14\text{-}349\\ 600)\overline{8749}\end{array}$	8. $\begin{array}{r}59\text{-}660\\ 900)\overline{53760}\end{array}$

16. Estimating Quotients in Long Division. The most difficult step in long division is to find the quotient figures. The process is shown in the following:

Example 1. Divide 567 by 92.

$$\begin{array}{r}6\\ 92)\overline{567}\\ 552\\ \hline 15\end{array}$$

Taking quotient of tens, $56 \div 9 = 6$ with a remainder. On multiplying, 6 is found to be the correct quotient.

Example 2. Divide 784 by 88.

$$\begin{array}{r}8\\ 88)\overline{784}\\ 704\\ \hline 80\end{array}$$

Taking quotients of tens, $78 \div 9 = 8$ with a remainder. On multiplying, 8 is found the correct quotient.

ORAL AND WRITTEN EXERCISES

Estimate quotients in the following and make a list of your estimates. After having estimated all of them, test to see whether they are right.

1. $\begin{array}{r}9\text{-}\\ 92)\overline{857}\end{array}$	2. $\begin{array}{r}5\text{-}\\ 78)\overline{394}\end{array}$	3. $\begin{array}{r}6\text{-}\\ 69)\overline{479}\end{array}$	4. $\begin{array}{r}8\text{-}\\ 53)\overline{427}\end{array}$
5. $\begin{array}{r}8\text{-}\\ 83)\overline{674}\end{array}$	6. $\begin{array}{r}6\text{-}\\ 74)\overline{498}\end{array}$	7. $\begin{array}{r}9\text{-}\\ 86)\overline{792}\end{array}$	8. $\begin{array}{r}8\text{-}\\ 79)\overline{687}\end{array}$

16. Three-Figure Divisors. Estimating the quotient figures when dividing by three-figure divisors may be done by taking the *quotient of hundreds*. This is illustrated in the following:

Example 1. Divide 4865 by 718.

$$\begin{array}{r} 6 \\ 718\overline{)4865} \\ 4308 \\ \hline 557 \end{array}$$

Taking quotient of hundreds, $48 \div 7 = 6$ with a remainder.

Is 6 the correct quotient? Why?

Example 2. Divide 7194 by 897.

$$\begin{array}{r} 7 \\ 897\overline{)7194} \\ 6279 \\ \hline 915 \end{array}$$

Taking quotient of hundreds, $71 \div 9 = 7$ with a remainder.

On multiplying, 7 is found *not* to be the correct quotient. Explain.

ORAL EXERCISES

1. Explain "taking quotient of tens" (*a*) when the second figure from the left in the divisor is small; (*b*) when this figure is large.

2. Explain "taking quotient of hundreds" (*a*) when the second figure from the left in the divisor is small; (*b*) when this figure is large.

3. May the process in example 1 above give too small a quotient? too large? Explain.

4. May the process in example 2 above give too small a quotient? too large? Explain.

Estimate and make a list of quotients in the following:

5. $794 \div 84$ 9	6. $8247 \div 987$ 8	7. $4795 \div 826$ 5
8. $917 \div 346$ 2	9. $1024 \div 199$ 5	10. $5947 \div 643$ 9

WRITTEN EXERCISES

Test the correctness of the quotients just estimated.

17. Dividing Numbers Representing Money. The division of a number representing money by a whole number is shown in the following examples:

Example 1. Divide \$786.40 by 16.

```
     $49.15
16 )$786.40
     64
     146
     144
       24
       16
        80
```

The division is carried out exactly as if there were no decimal point in the dividend. Then a decimal point is put into the quotient directly above the decimal point in the dividend.

Example 2. Divide \$1496.73 by 24.

```
     $62.36
24 )1496.73
    144
      56
      48
       87
       72
       153
       144
         9
```

In dividing we find a quotient of \$62.36 and a remainder of 9. Since 9 is less than one-half of 24 (the divisor), \$62.36 is the quotient correct to the nearest cent. If in this case we had a remainder of 12 or more (one-half the divisor) the quotient would be increased by one cent, that is, it would be \$62.37.

WRITTEN EXERCISES

1. Check example 1 above.

2. How do you check division when there is a remainder? Check example 2 above.

Divide and check each of the following:

186-129 3. 43839 ÷ 235	11-139 9. 3978 ÷ 349	205-326 15. 79456 ÷ 386
141-108 4. 59046 ÷ 418	90-511 10. 58741 ÷ 647	105-453 16. 87603 ÷ 830
111-201 5. 60807 ÷ 546	107-684 11. 94309 ÷ 875	122-676 17. 94006 ÷ 765
74-305 6. 59061 ÷ 794	60-366 12. 35046 ÷ 578	239-20 18. 98010 ÷ 410
102-333 7. 64083 ÷ 625	46-108 13. 37184 ÷ 806	40-8 19. 14968 ÷ 374
81-36 8. 43857 ÷ 541	147-275 14. 95384 ÷ 647	117-700 20. 97342 ÷ 826

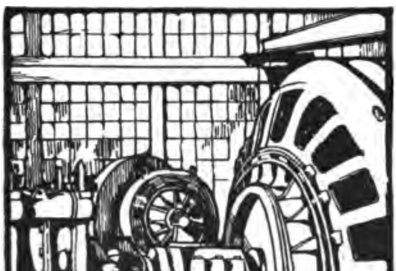
WRITTEN EXERCISES

1. In a house 18 tons of soft coal are used each year for heating. At $4.65 a ton, what is the cost of this coal? **$83.70**

2. The same house could be heated with 11 tons of hard coal, costing $8.45 a ton. Which would be more expensive, hard coal or soft coal, and how much? (See example 1.) **Hard, $9.25**

3. In the same house 3 cords of wood at $4.75 a cord and two loads of kindling at $3.25 a load are used in the fireplaces. What is the cost of this wood? **$20.75**

4. In this house the monthly bills for electric current are: $5.20, $6.10, $5.40, $5.10, $4.60, $3.25, $2.90, $3.70, $4.10, $4.80, $5.60, $5.40. What is the total yearly cost of lighting this house? **$56.15**



5. In the power station where the electric current is generated they burn 35 tons of coal a day. At $3.60 a ton what is the cost of this coal? **$126**

6. Gas for cooking is sold for 95 cents per thousand cubic feet. One month 5000 cubic feet are consumed. What is the gas bill for this month?

Suggestion: Since 5000 cubic feet are used, multiply 95 by 5. This will give the result in cents. **$4.75**

7. The monthly gas bills for the year are: $3.95, $3.75, $4.10, $4.70, $3.85, $4.10, $4.60, $5.80, $4.50, $3.70, $3.85, $4.15. What is the total gas bill for the year? **$51.05**

8. Find the cost of coal, gas, and electric current in your town, and make up problems on the cost of lighting and heating a house.

WRITTEN EXERCISES

1. John and Eleanor received 50 cents a week each from their mother for helping her during the summer vacation. How much did each of them receive in 14 weeks? **$7.00**

2. John bought a pair of skates for $2.10, a sweater for $2.85, a cap for 35 cents, and a pair of mittens for 50 cents. How much did these cost? How much did he have left of his $7.00? **$5.80, $1.20**

3. Eleanor bought a pair of skates for $1.95, a sweater for $3.50, and a cap for 75 cents. How much did she have left of her $7.00? **$6.20, $.80**

4. John received a pair of skis for Christmas, and Eleanor received a new sled. The skis cost $3.45, and the sled cost $2.85. How much did they both cost? **$6.30**

5. John has an electric train which cost $4.25, a set of struct iron which cost $3.75, and other toys costing $2.60. How much do all these cost? **$10.60**

6. Eleanor has a set of play dishes and a stove costing $2.35, a doll's carriage costing $1.75, a set of doll's furniture costing $2.85, and a doll costing $1.15. How much do all these cost? **$8.10**

18. Even and Odd Numbers. The numbers 0, 2, 4, 6, 8, 10, 12, and so on, are called *even numbers*. The numbers 1, 3, 5, 7, 9, and so on, are called *odd numbers*.

19. Divisibility of Numbers. A number is divisible by another number if it can be divided by that number giving a whole number for a quotient and no remainder. All even numbers are divisible by 2.

Thus, $0 \div 2 = 0$, $2 \div 2 = 1$, $4 \div 2 = 2$, $6 \div 2 = 3$, and so on.

Odd numbers are not divisible by 2.

20. Tests of Divisibility. We will now give some tests of divisibility.

I. A number is divisible by 2 if its last figure is an even number, otherwise not.

II. A number is divisible by 5 if its last figure is 5 or 0. It is divisible by 10 if the last figure is 0.

III. A number is divisible by 3 if the sum of all its digits is divisible by 3.

Thus, 5493624 is divisible by 3 because the sum $5+4+9+3+6+2+4=33$ is divisible by 3.

36424 is not divisible by 3 because the sum $3+6+4+2+4=19$ is not divisible by 3.

IV. A number is divisible by 6 if it is divisible by both 2 and 3, otherwise not.

ORAL EXERCISES

1. Which of the numbers 346, 75, 23690, 3007, 39504, are divisible by 2?

2. Which of the numbers 845, 75, 360, 307, 950, 745, are divisible by 5? Which are divisible by 10?

13, 26, 115, 23516, 29,700

3. For each of the numbers, 39, 78, 345, 5294, 70548, 9431, 89100, state whether it is divisible by 2, 3, 5, 6, 10.

WRITTEN EXERCISES

In cases of divisibility find the quotients in example 3 above.

(This page may be omitted if the teacher thinks best.)

21. **Multiplication and Division Compared.** In multiplication the multiplicand represents a group of things, and the multiplier states how many times this group is taken.

Multiplier × *multiplicand* = *product.*

A certain group taken so many times = *product.*

In division the product is always given. We also have given either the size of the groups, in which case we are required to find how many groups there are, or we have given the number of groups and are required to find the size of each group.

22. **Measuring and Grouping or Partitioning.** The following examples illustrate two kinds of division:

Example 1. At $3 a day, how many men can I pay for a day's work with $36?

The size of each group, $3, is given, and the question is as to the number of such groups in $36. We say we *measure* $36 by $3, and find that the measure is contained 12 times.

Example 2. If I have $36 to divide equally among 12 men, how much will each man get?

In this case the number of groups, 12, is given, and we are required to find the size of each group. The $36 are said to be divided or partitioned into 12 equal parts.

The process of division may therefore be regarded in some cases as a process of *measuring*, and in others as a process of *partitioning*. The work of carrying out the division is the same, however, in both cases.

If the size of the group is to be found, that is, in case of partitioning, the quotient represents the same kind of thing as the dividend. In case of measuring the quotient is abstract.

ORAL EXERCISES

Give examples in which division is to be regarded as measuring; others in which it is to be regarded as partitioning.

23. The Definition of Division and the Solution of Problems. In $3 \times 4 = 12$ we may leave out any one of the three numbers and thus obtain three distinct problems.

To find the missing number in $3 \times 4 = ?$ is a problem in multiplication.

To find the missing numbers in $3 \times ? = 12$, and $? \times 4 = 12$, are problems in division.

This simple idea forms the basis for the solution of a large number of problems.

If you know the length and the width of a rectangle how do you find its area?

The answer is that the area is found by multiplying the length by the width.

This may be written:

$$\textit{width} \times \textit{length} = \textit{area}.$$

If the area is not known we write:

$$\textit{width} \times \textit{length} = ?$$

We then have a problem in multiplication like $3 \times 4 = ?$

If the area and the width are known, but not the length, we write:

$$\textit{width} \times ? = \textit{area}.$$

To find the length is then a problem in division like finding the missing number in $4 \times ? = 12$.

If the area and the length are known, but not the width, we write:

$$? \times \textit{length} = \textit{area}.$$

To find the width is then a problem in division like finding the missing number in $? \times 3 = 12$.

You should make sure that you understand this page perfectly, since the same ideas will be used frequently hereafter. In the next few pages some of their simpler applications will be given.

It must be made clear that, before the rule *length* × *width* = *area* can be used, the length and width must be in the same units, and the unit of area must be a square of which the unit of length is a side.

WRITTEN EXERCISES

1. Find the area of a rectangle 60 feet wide and 80 feet long. **4800 sq. ft.**

2. A rectangle is 67 feet long, and its area is 3685 square feet. Find the width. **55 ft.**

3. A rectangle is 35 feet wide, and its area is 2240 square feet. Find its length. **64 ft.**

4. A lot which contains 2072 square feet is 74 feet deep. How wide is it? **28 ft.**

5. A field contains 3200 square rods. If the field is 40 rods wide, how long is it? **80 rd.**

6. A manufacturer is building a factory in which he wants a floor space of 2400 square feet on each floor. The width of his lot permits him to make the building just 30 feet wide inside. What must be the inside length of the building? **80 ft.**

7. A man wants 8000 square feet of land for a factory site. How wide a piece of land must he buy if it is 125 feet deep? **64 ft.**

8. A lot in the city of New York, 24 feet wide and 85 feet deep, sold for $1.20 per square foot. What was the selling price? **$2448**

9. A field 120 rods long contains 30 acres. How wide is it?
   Suggestion: First find the number of square rods in 30 acres. **40 rd.**

10. At 75 cents a square yard, how much will it cost to cover a lot 60 feet by 120 feet with black dirt? **$600**
   Suggestion: First find the number of square yards in the lot.

11. How long will it take a man to cut a field of grain 40 rods by 80 rods, if he cuts 8 acres a day? **2½ days**

A man bought 12 sheep at $9.00 a head. How much did they cost him?

If you know the number of things bought, and the price paid for each, how do you find the total cost? The answer may be written:

number of things bought × *price* = *total cost.*

Again, as in $3 \times 4 = 12$, we may leave out any one of the three numbers, thus obtaining three problems. That is,

(1) *number of things bought* × *price* = ?
(2) *number of things bought* × ? = *cost.*
(3) . . . . ? . . . . × *price* = *cost.*

ORAL EXERCISES

1. State each of these problems in words, and tell how to solve it. If you have difficulty in doing this, compare with the problems $3 \times 4 = ?$, $3 \times ? = 12$, and $? \times 4 = 12$.

2. How can you find the cost per acre of a piece of land, if you know the total cost and the number of acres?

3. How can you find the number of acres in a piece of land, if you know the total cost and the price per acre?

WRITTEN EXERCISES

Solve the following problems and state whether they come under (1), (2), or (3) of the forms given above:

1. A man bought 85 acres of land for $120 an acre. How much did the land cost him? **$10,200 (1)**

2. A man bought 175 head of sheep for $2450. How much did he pay per head for the sheep? **$14 (2)**

3. A man has $1170 with which to buy corn that is selling for 65 cents a bushel. How much corn can he buy? **1800 bu. (3)**

Suggestion: To find how many times 65 cents is contained in $1170, write the $1170 as 117000 cents and then divide.

WRITTEN EXERCISES

1. At $56 per acre, how many acres can be bought for $7840? 140

2. At $90 per head, how many cows can be bought for $1080? 12

3. A man buys a flock of sheep for $450. What is the price per head if there are 75 sheep in the flock? $6.00

4. At $45 per acre, how much land can a farmer buy for $3825? 85 acres

The total cost of a street railway system of 37 miles consists of the following items:

Track, $629,000; Power House, $148,000;
Overhead Trolley, $296,000; 21 cars at $2590.

5. What is the total cost of the 21 cars? $54,390

6. What is the cost per mile of the track? of the overhead trolley? $17,000; $8,000

7. What is the total cost of this railway system, including all four items? What is the total cost per mile of this system? $1,127,390; $30,470

8. A farmer buys a herd of 56 head of cattle for $3352, and later sells the herd for $4304. What is the increase in the price of the herd? What is the increase per head? $952; $17

9. In a large city a lot 30 feet by 90 feet sells for $3,000. What is the price per square foot? $1.11

10. At 90 cents per square foot, how many square feet of land can be bought for $12,600? 14,000

11. At $175 per acre, what is the value of a farm 120 rods wide and 216 rods long? $28,350

12. At an auction sale a lady bought a bolt of sheeting containing 65 yards for $12.35.. What was the cost per yard? 19c

13. Make other problems like those given on this page, and solve them by means of the forms given on page 26.

If you walk 9 miles in 3 hours, how many miles per hour do you walk?

If a driving horse goes 28 miles in 4 hours, how many miles per hour does he go?

The distance covered in a unit of time is called *speed*. The distance and the time may be expressed in many different units. When we say we walk 3 *miles per hour* the unit of distance is the mile, and the unit of time is the hour. When we say a railway train goes a *mile a minute*, the unit of time is the minute. A speed must always be given in certain units of time and distance.

ORAL EXERCISES

1. In each of the following, state in what units the speed is given: A rifle bullet goes 700 yards in a second; a passenger train goes 52 miles per hour; sound goes 1080 feet per second.

2. At an average speed of 20 miles an hour, how far will an automobile go in 4 hours?

3. If you know the speed and also the time, how do you find the distance covered? The answer may be written:

$$\textit{speed} \times \textit{time} = \textit{distance}.$$

As on the preceding pages, we may leave out any one of these three numbers, thus obtaining three problems:

(1) $\textit{speed} \times \textit{time} = ?$
(2) $\textit{speed} \times ? = \textit{distance}.$
(3) $? \times \textit{time} = \textit{distance}.$

State each of these problems in words, and tell how to solve it.

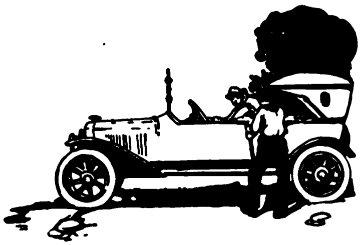
Notice that when you once know that

$$\textit{speed} \times \textit{time} = \textit{distance}$$

you need no further rule for finding any one of these members when the other two are given.

WRITTEN EXERCISES

1. How long does it require a train going 45 miles an hour to go 270 miles? **6 hr.**



2. What is the average speed of an auto that covers 360 miles in 20 hours? **18 miles per hour**

3. The Twentieth Century Limited of the New York Central goes from Chicago to New York in 20 hours. What is its average speed if the distance on the Central is 980 miles? **49 miles**

4. The Pennsylvania Limited goes from New York to Chicago in 20 hours. What is the average speed if the distance is 912 miles?

Solution:

$$\begin{array}{r} 45 \\ 20\,)\overline{912} \\ \underline{80} \\ 112 \\ \underline{100} \\ 12 \end{array}$$

Dividing 912 by 20 we get a quotient of 45 and a remainder 12. Hence, the speed is between 45 and 46 miles per hour. It is exactly $45\frac{3}{5}$ miles per hour.

5. In making the fastest journey ever made across the Atlantic Ocean, the steamship Mauretania steamed 2782 nautical miles in 107 hours. What was her average speed per hour? **26**

The nautical mile is longer than the ordinary English mile, since the English mile = 5280 feet, while the nautical mile is nearly 6080 feet; 60 nautical miles make nearly 69 English miles.

24. Drills. Frequent drills are necessary in order to develop and retain reasonable speed and accuracy in performing the fundamental operations. Much interest is added to such drills by using games of competition. The games described here should be used as indicated later on.

1. *Girls vs. Boys.* Exercises are dictated by the teacher and copied by the pupils (or else given the pupils in mimeographed or printed form). At a given signal the pupils go to work, and then stop working at another signal. The pupils exchange papers and mark them as the teacher reads the correct answers. The number of correct answers obtained by the girls are added, and also those obtained by the boys. The side having the largest score wins. If the numbers of boys and girls differ, the average score of each side is taken.

2. *One Side of Room vs. the Other.* Play just as above except that one side of the room plays against the other side.

3. *Field Meet.* The class is divided into several teams, which play as in No. 1. Captains are selected first, and they choose the teams. There are several events—one for each of several successive days. The total scores for all the events of the meet are added. Thus, a meet may consist of four events, using addition, subtraction, multiplication and division, respectively. The same teams play in all the events of the meet.

4. *Cross-Country Race.* The class is divided into several teams as in No. 3 When the first pupil finishes he holds up his hand and the teacher gives him the number 1, which he writes on his paper, and immediately turns it down on his desk. The pupil who finishes next is given the number 2 as his score, and so on. If two pupils hold up their hands at the same time, they are given the same score. When one-half or two-thirds of the class have finished the others stop working, and are all given the same score. The papers are marked by the pupils as in No. 1. If an example is not done at all, or if the answer is wrong, 5 is added to the score. If two are wrong 10 is added to the score, and so on. The team that gets the smallest total score wins.

The numbers on pages 32, 33, 46, 47 will be used for the games described on this page. The teacher will tell you just what numbers are to be used each time.

Even when trying very hard to work rapidly care must be taken to write the numbers reasonably well. If you get into the habit of writing numbers badly you will be sure to get mixed up and get wrong answers.

Add and check by adding each column both ways:

1.	2.	3.	4.	5.
67358	1642	1819	5416	5355
92100	18970	2797	3940	6113
84890	1748	10120	5764	1895
21924	9371	1914	5432	4974
33426	12463	6873	6789	7034
47597	7418	4422	1011	7149
55600	9214	4046	1315	2571
28230	17291	7970	3451	5118
64274	31000	9116	1718	3641
86450	10471	7474	8171	6618
91493	11495	2081	9456	1519
41765	12671	3841	5926	2406
715107	143754	62473	58389	54393

Multiply and check by interchanging multiplier and multiplicand:

6. 378	7. 917	8. 356	9. 319	10. 246
265	827	497	874	396
100170	758359	176932	278806	97416
11. 573	12. 719	13. 753	14. 413	15. 642
826	287	649	987	639
473298	206353	488697	407631	410238
16. 677	17. 385	18. 347	19. 384	20. 496
852	638	595	981	926
576804	245630	206465	376704	459296

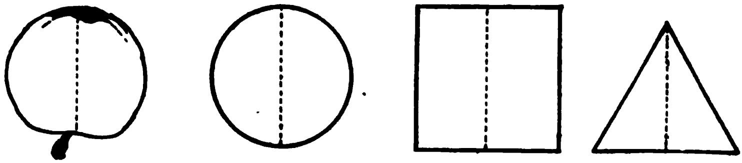
Divide and check by multiplying the quotient by the divisor and adding the remainder:

1673-1	17-181	114-807
21. 43)71940	27. 541)9378	33. 820)94287
1316-256	491-53	81-6480
22. 284)374000	28. 97)47680	34. 9320)761400
362-324	89-163	53-167
23. 548)198700	29. 217)19476	35. 329)17604
7804-48	79-451	59-550
24. 63)491700	30. 724)57647	36. 680)40670
815-180	81-91	199-337
25. 968)789100	31. 427)34678	37. 397)79340
228-568	80-266	60-410
26. 869)198700	32. 592)47626	38. 576)34970

		A	B	C	D	E
I.	1.	56,342	43,571	3492	315	26
	2.	37,802	29,634	2451	109	34
	3.	15,638	13,402	7631	410	45
	4.	78,281	56,491	1819	102	38
	5.	65,812	45,302	3402	560	25
II.	6.	34,291	32,615	4605	324	24
	7.	27,032	18,926	3204	116	39
	8.	45,604	39,324	8160	417	86
	9.	91,244	78,506	7251	352	59
	10.	63,007	58,029	3014	708	79
III.	11.	32,051	27,609	8407	342	63
	12.	69,107	54,346	1532	591	72
	13.	38,293	37,491	3040	682	17
	14.	62,812	47,394	2914	317	26
	15.	73,415	65,705	1857	827	83
IV.	16.	47,251	28,348	3756	326	87
	17.	75,432	29,256	2845	925	75
	18.	64,835	32,917	7156	273	65
	19.	65,374	44,512	4265	754	84
	20.	89,125	75,632	9463	934	67
V.	21.	35,637	33,981	7463	542	64
	22.	48,952	35,764	6173	384	89
	23.	78,334	56,473	5429	268	27
	24.	95,637	89,763	7638	712	38
	25.	84,732	84,648	5249	432	59
VI.	26.	59,481	51,627	2478	891	79
	27.	95,814	83,916	6071	673	82
	28.	89,184	34,670	3462	817	68
	29.	76,437	59,347	5432	231	53
	30.	77,347	71,408	7641	462	86

		A	B	C	D	E
VII.	31.	45,063	38,407	4536	238	57
	32.	89,603	72,635	7029	507	63
	33.	58,620	49,724	6385	920	84
	34.	72,806	35,209	1407	763	58
	35.	38,745	29,347	6819	647	78
VIII.	36.	58,674	38,579	5685	842	85
	37.	64,095	24,607	4328	567	79
	38.	85,306	77,845	9638	945	27
	39.	95,483	65,248	7536	847	48
	40.	67,538	64,729	6308	904	56
IX.	41.	95,632	49,745	8327	618	73
	42.	94,072	89,638	5429	527	45
	43.	35,608	33,849	6328	439	36
	44.	27,419	26,193	7437	508	40
	45.	38,516	27,498	6056	319	72
X.	46.	81,970	21,671	1982	598	92
	47.	39,181	39,842	3765	276	73
	48.	54,391	91,627	1917	782	86
	49.	24,217	84,207	7248	937	64
	50.	62,390	73,617	8294	876	27
XI.	51.	93,718	17,621	2891	895	29
	52.	27,938	24,893	5673	672	37
	53.	54,817	72,619	7191	287	68
	54.	61,902	70,248	8427	739	46
	55.	53,190	71,637	4928	678	72
XII.	56.	54,970	27,945	5416	342	31
	57.	21,929	92,912	2892	719	92
	58.	31,467	76,413	3767	243	87
	59.	84,891	19,848	6145	917	78
	60.	39,374	47,393	2982	826	29

25. Fractions of One Thing. Fractions are of frequent use in every-day life, and we are now to study them more fully than we have done before. We will first study fractions of one thing.

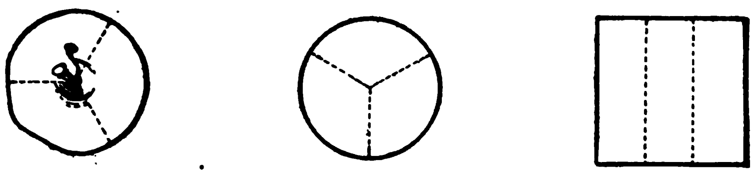


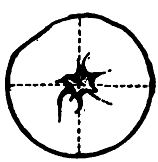
ORAL EXERCISES

1. If an apple is cut into two equal parts, what is each part called?

2. How many half circles are there in a whole circle?

3. If a square or a triangle is cut into two equal parts, what is each part called? How many halves are there in a whole?





4. If an apple is cut into three equal parts, what is each part called? How many thirds are there in a whole?

5. Point to $\frac{1}{3}$ of the circle, $\frac{1}{3}$ of the square, $\frac{2}{3}$ of the circle, $\frac{2}{3}$ of the square.

6. When a whole is divided into four equal parts, what is each part called?

7. Point to $\frac{1}{4}$ of the apple, $\frac{2}{4}$ of the apple.

8. Draw a circle and divide it into fourths. Point to $\frac{1}{2}$ of the circle, to $\frac{1}{4}$, to $\frac{2}{4}$, to $\frac{3}{4}$, to $\frac{4}{4}$.

26. Fractions of a Group of Things. To take a fraction of a group of things, the group must be divided into several equal groups. Then one or more of these groups must be taken.

ORAL EXERCISES

1. How many dots are there in this group? • •
   Point to $\frac{1}{2}$ of them, to $\frac{1}{4}$ of them. • •

2. How many dots are there in this group? • • •
   Point to $\frac{1}{2}$ of them, to $\frac{1}{3}$ of them. • • •

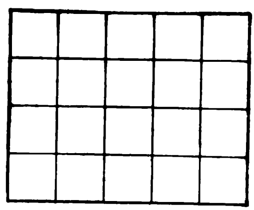
3. How many dots are there in this group? • • • •
   Point to $\frac{1}{2}$ of them, to $\frac{1}{4}$ of them. • • • •

4. How many dots are there in this group? • • • • • • • •
   Point to $\frac{1}{2}$ of them, to $\frac{1}{4}$, to $\frac{1}{8}$, to $\frac{1}{16}$. • • • • • • • •

5. Point to $\frac{3}{4}$ of these dots, to $\frac{5}{8}$, to $\frac{7}{8}$.

6. Point to $\frac{3}{16}$ of these dots, to $\frac{5}{16}$, to $\frac{7}{16}$, to $\frac{9}{16}$, to $\frac{11}{16}$, to $\frac{13}{16}$, to $\frac{15}{16}$.

7. Draw a rectangle like this. Show $\frac{1}{20}$ of it, $\frac{3}{20}$ of it, $\frac{4}{20}$ of it. Also show $\frac{1}{5}$ of the rectangle. How does $\frac{1}{5}$ compare with $\frac{4}{20}$?



8. Cut out a paper 3″ x 3″ and fold it to show $\frac{1}{9}$ of it. Also show $\frac{2}{9}$ of it, $\frac{3}{9}$. How does $\frac{1}{3}$ compare with $\frac{3}{9}$?

(3″ means 3 in., and 3″ x 3″ means 3 in. long and 3 in. wide.)

9. Cut out a paper 5″ x 2″ and fold it to show $\frac{1}{10}$ of it. Also show $\frac{3}{10}$ of it, and $\frac{5}{10}$ of it. How does $\frac{1}{2}$ compare with $\frac{5}{10}$?

10. Cut out a paper 4″ x 3″ and fold it to show $\frac{1}{12}$ of it. Also show $\frac{1}{3}$ of it and $\frac{1}{4}$ of it. Compare $\frac{1}{3}$ and $\frac{4}{12}$. Also compare $\frac{1}{4}$ and $\frac{3}{12}$.

27. Definitions. We are now able to understand the more formal statements given below. Such statements are called definitions.

A fraction represents one or more equal parts of anything.

In a fraction the number below the line is called the *denominator*, and *indicates into how many parts an object or group is divided.*

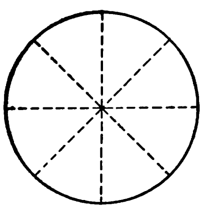
The number above the line is called the *numerator*, and *indicates how many of the equal parts are taken.*

Thus, the fraction $\frac{3}{8}$ indicates 3 of the 8 equal parts of one. It is read *three-eighths*, and also *three divided by eight.*

The numerator and denominator of a fraction are called the *terms of the fraction.*

28. Halves, Fourths, Eighths. Of all the fractions, halves, fourths, and eighths are in most frequent use.

1							
½				½			
¼		¼		¼		¼	
⅛	⅛	⅛	⅛	⅛	⅛	⅛	⅛



ORAL EXERCISES

1. How many halves are there in 1? how many 4ths? how many 8ths?

2. In the above figure point out $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$. How many 8ths make a 4th? a half? How many 4ths make a half?

3. Show $\frac{1}{2}$ of the circle, $\frac{1}{4}$ of it, and $\frac{1}{8}$ of it.

4. How many 8ths are there in $\frac{2}{4}$? in $\frac{3}{4}$?

5. Read each fraction on this page in two ways.

29. Thirds, Sixths, Twelfths. From the figure we see that a third may be changed into sixths and twelfths.

1											
1/3				1/3				1/3			
1/6		1/6		1/6		1/6		1/6		1/6	
1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12

ORAL EXERCISES

1. How many 3rds are there in 1? how many 6ths? how many 12ths?

2. In the figure point out $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{12}$. How many 6ths make a 3rd? How many 12ths make a 6th? How many 12ths make a 3rd?

3. Supply the missing numbers in the following: $\frac{1}{3}=\frac{?}{6}$, $\frac{1}{6}=\frac{?}{12}$, $\frac{1}{3}=\frac{?}{12}$.

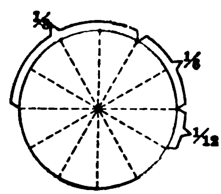
4. In the figure point out $\frac{2}{3}$. How many 6ths are there in $\frac{2}{3}$?

5. How many 12ths are there in $\frac{2}{3}$?

6. In the figure point out $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$.

7. How many 12ths are there in $\frac{5}{6}$?

8. Which is larger, $\frac{1}{3}$ or $\frac{1}{6}$? How many 6ths are needed to make one 3d?



9. Which is larger, $\frac{1}{6}$ or $\frac{1}{12}$? How many 12ths are needed to make one 6th?

10. Which is larger, $\frac{1}{3}$ or $\frac{1}{12}$? How many 12ths are needed to make one 3d?

11. In the circle point out $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{12}$.

12. Show $\frac{1}{2}$ of $\frac{1}{3}$. also $\frac{1}{2}$ of $\frac{1}{6}$. How many 12ths are there in $\frac{1}{2}$ of $\frac{1}{3}$? how many 6ths? How many 12ths are there in $\frac{1}{2}$ of $\frac{1}{6}$?

Helen's mother sent her to the store to buy spices. She bought a 2-ounce can of paprica, a 4-ounce can of pepper, and an 8-ounce can of cinnamon.

ORAL EXERCISES

1. How many ounces are there in one pound? One ounce is what fraction of a pound?

2. How many 16ths of a pound are there in 2 ounces? in 4 ounces? in 8 ounces?

3. How many ounces are there in $\frac{1}{2}$ of a pound? $\frac{1}{2}$ is equal to how many 16ths?

4. How many 4-ounce cans will make a pound? $\frac{1}{4}$ of a pound is how many ounces? $\frac{1}{4}$ is equal to how many 16ths?

5. How many 2's are there in 16? How many 2-ounce cans will make a pound? $\frac{1}{8}$ of a pound is how many ounces? $\frac{1}{8}$ is equal to how many 16ths?

6. Supply the missing numbers in the following:

$$\frac{1}{2}=\frac{?}{16}, \quad \frac{1}{4}=\frac{?}{16}, \quad \frac{1}{8}=\frac{?}{16}.$$

7. One dozen is how many? One is what fraction of a dozen.

8. How many in $\frac{1}{2}$ of a dozen? in $\frac{1}{3}$ of a dozen? in $\frac{1}{4}$ of a dozen? in $\frac{1}{6}$ of a dozen? in $\frac{1}{12}$ of a dozen.

9. Supply the missing numbers in the following:

$$\frac{1}{2}=\frac{?}{12}, \quad \frac{1}{3}=\frac{?}{12}, \quad \frac{1}{4}=\frac{?}{12}, \quad \frac{1}{6}=\frac{?}{12}.$$

10. Read each of the following fractions in two ways:

$$\frac{1}{2}, \ \frac{1}{3}, \ \frac{1}{4}, \ \frac{2}{3}, \ \frac{1}{6}, \ \frac{5}{8}, \ \frac{1}{8}, \ \frac{3}{8}, \ \frac{5}{8}, \ \frac{7}{8}.$$

30. Multiplying Both Terms of a Fraction. We have found that $\frac{1}{2}=\frac{2}{4}$, $\frac{1}{2}=\frac{4}{8}$, $\frac{1}{3}=\frac{2}{6}$, $\frac{1}{6}=\frac{2}{12}$, $\frac{1}{3}=\frac{4}{12}$, and so on.

In each of these the second fraction may be obtained by multiplying both terms of the first fraction by the same number.

That is, $\frac{1}{2}=\frac{2\times 1}{2\times 2}=\frac{2}{4}$, $\frac{1}{2}=\frac{4\times 1}{4\times 2}=\frac{4}{8}$, $\frac{1}{3}=\frac{4\times 1}{4\times 3}=\frac{4}{12}$

These examples illustrate the following rule:

> *Both terms of a fraction may be multiplied by the same number without changing the value of the fraction.*

ORAL EXERCISES

Supply the missing numbers:

1. $\frac{1}{3}=\frac{?}{9}$ $\quad\frac{1}{4}=\frac{?}{8}$ $\quad\frac{3}{5}=\frac{?}{15}$ $\quad\frac{2}{3}=\frac{?}{12}$ $\quad\frac{5}{6}=\frac{?}{18}$
2. $\frac{3}{7}=\frac{?}{21}$ $\quad\frac{6}{7}=\frac{?}{28}$ $\quad\frac{3}{8}=\frac{?}{24}$ $\quad\frac{5}{9}=\frac{?}{18}$ $\quad\frac{4}{9}=\frac{?}{27}$
3. $\frac{4}{5}=\frac{?}{25}$ $\quad\frac{5}{8}=\frac{?}{56}$ $\quad\frac{3}{11}=\frac{?}{44}$ $\quad\frac{5}{12}=\frac{?}{60}$ $\quad\frac{7}{8}=\frac{?}{80}$

31. Dividing both Terms of a Fraction. Since $\frac{1}{4}=\frac{3}{12}$ we can change $\frac{3}{12}$ to $\frac{1}{4}$. This is done by dividing both terms of $\frac{3}{12}$ by 3.

The following is a general rule:

> *Both terms of a fraction may be divided by the same number without changing the value of the fraction.*

ORAL EXERCISES

Read the following and supply the missing numbers:

1. $\frac{3}{9}=\frac{?}{3}$ $\quad\frac{4}{10}=\frac{?}{5}$ $\quad\frac{4}{12}=\frac{?}{3}$ $\quad\frac{8}{16}=\frac{?}{2}$ $\quad\frac{6}{18}=\frac{?}{3}$
2. $\frac{3}{12}=\frac{?}{4}$ $\quad\frac{8}{12}=\frac{?}{3}$ $\quad\frac{6}{12}=\frac{?}{6}$ $\quad\frac{5}{20}=\frac{?}{4}$ $\quad\frac{6}{18}=\frac{?}{9}$
3. $\frac{6}{12}=\frac{?}{4}$ $\quad\frac{4}{12}=\frac{?}{3}$ $\quad\frac{6}{12}=\frac{?}{2}$ $\quad\frac{4}{18}=\frac{?}{9}$ $\quad\frac{3}{21}=\frac{?}{7}$
4. $\frac{6}{10}=\frac{?}{5}$ $\quad\frac{8}{10}=\frac{?}{5}$ $\quad\frac{2}{10}=\frac{?}{5}$ $\quad\frac{6}{16}=\frac{?}{8}$ $\quad\frac{6}{30}=\frac{?}{10}$

32. A Factor of a Number. *A whole number which exactly divides another number is a factor of that number.*

Thus, 2 is a factor of 4, 6, 8, 10, and also of 2. 2 is not a factor of 1, 3, 5, 7, 9. Similarly, 3 is a factor of 3, 12, 15, 18, but not of 8, 14 or 17.

33. Common Factors. *Two numbers which have the same factor are said to have a common factor.*

Thus, 2 and 4 have the common factor of 2, and 6 and 9 have the common factor 3.

The numbers 8 and 9 have no common factor except 1. We usually disregard the factor 1, and say that these numbers have no common factor.

34. Fractions in Lowest Terms. *A fraction is in its lowest terms if the numerator and denominator have no common factor.*

The following rule is important:

> *A fraction may be reduced to its lowest terms by dividing the numerator and denominator by common factors.*

Fractions in a final result should be reduced to the lowest terms.

ORAL EXERCISES

Which of the following fractions are in their lowest terms? Reduce the others to lowest terms.

1. $\frac{3}{9}$	7. $\frac{3}{12}$	13. $\frac{3}{15}$	19. $\frac{7}{19}$	25. $\frac{8}{28}$
2. $\frac{7}{8}$	8. $\frac{5}{20}$	14. $\frac{4}{14}$	20. $\frac{8}{48}$	26. $\frac{8}{30}$
3. $\frac{6}{16}$	9. $\frac{4}{18}$	15. $\frac{2}{17}$	21. $\frac{9}{45}$	27. $\frac{8}{34}$
4. $\frac{4}{9}$	10. $\frac{4}{21}$	16. $\frac{4}{15}$	22. $\frac{9}{33}$	28. $\frac{12}{14}$
5. $\frac{2}{10}$	11. $\frac{7}{25}$	17. $\frac{8}{32}$	23. $\frac{8}{24}$	29. $\frac{12}{18}$
6. $\frac{4}{12}$	12. $\frac{5}{24}$	18. $\frac{6}{18}$	24. $\frac{8}{26}$	30. $\frac{12}{24}$

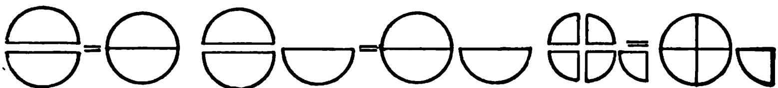
Drill in Fundamentals. Play game No. 4, page 30, using examples in multiplication.

35. Proper and Improper Fractions. A fraction which is less than 1 is called a *proper fraction*, while other fractions are called *improper fractions*.

Thus, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$ are proper fractions, while $\frac{2}{2}$, $\frac{4}{4}$, $\frac{3}{2}$, $\frac{6}{5}$, are improper fractions.

36. Mixed Numbers. A number consisting of a whole number and a fraction is called a *mixed number*.

Thus, $1\frac{1}{2}$, $2\frac{1}{3}$, $3\frac{5}{6}$, are mixed numbers.



$\frac{2}{2}=1$ $\qquad$ $\frac{3}{2} = 1\frac{1}{2}$ $\qquad$ $\frac{5}{4}=1\frac{1}{4}$

An improper fraction may always be reduced to an integer or to a mixed number, as in $\frac{2}{2}=1$, $\frac{4}{4}=1$, $\frac{3}{2}=1\frac{1}{2}$, $\frac{5}{4}=1\frac{1}{4}$.

A number is in the *simplest form* if it is an integer or if its fractional part is a proper fraction in its lowest terms.

Example 1. Reduce $\frac{72}{4}$.

$$4\overline{)72}\quad 18$$

Since $\frac{72}{4}$ means 72 divided by 4, we carry out the indicated division. The quotient is 18, with no remainder. Hence the result is 18.

Example 2. Reduce $\frac{64}{5}$.

$$5\overline{)64}\quad 12\tfrac{4}{5}$$

Dividing 64 by 5, the quotient is 12, and the remainder is 4. Dividing 4 by 5, we have $\frac{4}{5}$. Hence the result is $12\frac{4}{5}$.

ORAL EXERCISES

Which of the following are in the simplest form? Reduce the others to the simplest form:

1. $1\frac{2}{3}$	4. $\frac{16}{3}$	7. $\frac{12}{5}$	10. $\frac{45}{6}$	13. $3\frac{7}{21}$
2. $\frac{4}{8}$	5. $\frac{24}{7}$	8. $\frac{24}{5}$	11. $\frac{57}{9}$	14. $3\frac{9}{6}$
3. $\frac{8}{4}$	6. $\frac{26}{8}$	9. $\frac{34}{8}$	12. $1\frac{4}{12}$	15. $8\frac{9}{21}$

37. Fractions Having a Common Denominator. Fractions which have the same denominator are said to have a *common denominator*. Such fractions are also called *similar* or *like fractions*. Thus, $\frac{3}{8}$ and $\frac{5}{8}$ are similar, or like, while $\frac{3}{8}$ and $\frac{7}{12}$ are not similar or like.

38. Addition of Fractions Having a Common Denominator. In the manner shown in the diagram, any two fractions having the same denominator may be added.

$$\underbrace{1/8 \mid 1/8}_{2/8} \mid \underbrace{1/8 \mid 1/8 \mid 1/8}_{+\ 3/8\ =\ 5/8} \mid 1/8 \mid 1/8 \mid 1/8$$

ORAL EXERCISES

Add the following and reduce sums to simplest forms:

1.	$\frac{1}{2}+\frac{1}{2}$	$\frac{1}{3}+\frac{1}{3}$	$\frac{1}{4}+\frac{3}{4}$	$\frac{1}{5}+\frac{3}{5}$	$\frac{1}{6}+\frac{5}{6}$
2.	$\frac{3}{8}+\frac{5}{8}$	$\frac{1}{8}+\frac{5}{8}$	$\frac{1}{10}+\frac{1}{10}$	$\frac{2}{9}+\frac{4}{9}$	$\frac{5}{9}+\frac{1}{9}$
3.	$\frac{1}{16}+\frac{3}{16}$	$\frac{3}{16}+\frac{5}{16}$	$\frac{5}{16}+\frac{7}{16}$	$\frac{3}{16}+\frac{7}{16}$	$\frac{1}{16}+\frac{7}{16}$
4.	$\frac{5}{32}+\frac{3}{32}$	$\frac{5}{32}+\frac{7}{32}$	$\frac{7}{32}+\frac{9}{32}$	$\frac{5}{32}+\frac{11}{32}$	$\frac{9}{32}+\frac{11}{32}$

39. Addition of Fractions Not Having a Common Denominator. To add the fractions $\frac{1}{2}$ and $\frac{1}{4}$, the $\frac{1}{2}$ must be reduced to 4ths. Thus, $\frac{1}{2}+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$.

Again, to add $\frac{1}{2}$ and $\frac{1}{6}$, the $\frac{1}{2}$ must be reduced to 6ths. Thus, $\frac{1}{2}+\frac{1}{6}=\frac{3}{6}+\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$.

ORAL EXERCISES

Add the following and reduce sums to simplest forms:

1.	$\frac{1}{2}+\frac{1}{4}$	$\frac{1}{2}+\frac{1}{8}$	$\frac{1}{2}+\frac{3}{8}$	$\frac{1}{4}+\frac{1}{8}$	$\frac{1}{4}+\frac{3}{8}$
2.	$\frac{1}{8}+\frac{1}{16}$	$\frac{1}{8}+\frac{3}{16}$	$\frac{1}{8}+\frac{5}{16}$	$\frac{1}{8}+\frac{7}{16}$	$\frac{3}{8}+\frac{1}{16}$
3.	$\frac{7}{8}+\frac{1}{16}$	$\frac{3}{8}+\frac{3}{16}$	$\frac{3}{8}+\frac{5}{16}$	$\frac{3}{8}+\frac{7}{16}$	$\frac{5}{8}+\frac{5}{16}$
4.	$\frac{2}{3}+\frac{1}{6}$	$\frac{1}{3}+\frac{1}{9}$	$\frac{2}{3}+\frac{1}{9}$	$\frac{1}{3}+\frac{2}{9}$	$\frac{2}{3}+\frac{2}{9}$

Example. Add $\frac{1}{2}$ and $\frac{2}{3}$.

$$\frac{1}{2}=\frac{3}{6}$$
$$\frac{2}{3}=\frac{4}{6}$$
$$\frac{7}{6}=1\frac{1}{6}$$

We see at once that $\frac{1}{2}$ and $\frac{2}{3}$ may be reduced to 6ths. Multiply both terms of $\frac{1}{2}$ by 3, and both terms of $\frac{2}{3}$ by 2. Finally, add $\frac{3}{6}$ and $\frac{4}{6}$.

ORAL EXERCISES

Add and reduce each sum to the simplest form:

1.	$\frac{1}{2}+\frac{1}{3}$	$\frac{1}{4}+\frac{1}{3}$	$\frac{3}{4}+\frac{1}{3}$	$\frac{3}{4}+\frac{2}{3}$	$\frac{1}{4}+\frac{1}{6}$
2.	$\frac{1}{3}+\frac{1}{5}$	$\frac{2}{3}+\frac{3}{5}$	$\frac{2}{3}+\frac{4}{5}$	$\frac{1}{6}+\frac{1}{8}$	$\frac{5}{6}+\frac{1}{8}$
3.	$\frac{1}{2}+\frac{1}{5}$	$\frac{1}{2}+\frac{3}{5}$	$\frac{1}{2}+\frac{1}{7}$	$\frac{1}{2}+\frac{3}{7}$	$\frac{1}{2}+\frac{6}{7}$
4.	$\frac{2}{3}+\frac{1}{8}$	$\frac{1}{3}+\frac{3}{8}$	$\frac{1}{3}+\frac{5}{8}$	$\frac{1}{6}+\frac{1}{9}$	$\frac{5}{6}+\frac{1}{9}$

Example. Add $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$.

$$\frac{1}{2}=\frac{6}{12}$$
$$\frac{1}{3}=\frac{4}{12}$$
$$\frac{1}{4}=\frac{3}{12}$$
$$\frac{13}{12}=1\frac{1}{12}$$

We see at once that $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ may all be reduced to 12ths. We multiply both terms of $\frac{1}{2}$ by 6, both terms of $\frac{1}{3}$ by 4, and both terms of $\frac{1}{4}$ by 3. We know that we must multiply both terms of $\frac{1}{2}$ by 6 because $6\times2=12$.

WRITTEN EXERCISES

Add and reduce each sum to the simplest form:

1.	$\frac{1}{2}+\frac{1}{3}+\frac{3}{4}$	$1\frac{7}{12}$	7.	$\frac{1}{3}+\frac{1}{4}+\frac{1}{6}$	$\frac{3}{4}$	13.	$\frac{1}{4}+\frac{1}{8}+\frac{1}{16}$	$\frac{7}{16}$
2.	$\frac{1}{2}+\frac{2}{3}+\frac{1}{4}$	$1\frac{5}{12}$	8.	$\frac{2}{3}+\frac{1}{4}+\frac{5}{6}$	$1\frac{3}{4}$	14.	$\frac{1}{4}+\frac{3}{4}+\frac{3}{16}$	$1\frac{3}{16}$
3.	$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}$	$1\frac{11}{12}$	9.	$\frac{1}{3}+\frac{3}{4}+\frac{1}{6}$	$1\frac{1}{4}$	15.	$\frac{1}{4}+\frac{5}{8}+\frac{5}{16}$	$1\frac{3}{16}$
4.	$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$	$\frac{7}{8}$	10.	$\frac{2}{3}+\frac{3}{4}+\frac{5}{6}$	$2\frac{1}{4}$	16.	$\frac{3}{4}+\frac{7}{8}+\frac{1}{16}$	$1\frac{11}{16}$
5.	$\frac{1}{2}+\frac{1}{4}+\frac{3}{8}$	$1\frac{1}{8}$	11.	$\frac{2}{3}+\frac{3}{4}+\frac{5}{12}$	$1\frac{5}{6}$	17.	$\frac{3}{4}+\frac{5}{8}+\frac{7}{16}$	$1\frac{13}{16}$
6.	$\frac{1}{2}+\frac{3}{4}+\frac{1}{8}$	$1\frac{3}{8}$	12.	$\frac{1}{3}+\frac{1}{4}+\frac{5}{6}$	$1\frac{5}{12}$	18.	$\frac{3}{4}+\frac{3}{8}+\frac{9}{16}$	$1\frac{11}{16}$

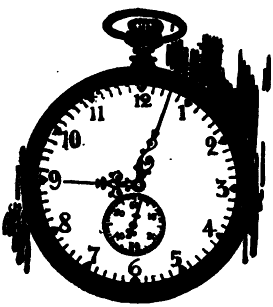
19. A board $\frac{5}{8}$ inches thick is covered with a veneer $\frac{3}{16}$ inches thick. How thick is the veneered board? $\frac{13}{16}$ in.

ORAL EXERCISES

1. Tell how many inches there are in each of the following:

$\frac{1}{2}$ of one foot
$\frac{1}{3}$ of one foot
$\frac{2}{3}$ of one foot
$\frac{1}{4}$ of one foot
$\frac{3}{4}$ of one foot

$\frac{1}{6}$ of one foot
$\frac{5}{6}$ of one foot
$\frac{1}{12}$ of one foot
$\frac{5}{12}$ of one foot
$\frac{7}{12}$ of one foot



2. Tell how many minutes there are in each of the following:

$\frac{1}{2}$ of one hour
$\frac{1}{3}$ of one hour
$\frac{2}{3}$ of one hour
$\frac{1}{4}$ of one hour

$\frac{1}{5}$ of one hour
$\frac{2}{5}$ of one hour
$\frac{3}{5}$ of one hour
$\frac{4}{5}$ of one hour

3. Tell how many hours there are in each of the following, 24 hours being one day:

$\frac{1}{2}$ of one day
$\frac{1}{3}$ of one day
$\frac{2}{3}$ of one day

$\frac{1}{4}$ of one day
$\frac{3}{4}$ of one day
$\frac{1}{6}$ of one day

4. If 52 weeks are regarded as one year, tell how many weeks there are in each of the following:

$\frac{1}{2}$ of one year
$\frac{1}{4}$ of one year

$\frac{3}{4}$ of one year
$\frac{1}{13}$ of one year

5. If a school year is 40 weeks, tell how many weeks there are in each of the following:

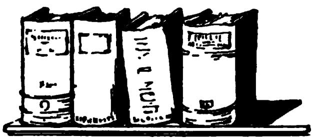
$\frac{1}{2}$ of a school year
$\frac{1}{4}$ of a school year
$\frac{3}{4}$ of a school year

$\frac{1}{5}$ of a school year
$\frac{2}{5}$ of a school year
$\frac{3}{5}$ of a school year

Drill in Fundamentals. Run a cross-country race. (See page 30.) Use examples in long division.

PROBLEMS

1. Three boards are laid on top of one another. How thick are they altogether if the boards are $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{3}{16}$ of an inch thick respectively? **$1\frac{13}{16}$**



2. Four books stand together on a shelf. They are $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{2}$, and $\frac{7}{8}$ of an inch thick respectively. How wide a space do they occupy on the shelf? **$2\frac{3}{4}$ in.**

3. Add $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{5}$ of a dollar. Then reduce each of these fractions of a dollar to cents and add. Compare the results. **$\frac{19}{20}$; 95c**

4. Add $\frac{1}{4}$, $\frac{13}{16}$, and $\frac{5}{8}$ of a pound. Then reduce each of these fractions of a pound to ounces and add. Compare the results. **$1\frac{11}{16}$ lbs.; 27 oz.**

5. Add $\frac{5}{6}$, $\frac{3}{4}$, and $\frac{2}{3}$ of a foot. Then reduce each of these fractions of a foot to inches and add. Compare the results. **$2\frac{1}{4}$ ft.; 27 in.**

6. Add $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$ of a peck. Then reduce each of these fractions of a peck to quarts and add. Compare the results. **$1\frac{7}{8}$ pk.; 15 qt.**

7. Add $\frac{3}{4}$, $\frac{7}{16}$, and $\frac{11}{32}$ of a bushel. Then reduce each of these fractions of a bushel to quarts and add. Compare the results. **$1\frac{17}{32}$ bu.; 49 qt.**

8. Add $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{5}{6}$ of an hour. Then reduce each of these fractions of an hour to minutes and add. Compare the results. **$1\frac{7}{12}$ hr.; 95 min.**

9. In a truck garden $\frac{3}{8}$ of an acre is in corn, $\frac{1}{4}$ of an acre in beets, and $\frac{1}{2}$ of an acre in beans. How much land is used for all these? **$1\frac{1}{8}$ acres**

Look over the problems on this page again to see how many you can do without using pencil and paper.

		A	B	C	D	E	F	G	K
I.	1.	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{8}$	2
	2.	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{5}{8}$	$\frac{7}{16}$	3
	3.	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{5}{12}$	$\frac{1}{36}$	$\frac{7}{8}$	$\frac{5}{16}$	6
	4.	$\frac{15}{16}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{7}{12}$	$\frac{1}{9}$	$\frac{5}{16}$	$\frac{7}{32}$	8
	5.	$\frac{11}{12}$	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{5}{8}$	$\frac{1}{16}$	$\frac{27}{32}$	$\frac{2}{9}$	12
II.	6.	$\frac{14}{15}$	$\frac{4}{5}$	$\frac{7}{10}$	$\frac{3}{5}$	$\frac{1}{20}$	$\frac{9}{10}$	$\frac{3}{10}$	15
	7.	$\frac{11}{18}$	$\frac{5}{9}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{9}$	$\frac{1}{8}$	$\frac{5}{12}$	18
	8.	$\frac{13}{16}$	$\frac{3}{4}$	$\frac{7}{10}$	$\frac{2}{3}$	$\frac{1}{12}$	$\frac{3}{16}$	$\frac{9}{16}$	8
	9.	$\frac{25}{32}$	$\frac{13}{16}$	$\frac{3}{5}$	$\frac{7}{10}$	$\frac{1}{2}$	$\frac{4}{5}$	$\frac{14}{35}$	6
	10.	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{5}{8}$	$\frac{9}{16}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{15}{32}$	12
III.	11.	$4\frac{5}{6}$	$5\frac{3}{8}$	$10\frac{1}{5}$	$5\frac{1}{4}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{7}{8}$	6
	12.	$3\frac{5}{8}$	$4\frac{2}{3}$	$8\frac{2}{3}$	$3\frac{2}{3}$	$\frac{5}{6}$	$\frac{5}{8}$	$\frac{3}{4}$	4
	13.	$4\frac{3}{4}$	$6\frac{3}{4}$	$4\frac{3}{5}$	$6\frac{1}{3}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{5}$	5
	14.	$7\frac{2}{3}$	$2\frac{1}{2}$	$6\frac{7}{8}$	$5\frac{3}{8}$	$\frac{4}{5}$	$\frac{3}{8}$	$\frac{7}{12}$	3
	15.	$8\frac{7}{8}$	$5\frac{1}{6}$	$12\frac{1}{2}$	$4\frac{7}{8}$	$\frac{1}{2}$	$\frac{5}{12}$	$\frac{11}{20}$	2
IV.	16.	$12\frac{2}{3}$	$18\frac{2}{3}$	$6\frac{1}{2}$	$5\frac{1}{8}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{5}{16}$	8
	17.	$8\frac{3}{4}$	$24\frac{1}{8}$	$7\frac{3}{4}$	$12\frac{3}{5}$	$\frac{5}{12}$	$\frac{5}{6}$	$\frac{2}{8}$	12
	18.	$5\frac{1}{6}$	$3\frac{5}{8}$	$8\frac{2}{3}$	$6\frac{7}{8}$	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{3}{4}$	5
	19.	$7\frac{1}{8}$	$6\frac{2}{3}$	$5\frac{1}{6}$	$15\frac{2}{3}$	$\frac{5}{8}$	$\frac{5}{6}$	$\frac{1}{2}$	3
	20.	$3\frac{5}{6}$	$3\frac{5}{9}$	$12\frac{1}{4}$	$18\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{16}$	$\frac{5}{8}$	4
V.	21.	$8\frac{3}{5}$	$2\frac{3}{8}$	$1\frac{4}{5}$	$3\frac{5}{8}$	$\frac{5}{9}$	$\frac{2}{5}$	$\frac{5}{16}$	6
	22.	$6\frac{2}{3}$	$4\frac{3}{4}$	$1\frac{7}{8}$	$2\frac{5}{16}$	$\frac{3}{7}$	$\frac{3}{5}$	$\frac{7}{16}$	12
	23.	$4\frac{5}{8}$	$5\frac{7}{8}$	$2\frac{5}{9}$	$6\frac{7}{16}$	$\frac{7}{9}$	$\frac{5}{8}$	$\frac{2}{3}$	9
	24.	$6\frac{3}{8}$	$7\frac{3}{16}$	$3\frac{7}{16}$	$9\frac{9}{16}$	$\frac{5}{9}$	$\frac{7}{8}$	$\frac{3}{7}$	8
	25.	$5\frac{5}{6}$	$4\frac{9}{16}$	$4\frac{5}{16}$	$7\frac{3}{16}$	$\frac{3}{7}$	$\frac{3}{16}$	$\frac{5}{8}$	7

		A	B	C	D	E
I.	1.	2434.540	24.684	24.501	.168	.25
	2.	1426.300	323.470	32.480	1.238	.24
	3.	5368.130	53.760	54.164	3.740	6.70
	4.	7021.500	564.900	78.130	2.630	5.60
	5.	6842.750	293.813	65.127	.784	.93
II.	6.	9025.600	312.560	72.180	.685	.74
	7.	156.320	14.785	9.337	.024	1.20
	8.	6024.500	473.290	12.080	4.130	.32
	9.	5803.600	513.240	85.610	.632	.43
	10.	4150.300	37.292	7.142	.029	.26
III.	11.	5632.400	415.240	48.250	.136	.27
	12.	146.450	36.365	7.362	.024	.38
	13.	789.070	718.050	6.045	.167	9.70
	14.	824.800	613.340	19.240	2.320	5.50
	15.	5813.700	329.050	8.307	5.480	.08
IV.	16.	348.070	315.620	25.320	.407	.32
	17.	1456.380	14.607	7.015	3.260	5.40
	18.	3406.790	215.670	63.240	2.050	.38
	19.	8019.630	3152.480	5.038	3.140	.27
	20.	7905.480	728.540	12.480	.508	4.70
V.	21.	453.120	329.180	63.420	.307	.73
	22.	6713.500	5048.700	8.597	1.420	.09
	23.	24.324	18.052	71.062	5.460	1.48
	24.	613.980	508.380	83.507	7.050	2.07
	25.	532.460	407.220	19.207	.432	5.30
VI.	26.	817.150	51.840	139.400	5.102	0.87
	27.	1147.080	621.290	8270.820	1.046	1.93
	28.	29.671	71.640	934.810	0.780	8.46
	29.	387.802	37.800	642.920	0.194	2.31
	30.	819.270	2.590	21.310	8.191	8.09

40. Multiples of Numbers. The numbers 10, 15, 20, 25 are said to be *multiples* of 5 because they are products obtained by multiplying 5 by integers. The table of 5's gives all multiples of 5 up to $10\times5=50$. Similarly the table of 8's gives all multiples of 8 up to $10\times8=80$.

41. The Least Common Multiple. The number 24 is said to be a *common multiple* of 6 and 8 because it is a multiple of 6 and also of 8. 48 is also a common multiple of 6 and 8. 24 is *their least common multiple.*

Instead of least common multiple we write L. C. M.

ORAL EXERCISES

Find the L. C. M. of each of the following:

1. 2, 3, 4.	2. 2, 5, 10.	3. 3, 4, 6.
4. 2, 6, 12.	5. 4, 8, 16.	6. 5, 10, 4.

In most practical problems the L. C. M. may be found directly by inspection as in the above exercises. In a few more complicated cases the method shown in the following example may be used:

Example. Find the L. C. M. of 6, 8, 20.

Solution: Write down a series of multiples of the largest number, 20, and notice the smallest of these which is a multiple of the others.

Thus, in the series 40, 60, 80, 100, 120, 140, we notice that 120 is the smallest which is a multiple of both 6 and 8. Hence 120 is the L. C. M. of 6, 8, 20.

WRITTEN EXERCISES

Find the L. C. M. of each of the following:

1. 3, 5, 8.	120	4. 5, 8, 12.	120	7. 8, 9, 36.	72
2. 15, 18.	90	5. 2, 7, 9.	126	8. 5, 9, 12.	180
3. 8, 12, 16.	48	6. 4, 6, 9.	36	9. 6, 9, 12.	36

42. The Least Common Denominator. Two fractions such as $\frac{1}{2}$ and $\frac{1}{3}$ may be reduced to 6ths, or to 12ths, 18ths, etc. The smallest of these is the least common denominator of $\frac{1}{2}$ and $\frac{1}{3}$. Instead of least common denominator we write L. C. D. Notice that 6 is the L. C. M. of 2 and 3, and that 6 is also the L. C. D. of $\frac{1}{2}$ and $\frac{1}{3}$.

The general rule is:

To find the L. C. D. of fractions find the L. C. M. of their denominators.

The process to be used in more difficult examples is shown in the following:

Example. Reduce $\frac{3}{4}$, $\frac{5}{9}$ and $\frac{7}{16}$ to a common denominator.

$$\frac{3}{4}=\frac{36\times 3}{36\times 4}=\frac{108}{144}$$

$$\frac{5}{9}=\frac{16\times 5}{16\times 9}=\frac{80}{144}$$

$$\frac{7}{16}=\frac{9\times 7}{9\times 16}=\frac{63}{144}$$

(1) Find by the method shown on page 48 that 144 is the L. C. M. of 4, 9, 16. Hence the fractions must be reduced to 144ths.

(2) Divide 144 by 4 and multiply both terms of $\frac{3}{4}$ by the quotient.

(3) Divide 144 by 9 and multiply both terms of $\frac{5}{9}$ by the quotient.

(4) Divide 144 by 16 and multiply both terms of $\frac{7}{16}$ by the quotient.

WRITTEN EXERCISES

Reduce the following fractions to a common denominator and add them. Find the L. C. D. by inspection when you can.

1. $\frac{5}{6}+\frac{3}{8}+\frac{7}{12}$ $1\frac{19}{24}$
2. $\frac{1}{6}+\frac{7}{8}+\frac{3}{10}$ $1\frac{41}{120}$
3. $\frac{11}{12}+\frac{5}{18}+\frac{1}{8}$ $1\frac{23}{72}$
4. $\frac{3}{8}+\frac{7}{16}+\frac{5}{32}$ $\frac{31}{32}$
5. $\frac{5}{12}+\frac{7}{16}+\frac{3}{8}$ $1\frac{11}{48}$
6. $\frac{1}{4}+\frac{3}{32}+\frac{5}{64}$ $\frac{27}{64}$
7. $\frac{3}{5}+\frac{5}{6}+\frac{7}{10}$ $2\frac{2}{15}$
8. $\frac{3}{7}+\frac{1}{4}+\frac{3}{16}$ $\frac{97}{112}$
9. $\frac{5}{8}+\frac{5}{12}+\frac{3}{16}$ $1\frac{11}{48}$

Drill in Fundamentals. Play game No. 4, page 30. Use examples in adding fractions.

Example. Add $1\frac{1}{2}+3\frac{2}{3}+2\frac{1}{4}$.

$$\begin{array}{ll} 1\frac{1}{2} & \frac{3}{12} \\ 3\frac{2}{3} & \frac{8}{12} \\ 2\frac{1}{4} & \frac{3}{12} \\ \hline 7\frac{5}{12} & \frac{17}{12} \text{ or } 1\frac{5}{12} \end{array}$$

First add the fractions separately, and then the integers. If possible, the fractions should be added mentally, since that saves one copying of the numbers. The sum of the fractions is $\frac{17}{12}$ or $1\frac{5}{12}$. The 1 is added to the integers.

WRITTEN EXERCISES

In this manner add each of the following:

1. $3\frac{1}{4}+2\frac{3}{8}+1\frac{1}{2}$ $7\frac{1}{8}$
4. $6\frac{1}{2}+2\frac{3}{8}+4\frac{3}{4}$ $13\frac{5}{8}$
7. $4\frac{3}{7}+2\frac{1}{3}+4\frac{5}{14}$ $11\frac{5}{42}$
2. $5\frac{3}{4}+4\frac{5}{6}+\frac{1}{3}$ $10\frac{11}{12}$
5. $1\frac{5}{12}+4\frac{5}{6}+2\frac{1}{2}$ $8\frac{3}{4}$
8. $3\frac{5}{9}+2\frac{2}{3}+3\frac{5}{6}$ $10\frac{1}{18}$
3. $2\frac{4}{5}+3\frac{7}{10}+2\frac{1}{2}$ 9
6. $5\frac{1}{4}+2\frac{7}{8}+4\frac{9}{16}$ $12\frac{11}{16}$
9. $4\frac{1}{4}+2\frac{1}{8}+4\frac{1}{16}$ $10\frac{7}{16}$

10. In a sewing class a girl made an apron, a cap and a pair of loose sleeves. She used $1\frac{1}{16}$ yards of cloth for the apron, $\frac{1}{2}$ yard for the cap and $\frac{1}{3}$ yard for the sleeves. How many yards of cloth did she use? $1\frac{43}{48}$

11. She does $3\frac{3}{4}$ yards of hemming on the apron, and $\frac{3}{4}$ yards of hemming on each sleeve. How many yards of hemming does she do on the sleeves and apron? $5\frac{1}{4}$

12. One girl in the sewing class made four lace handkerchiefs for Christmas. For each of two handkerchiefs she used $1\frac{1}{4}$ yards of lace, and for each of the other two she used $1\frac{1}{3}$ yards of lace. How many yards of lace did she use in all? $5\frac{1}{6}$

43. Subtraction of Fractions. When two fractions have a common denominator, it is clear that we can subtract one from the other by subtracting the numerators, leaving the denominator unchanged.

That is, $\frac{7}{12}-\frac{5}{12}=\frac{2}{12}=\frac{1}{6}$.

ORAL EXERCISES

In each of the following examples give the remainder reduced to the simplest form.

1. $\frac{3}{8}-\frac{1}{8}$	7. $\frac{7}{16}-\frac{3}{16}$	13. $\frac{5}{12}-\frac{3}{12}$
2. $\frac{3}{4}-\frac{1}{4}$	8. $\frac{9}{16}-\frac{4}{16}$	14. $\frac{7}{12}-\frac{3}{12}$
3. $\frac{7}{8}-\frac{1}{8}$	9. $\frac{13}{16}-\frac{5}{16}$	15. $\frac{7}{15}-\frac{2}{15}$
4. $\frac{5}{8}-\frac{3}{8}$	10. $\frac{15}{16}-\frac{7}{16}$	16. $\frac{11}{15}-\frac{4}{15}$
5. $\frac{7}{8}-\frac{3}{8}$	11. $\frac{11}{16}-\frac{5}{16}$	17. $\frac{17}{32}-\frac{9}{32}$
6. $\frac{4}{5}-\frac{1}{5}$	12. $\frac{13}{16}-\frac{3}{16}$	18. $\frac{27}{32}-\frac{3}{32}$

If the minuend and subtrahend are not like fractions, they must first be reduced to such fractions.

Example. From $\frac{1}{3}$ subtract $\frac{1}{4}$.

$\frac{1}{3}=\frac{4}{12}$ First reduce $\frac{1}{3}$ and $\frac{1}{4}$ to 12ths and then subtract as
$\frac{1}{4}=\frac{3}{12}$ shown.
$\frac{1}{12}$ remainder.

WRITTEN EXERCISES

Find the remainders in the following:

1. $\frac{1}{2}-\frac{1}{4}$ $\frac{1}{4}$	6. $\frac{3}{4}-\frac{5}{8}$ $\frac{1}{8}$	11. $\frac{13}{16}-\frac{3}{4}$ $\frac{1}{16}$
2. $\frac{3}{4}-\frac{1}{2}$ $\frac{1}{4}$	7. $\frac{5}{6}-\frac{1}{4}$ $\frac{7}{12}$	12. $\frac{15}{16}-\frac{5}{8}$ $\frac{5}{16}$
3. $\frac{1}{3}-\frac{1}{6}$ $\frac{1}{6}$	8. $\frac{7}{8}-\frac{3}{4}$ $\frac{1}{8}$	13. $\frac{13}{16}-\frac{5}{8}$ $\frac{3}{16}$
4. $\frac{1}{4}-\frac{1}{8}$ $\frac{1}{8}$	9. $\frac{9}{16}-\frac{1}{2}$ $\frac{1}{16}$	14. $\frac{15}{32}-\frac{3}{8}$ $\frac{3}{32}$
5. $\frac{1}{2}-\frac{1}{8}$ $\frac{3}{8}$	10. $\frac{11}{16}-\frac{1}{4}$ $\frac{7}{16}$	15. $\frac{25}{32}-\frac{7}{16}$ $\frac{11}{32}$

ORAL EXERCISES

Give the remainders in the following. Also add each pair of fractions.

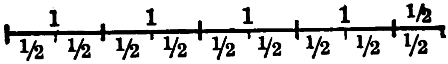
1. $\frac{1}{2}-\frac{1}{3}$
2. $\frac{2}{3}-\frac{1}{2}$
3. $\frac{1}{3}-\frac{1}{4}$
4. $\frac{2}{3}-\frac{1}{4}$
5. $\frac{3}{4}-\frac{1}{3}$
6. $\frac{3}{4}-\frac{2}{3}$
7. $\frac{1}{4}-\frac{1}{6}$
8. $\frac{3}{4}-\frac{1}{6}$
9. $\frac{5}{6}-\frac{1}{4}$
10. $\frac{5}{6}-\frac{3}{4}$
11. $\frac{1}{3}-\frac{1}{5}$
12. $\frac{2}{3}-\frac{1}{5}$
13. $\frac{2}{3}-\frac{2}{5}$
14. $\frac{2}{3}-\frac{3}{5}$
15. $\frac{3}{5}-\frac{1}{3}$
16. $\frac{4}{5}-\frac{1}{3}$
17. $\frac{4}{5}-\frac{2}{3}$
18. $\frac{1}{6}-\frac{1}{8}$
19. $\frac{3}{8}-\frac{1}{6}$
20. $\frac{5}{8}-\frac{1}{6}$
21. $\frac{7}{8}-\frac{1}{6}$
22. $\frac{7}{8}-\frac{5}{6}$
23. $\frac{5}{6}-\frac{3}{8}$
24. $\frac{5}{6}-\frac{5}{8}$

WRITTEN EXERCISES

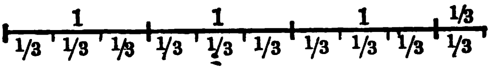
In the following subtract and reduce all results to the simplest form. Also add each pair of fractions.

25. $\frac{4}{5}-\frac{1}{6}$ $\frac{19}{30}$; $\frac{29}{30}$
26. $\frac{3}{5}-\frac{3}{8}$ $\frac{9}{40}$; $\frac{39}{40}$
27. $\frac{5}{6}-\frac{3}{8}$ $\frac{11}{24}$; $1\frac{5}{24}$
28. $\frac{5}{8}-\frac{2}{5}$ $\frac{9}{40}$; $1\frac{1}{40}$
29. $\frac{7}{8}-\frac{2}{3}$ $\frac{5}{24}$; $1\frac{13}{24}$
30. $\frac{3}{7}-\frac{1}{9}$ $\frac{20}{63}$; $\frac{34}{63}$
31. $\frac{5}{9}-\frac{2}{5}$ $\frac{7}{45}$; $\frac{43}{45}$
32. $\frac{4}{7}-\frac{2}{5}$ $\frac{6}{35}$; $\frac{34}{35}$
33. $\frac{7}{8}-\frac{4}{5}$ $\frac{3}{40}$; $1\frac{27}{40}$
34. $\frac{3}{7}-\frac{2}{5}$ $\frac{1}{35}$; $\frac{29}{35}$
35. $\frac{3}{8}-\frac{2}{9}$ $\frac{11}{72}$; $\frac{43}{72}$
36. $\frac{3}{10}-\frac{2}{9}$ $\frac{7}{90}$; $\frac{47}{90}$
37. $\frac{1}{7}-\frac{1}{8}$ $\frac{1}{56}$; $\frac{15}{56}$
38. $\frac{3}{7}-\frac{1}{8}$ $\frac{17}{56}$; $\frac{31}{56}$
39. $\frac{5}{7}-\frac{5}{8}$ $\frac{5}{56}$; $1\frac{19}{56}$
40. $\frac{6}{7}-\frac{3}{8}$ $\frac{27}{56}$; $1\frac{13}{56}$
41. $\frac{7}{8}-\frac{6}{7}$ $\frac{1}{56}$; $1\frac{41}{56}$
42. $\frac{1}{8}-\frac{1}{12}$ $\frac{1}{24}$; $\frac{5}{24}$
43. $\frac{5}{8}-\frac{7}{12}$ $\frac{1}{24}$; $1\frac{5}{24}$
44. $\frac{5}{12}-\frac{3}{8}$ $\frac{1}{24}$; $\frac{19}{24}$
45. $\frac{7}{8}-\frac{5}{12}$ $\frac{11}{24}$; $1\frac{7}{24}$
46. $\frac{11}{12}-\frac{7}{8}$ $\frac{1}{24}$; $1\frac{19}{24}$
47. $\frac{1}{6}-\frac{1}{9}$ $\frac{1}{18}$; $\frac{5}{18}$
48. $\frac{5}{6}-\frac{2}{9}$ $\frac{11}{18}$; $1\frac{1}{18}$
49. $\frac{2}{9}-\frac{1}{6}$ $\frac{1}{18}$; $\frac{7}{18}$
50. $\frac{5}{6}-\frac{4}{9}$ $\frac{7}{18}$; $1\frac{5}{18}$
51. $\frac{1}{12}-\frac{1}{16}$ $\frac{1}{48}$; $\frac{7}{48}$
52. $\frac{5}{12}-\frac{3}{16}$ $\frac{11}{48}$; $\frac{29}{48}$
53. $\frac{7}{12}-\frac{5}{16}$ $\frac{13}{48}$; $\frac{43}{48}$
54. $\frac{9}{16}-\frac{5}{12}$ $\frac{7}{48}$; $\frac{47}{48}$
55. $\frac{1}{8}-\frac{1}{9}$ $\frac{1}{72}$; $\frac{17}{72}$
56. $\frac{5}{8}-\frac{4}{9}$ $\frac{13}{72}$; $1\frac{5}{72}$
57. $\frac{5}{9}-\frac{3}{8}$ $\frac{13}{72}$; $\frac{67}{72}$
58. $\frac{7}{8}-\frac{5}{9}$ $\frac{23}{72}$; $1\frac{31}{72}$
59. $\frac{8}{9}-\frac{7}{8}$ $\frac{1}{72}$; $1\frac{55}{72}$
60. $\frac{5}{6}-\frac{13}{16}$ $\frac{1}{48}$; $1\frac{31}{48}$

ORAL EXERCISES



1. How many halves are there in 1? In $1\frac{1}{2}$? In 2? In $2\frac{1}{2}$? In 3? In $3\frac{1}{2}$? In 4? In $4\frac{1}{2}$? Point to these in the figure.



2. How many 3ds are there in 1? In $1\frac{1}{3}$? In $1\frac{2}{3}$? In 2? In $2\frac{1}{3}$? In $2\frac{2}{3}$? In $3\frac{1}{3}$? Point to these in the figure.

3. How many 4ths are there in 1? In 4? In 8? In $2\frac{1}{4}$? In $3\frac{3}{4}$? In $10\frac{3}{4}$?

4. Give a rule for reducing a whole number to 3ds, to 4ths, to 5ths, to 6ths.

5. Reduce to improper fractions: $4\frac{1}{3}$, $3\frac{4}{5}$, $5\frac{3}{8}$, $4\frac{3}{4}$, $7\frac{3}{4}$, $2\frac{7}{16}$, $3\frac{5}{12}$, $4\frac{7}{7}$.

Sometimes it is necessary to reduce only part of a whole number to a fraction. Thus, $4=3\frac{6}{6}$, $2\frac{3}{4}=1\frac{7}{4}$, $5\frac{7}{8}=4\frac{15}{8}$.

6. $3\frac{1}{4}$ equals 2 and how many 4ths?

7. $4\frac{2}{3}$ equals 3 and how many 3ds?

8. $5\frac{2}{5}$ equals 4 and how many 5ths?

9. $6\frac{3}{7}$ equals 5 and how many 7ths?

10. $5\frac{3}{8}$ equals 4 and how many 8ths?

11. Give the missing numbers in $4\frac{7}{8}=3\frac{?}{8}$, $8\frac{5}{7}=7\frac{?}{7}$, $4\frac{3}{16}=3\frac{?}{16}$.

Drill in Fundamentals. On three succeeding days run off a field meet. Use examples in addition for the first event, examples in subtraction for the second, and examples in multiplication for the third.

Example 1. From $12\frac{3}{5}$ subtract 8.

$12\frac{3}{5}$	Subtract 8 from 12 and bring down the $\frac{3}{5}$.
8	
$4\frac{3}{5}$	

Example 2. From 2 subtract $\frac{3}{4}$.

$2\frac{4}{4}$	Add 1 or $\frac{4}{4}$ to minuend. Then subtract $\frac{3}{4}$ from $\frac{4}{4}$ and carry 1 to subtrahend.	First method
$\frac{3}{4}$		
$1\frac{1}{4}$		
$1\frac{4}{4}$	Change 2 to $1\frac{4}{4}$, then subtract $\frac{3}{4}$ from $\frac{4}{4}$.	Second method.
$\frac{3}{4}$		
$1\frac{1}{4}$		

ORAL EXERCISES

1. $5\frac{3}{4}-2$	5. $2-\frac{2}{3}$	9. $8-\frac{5}{9}$	13. $8-\frac{13}{16}$
2. $9\frac{7}{9}-4$	6. $4-\frac{3}{4}$	10. $7-\frac{5}{12}$	14. $9-\frac{7}{12}$
3. $8\frac{3}{7}-6$	7. $8-\frac{6}{7}$	11. $6-\frac{9}{16}$	15. $10-\frac{5}{7}$
4. $6\frac{2}{3}-4$	8. $9-\frac{4}{5}$	12. $3-\frac{9}{10}$	16. $14-\frac{13}{16}$

Example 3. From $3\frac{3}{4}$ subtract $1\frac{1}{4}$.

$3\frac{3}{4}$	Subtract the fractions and then the whole numbers.
$1\frac{1}{4}$	
$2\frac{2}{4} = 2\frac{1}{2}$	

Example 4. From 4 subtract $1\frac{4}{5}$.

Suggestion: Add 1 or $\frac{5}{5}$ to minuend and carry 1 to subtrahend, or change 4 into $3\frac{5}{5}$. Then subtract as in example 3.

WRITTEN EXERCISES

1. $2-1\frac{1}{2}$	$\frac{1}{2}$	5. $5-1\frac{5}{7}$	$3\frac{2}{7}$	9. $12-7\frac{4}{7}$	$4\frac{3}{7}$	13. $14-7\frac{4}{9}$	$6\frac{5}{9}$
2. $3-1\frac{3}{4}$	$1\frac{1}{4}$	6. $9-4\frac{3}{8}$	$4\frac{5}{8}$	10. $14-8\frac{5}{9}$	$5\frac{4}{9}$	14. $16-6\frac{7}{8}$	$9\frac{1}{8}$
3. $4-2\frac{7}{8}$	$1\frac{1}{8}$	7. $8-3\frac{7}{32}$	$4\frac{25}{32}$	11. $23-17\frac{3}{10}$	$5\frac{7}{10}$	15. $21-14\frac{9}{16}$	$6\frac{7}{16}$
4. $5-2\frac{7}{16}$	$2\frac{9}{16}$	8. $10-2\frac{9}{32}$	$7\frac{23}{32}$	12. $23-16\frac{3}{16}$	$6\frac{13}{16}$	16. $42-28\frac{7}{24}$	$13\frac{17}{24}$

Example 1. From $5\frac{1}{3}$ subtract $2\frac{1}{4}$.

$5\frac{1}{3}=5\frac{4}{12}$	First subtract the fractions separately. If possible, do this mentally.
$2\frac{1}{4}=2\frac{3}{12}$	
$3\frac{1}{12}$	

Example 2. From $3\,{}^1\!/_3$ subtract $1\,{}^1\!/_2$.

$3\frac{1}{3}=3\frac{2}{6}$	$3\frac{8}{6}$	After reducing $\frac{1}{3}$ and $\frac{1}{2}$ to 6ths we find that $\frac{2}{6}$ is less than $\frac{3}{6}$. Hence add $\frac{6}{6}$ to minuend and carry 1 to subtrahend.	First method.
$1\frac{1}{2}=1\frac{3}{6}$	$2\frac{3}{6}$		
	$1\frac{5}{6}$		
$3\frac{1}{3}=3\frac{2}{6}$	$2\frac{8}{6}$	Change $3\frac{2}{6}$ to $2\frac{8}{6}$ and subtract as in example 1.	Second method.
$1\frac{1}{2}=1\frac{3}{6}$	$1\frac{3}{6}$		
	$1\frac{5}{6}$		

WRITTEN EXERCISES

In the manner of example 1 subtract the following:

1.	$3\frac{1}{3}-1\frac{1}{12}$	$2\frac{1}{4}$	6.	$6\frac{1}{3}-2\frac{1}{8}$	$4\frac{5}{24}$	11.	$12\frac{7}{8}-4\frac{3}{4}$	$8\frac{1}{8}$
2.	$5\frac{2}{3}-2\frac{1}{4}$	$3\frac{5}{12}$	7.	$9\frac{1}{4}-3\frac{3}{16}$	$6\frac{1}{16}$	12.	$18\frac{5}{8}-12\frac{1}{3}$	$6\frac{7}{24}$
3.	$6\frac{3}{4}-3\frac{5}{8}$	$3\frac{1}{8}$	8.	$15\frac{2}{3}-7\frac{1}{6}$	$8\frac{1}{2}$	13.	$25\frac{7}{16}-4\frac{13}{32}$	$21\frac{1}{32}$
4.	$7\frac{2}{3}-4\frac{3}{5}$	$3\frac{1}{15}$	9.	$25\frac{3}{5}-12\frac{7}{15}$	$13\frac{2}{15}$	14.	$108\frac{3}{10}-45\frac{1}{4}$	$63\frac{1}{20}$
5.	$5\frac{3}{4}-1\frac{3}{8}$	$4\frac{3}{8}$	10.	$35\frac{4}{7}-21\frac{5}{14}$	$14\frac{3}{14}$	15.	$5\frac{3}{4}-2\frac{1}{3}$	$3\frac{5}{12}$

In the manner of example 2 subtract the following:

16.	$7\frac{1}{3}-2\frac{3}{4}$	$4\frac{7}{12}$	21.	$28\frac{5}{8}-13\frac{5}{6}$	$14\frac{19}{24}$	26.	$124\frac{7}{12}-94\frac{5}{6}$	$29\frac{3}{4}$
17.	$9\frac{2}{3}-4\frac{3}{4}$	$4\frac{11}{12}$	22.	$19\frac{7}{8}-6\frac{15}{16}$	$12\frac{15}{16}$	27.	$67\frac{5}{12}-42\frac{11}{24}$	$24\frac{23}{24}$
18.	$12\frac{4}{5}-6\frac{9}{10}$	$5\frac{9}{10}$	23.	$17\frac{3}{8}-4\frac{4}{9}$	$12\frac{67}{72}$	28.	$39\frac{3}{8}-14\frac{7}{10}$	$24\frac{27}{40}$
19.	$15\frac{3}{4}-4\frac{7}{8}$	$10\frac{7}{8}$	24.	$49\frac{3}{7}-21\frac{2}{3}$	$27\frac{16}{21}$	29.	$64\frac{2}{3}-36\frac{7}{8}$	$27\frac{19}{24}$
20.	$17\frac{3}{5}-4\frac{2}{3}$	$12\frac{14}{15}$	25.	$59\frac{4}{5}-31\frac{6}{7}$	$27\frac{33}{35}$	30.	$81\frac{3}{7}-47\frac{5}{6}$	$33\frac{25}{42}$

31. In a sewing class a girl is to do 7 yards of hemming. How many yards has she left to do after hemming $3\frac{5}{8}$ yards? $3\frac{3}{8}$

Example 1. Perform the indicated operations in $1\frac{1}{2}+3\frac{3}{4}-1\frac{1}{5}$.

Add: $1\frac{1}{2}=1\frac{2}{4}$ *Solution:* First add $1\frac{1}{2}$ and $3\frac{3}{4}$, obtaining $5\frac{1}{4}$ as
$3\frac{3}{4}=3\frac{3}{4}$ the sum.

$$4\frac{5}{4}=5\frac{1}{4}$$

Subtract: $5\frac{1}{4}=5\frac{5}{20}$ From $5\frac{1}{4}$ subtract $1\frac{1}{5}$. The result is $4\frac{1}{20}$.
$1\frac{1}{5}=1\frac{4}{20}$

$$4\frac{1}{20}$$

Example 2. Perform the indicated operations in $\frac{3}{4}+5\frac{2}{3}-2\frac{1}{3}$.

Subtract: $5\frac{2}{3}$ Since $5\frac{2}{3}$ and $2\frac{1}{3}$ contain like fractions it is
$2\frac{1}{3}$ simpler to first subtract $2\frac{1}{3}$ from $5\frac{2}{3}$ and
$3\frac{1}{3}$ then add $\frac{3}{4}$.

Add: $3\frac{1}{3}=3\frac{4}{12}$
$\frac{3}{4}=\frac{9}{12}$

$$4\frac{1}{12}$$

WRITTEN EXERCISES

In the following perform the indicated operations in the simplest way as suggested above:

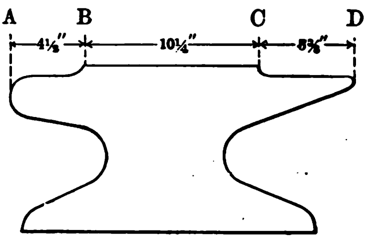
1. $1\frac{2}{3}+4\frac{3}{4}-2\frac{1}{6}$ $4\frac{1}{4}$
2. $1\frac{1}{2}+4\frac{1}{3}-5\frac{1}{4}$ $\frac{7}{12}$
3. $5\frac{3}{4}+2\frac{1}{2}-3\frac{1}{3}$ $4\frac{11}{12}$
4. $7\frac{1}{4}+5\frac{5}{6}-9\frac{3}{4}$ $3\frac{1}{3}$
5. $24\frac{7}{8}+12\frac{3}{4}-30\frac{1}{2}$ $7\frac{1}{8}$
6. $7\frac{2}{3}+7\frac{5}{6}-8\frac{5}{8}$ $6\frac{7}{8}$
7. $35\frac{3}{8}-16\frac{1}{4}+2\frac{1}{2}$ $21\frac{5}{8}$
8. $16\frac{1}{3}-4\frac{2}{3}+2\frac{5}{6}$ $14\frac{1}{2}$
9. $31\frac{3}{7}+5\frac{1}{2}-28\frac{6}{7}$ $8\frac{1}{14}$
10. $47\frac{3}{5}+25\frac{2}{3}-3\frac{1}{2}$ $69\frac{23}{30}$
11. $14\frac{2}{3}+6\frac{3}{4}-2\frac{1}{6}$ $19\frac{1}{4}$
12. $71\frac{5}{6}-19\frac{2}{3}+8\frac{1}{2}$ $60\frac{2}{3}$
13. $41\frac{5}{8}-17\frac{3}{4}-3\frac{1}{2}$ $20\frac{3}{8}$
14. $29\frac{1}{4}-6\frac{1}{3}+5\frac{1}{12}$ 28
15. $13\frac{3}{8}-8\frac{3}{4}+2\frac{7}{16}$ $7\frac{1}{16}$
16. $58\frac{7}{12}-8\frac{5}{8}+16\frac{5}{6}$ $66\frac{19}{24}$

Drill in Fundamentals. Drill on denominate number facts, as 1 foot = ? inches. Let the boys play against the girls.

The signs ′ and ″ are often used to indicate *feet* and *inches*. Thus, 3′ 6″ means 3 feet 6 inches.

1. A picture is $6\frac{1}{4}''$ by $8\frac{3}{4}''$ inside the frame, and the frame is $\frac{5}{8}''$ wide. What are the dimensions of the picture, including the frame? **$7\frac{1}{2}'' \times 10''$**

2. From the data given in the picture, find the length from A to D. **$20\frac{1}{8}''$**



3. A sheet of letter paper is $6\frac{1}{4}$ inches wide. How long must the envelope be to allow $\frac{1}{8}$ of an inch at each end? **$6\frac{1}{2}''$**

4. A lady bought $1\frac{1}{4}$ pounds of meat on Monday, $\frac{7}{8}$ of a pound on Tuesday, $2\frac{1}{2}$ pounds on Wednesday, $1\frac{5}{8}$ pounds on Friday, and $3\frac{1}{2}$ pounds on Saturday. How many pounds of meat did she buy during the whole week? **$9\frac{3}{4}$ lb.**

5. A lady bought $\frac{1}{2}$ of a yard of velvet for trimming her coat, $\frac{3}{4}$ of a yard for her dress, and $\frac{7}{8}$ of a yard for her hat. How many yards of velvet did she buy all together? **$2\frac{1}{8}$ yd.**

6. Mrs. West bought $8\frac{1}{2}$ yards of goods for school dresses for her daughters Mary and Elsie. For Mary's dress she used $3\frac{1}{2}$ yards, and for Elsie's $3\frac{3}{4}$ yards. How much goods was left? Give the result in yards and inches. **1 yd., 9″**

7. A board $\frac{7}{8}$ inches thick is screwed to a plank $3\frac{1}{4}$ inches thick. How thick are the two together? By how much does a screw $2\frac{1}{2}$ inches long fail to reach through the two? **$4\frac{1}{8}''$; $1\frac{5}{8}''$**

8. A lady bought $\frac{5}{8}$ of a yard of silk. Is this more or less than $\frac{2}{3}$ of a yard? how much? Reduce the difference to inches. **Less $\frac{1}{24}$ yd.; $1\frac{1}{2}''$**

44. A Fraction Multiplied by an Integer. To multiply a number by an integer gives the same result as adding it as many times as the multiplier contains one.

Thus, since $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$, we have $2\times\frac{1}{3}=\frac{2}{3}$,
and $3\times\frac{2}{7}=\frac{6}{7}$, because $\frac{2}{7}+\frac{2}{7}+\frac{2}{7}=\frac{6}{7}$.

Examples. $7\times\frac{2}{3}=\frac{14}{3}=4\frac{2}{3}$, and $8\times\frac{5}{7}=\frac{40}{7}=5\frac{5}{7}$.

ORAL EXERCISES

In this manner find the products in the following and reduce each to the simplest form:

1. $3\times\frac{3}{4}$	5. $5\times\frac{3}{4}$	9. $9\times\frac{2}{5}$	13. $6\times\frac{4}{5}$
2. $4\times\frac{1}{3}$	6. $3\times\frac{5}{8}$	10. $4\times\frac{5}{7}$	14. $4\times\frac{5}{9}$
3. $5\times\frac{2}{3}$	7. $3\times\frac{2}{5}$	11. $9\times\frac{3}{4}$	15. $8\times\frac{3}{5}$
4. $4\times\frac{3}{5}$	8. $5\times\frac{3}{8}$	12. $5\times\frac{7}{8}$	16. $7\times\frac{3}{8}$

45. Multiplying an Integer by a Fraction. We have frequently noticed that when we multiply two numbers together the order in which they are taken does not affect the product. That is, $2\times4=4\times2$, and $3\times5=5\times3$. In the same manner $\frac{3}{4}\times5=5\times\frac{3}{4}=\frac{15}{4}=3\frac{3}{4}$, and $\frac{5}{7}\times4=4\times\frac{5}{7}=\frac{20}{7}=2\frac{6}{7}$.

ORAL EXERCISES

In this manner find the products in the following:

1. $\frac{4}{5}\times2$	3. $\frac{7}{8}\times3$	5. $\frac{4}{9}\times5$	7. $\frac{6}{7}\times6$
2. $\frac{5}{8}\times5$	4. $\frac{5}{12}\times5$	6. $\frac{2}{5}\times6$	8. $\frac{3}{5}\times8$

Examples like those given on this page may all be solved by using the following rule:

To find the product of a fraction and an integer, multiply the numerator of the fraction by the integer and reduce the result to the simplest form.

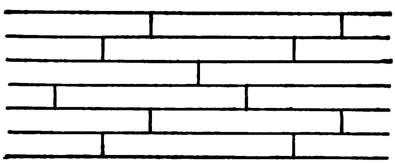
Drill in Fundamentals. Play a game like No. 1 on page 30, using examples in multiplying fractions by integers.

In the following reduce the products to the simplest form:

1. $28\times\frac{4}{5}$ $22\frac{2}{5}$	8. $\frac{7}{16}\times 81$ $35\frac{7}{16}$	15. $75\times\frac{8}{4}$ 150
2. $\frac{7}{32}\times 95$ $20\frac{25}{32}$	9. $49\times\frac{9}{16}$ $27\frac{9}{16}$	16. $35\times\frac{7}{8}$ $30\frac{5}{8}$
3. $\frac{9}{16}\times 43$ $24\frac{3}{16}$	10. $\frac{13}{16}\times 49$ $39\frac{13}{16}$	17. $14\times\frac{3}{5}$ $8\frac{2}{5}$
4. $\frac{7}{24}\times 73$ $21\frac{7}{24}$	11. $49\times\frac{5}{32}$ $7\frac{21}{32}$	18. $24\times\frac{7}{13}$ $12\frac{12}{13}$
5. $64\times\frac{9}{31}$ $18\frac{18}{31}$	12. $63\times\frac{9}{16}$ $35\frac{7}{16}$	19. $15\times\frac{3}{7}$ $6\frac{3}{7}$
6. $\frac{25}{32}\times 9$ $7\frac{1}{32}$	13. $\frac{11}{12}\times 53$ $48\frac{7}{12}$	20. $41\times\frac{3}{5}$ $24\frac{3}{5}$
7. $\frac{9}{16}\times 41$ $23\frac{1}{16}$	14. $35\times\frac{7}{32}$ $7\frac{21}{32}$	21. $79\times\frac{2}{3}$ $52\frac{2}{3}$

22. In a sewing class there are 25 girls. Each girl makes an apron, a cap and a pair of sleeves, using $\frac{7}{8}$ of a yard of cloth for the apron, $\frac{1}{2}$ of a yard for the cap, and $\frac{3}{8}$ of a yard for the sleeves. How many yards does the class use for aprons? For caps? For sleeves? $21\frac{7}{8}$; $12\frac{1}{2}$; $9\frac{3}{8}$

23. The facing bricks used in this wall are $1\frac{1}{2}$ inches thick. If each course of mortar is $\frac{1}{8}$ of an inch thick, what will be the height of 6 such bricks and 6 layers of mortar? $9\frac{3}{4}''$



Suggestion: The thickness of one brick and one layer of mortar is $1\frac{5}{8}$ inches. Hence find $6\times\frac{13}{8}$.

24. The length of these bricks is $12\frac{1}{2}$ inches, and the thickness of the mortar between the ends is $\frac{3}{16}$ of an inch. What is the total length of 3 such bricks and 3 layers of mortar?

Suggestion: The length of one brick including one layer of mortar in inches is $12\frac{1}{2}+\frac{3}{16}=12\frac{11}{16}$. $38\frac{1}{16}''$

25. The width of these bricks is 4 inches. How thick is a wall made of 3 widths of brick and two $\frac{1}{4}$-inch layers of mortar? $12\frac{1}{2}''$

46. Cancellation. We have already seen that both terms of a fraction may be divided by the same number without changing the value of the fraction.

That is, $\frac{3}{6}=\frac{1}{2}$, $\frac{4}{12}=\frac{1}{3}$, $\frac{10}{15}=\frac{2}{3}$.

In each of these cases the numerator and denominator have a common factor which is removed by division. We call this process *cancelling*. In $\frac{3}{6}=\frac{1}{2}$ the factor 3 is cancelled. In $\frac{4}{12}=\frac{1}{3}$ the factor 4 is cancelled. In $\frac{10}{15}=\frac{2}{3}$ the factor 5 is cancelled.

> The taking out of a common factor of the numerator and denominator of a fraction is called *cancelling*.

The rule above may now be stated as follows:

A common factor in the numerator and denominator of a fraction may be cancelled without changing the value of the fraction.

Consider now the example $6\times\frac{4}{15}=\frac{24}{15}=\frac{8}{5}=1\frac{3}{5}$. In this case 3 is cancelled after multiplying. In practice, however, the 3 should be cancelled *before* multiplying as follows:

$$\overset{2}{\cancel{6}}\times\frac{4}{\underset{5}{\cancel{15}}}=\frac{8}{5}=1\frac{3}{5}$$

When the common factor 3 is cancelled 6 is divided by 3, giving a quotient 2, which is written above the 6, and 15 is divided by 3, giving a quotient 5, which is written below the 15.

WRITTEN EXERCISES

Find the products indicated below in two ways: (1) multiply the numerator by the integer before cancelling, (2) cancel before multiplying as in the example just given.

1. $8\times\frac{7}{12}$ $4\frac{2}{3}$	5. $4\times\frac{5}{6}$ $3\frac{1}{3}$	9. $12\times\frac{4}{9}$ $5\frac{1}{3}$
2. $6\times\frac{5}{8}$ $3\frac{3}{4}$	6. $3\times\frac{4}{9}$ $1\frac{1}{3}$	10. $16\times\frac{3}{10}$ $4\frac{4}{5}$
3. $9\times\frac{7}{15}$ $4\frac{1}{5}$	7. $8\times\frac{5}{12}$ $3\frac{1}{3}$	11. $24\times\frac{7}{16}$ $10\frac{1}{2}$
4. $4\times\frac{3}{10}$ $1\frac{1}{5}$	8. $6\times\frac{3}{16}$ $1\frac{1}{8}$	12. $30\times\frac{5}{12}$ $12\frac{1}{2}$

In an example like $5\times\frac{7}{10}$ the multiplier, 5, cancels out, leaving 1 as the multiplier. That is, $\overset{1}{\cancel{5}}\times\frac{7}{\underset{2}{\cancel{10}}}=1\times\frac{7}{2}=3\frac{1}{2}$.

In $8\times\frac{3}{4}$ the 4 cancels out, leaving 1 as the denominator. That is, $\overset{2}{\cancel{8}}\times\frac{3}{\underset{1}{\cancel{4}}}=2\times\frac{3}{1}=6$.

In practice we omit the 1's and write

$$\cancel{5}\times\frac{7}{\underset{2}{\cancel{10}}}=\frac{7}{2}=3\frac{1}{2} \text{ and } \overset{2}{\cancel{8}}\times\frac{3}{\cancel{4}}=6.$$

WRITTEN EXERCISES

In this manner find the products of the following:

1. $8\times\frac{7}{24}$ $2\frac{1}{3}$	4. $3\times\frac{4}{9}$ $1\frac{1}{3}$	7. $7\times\frac{12}{35}$ $2\frac{2}{5}$	10. $12\times\frac{5}{6}$ 10
2. $5\times\frac{3}{10}$ $1\frac{1}{2}$	5. $\frac{4}{5}\times 30$ 24	8. $6\times\frac{5}{12}$ $2\frac{1}{2}$	11. $48\times\frac{7}{12}$ 28
3. $4\times\frac{17}{16}$ $4\frac{1}{4}$	6. $\frac{3}{4}\times 40$ 30	9. $9\times\frac{2}{3}$ 6	12. $\frac{5}{6}\times 18$ 15

47. **Product of an Integer and a Mixed Number.** To multiply a mixed number by an integer, we multiply the whole number and the fraction separately by the integer, and add the products.

Examples. Multiply $2\frac{1}{2}$ by 5, and $3\frac{3}{4}$ by 6.

$$\begin{array}{r} 2\frac{1}{2} \\ \underline{5} \\ 10\frac{5}{2}=12\frac{1}{2} \end{array} \qquad\qquad \begin{array}{r} 3\frac{3}{4} \\ \underline{6} \\ 18\frac{9}{2}=22\frac{1}{2} \end{array}$$

WRITTEN EXERCISES

In this manner find the products of the following:

1. $5\times 1\frac{1}{3}$ $6\frac{2}{3}$	4. $1\frac{5}{6}\times 4$ $7\frac{1}{3}$	7. $5\times 4\frac{3}{4}$ $23\frac{3}{4}$	10. $16\frac{2}{3}\times 6$ 100
2. $7\times 2\frac{3}{5}$ $18\frac{1}{5}$	5. $2\frac{3}{7}\times 5$ $12\frac{1}{7}$	8. $12\frac{1}{2}\times 6$ 75	11. $33\frac{1}{3}\times 6$ 200
3. $6\times 3\frac{1}{3}$ 20	6. $6\times 2\frac{3}{8}$ $14\frac{1}{4}$	9. $3\frac{1}{3}\times 12$ 40	12. $6\frac{1}{4}\times 8$ 50

WRITTEN WORK

By turning a screw completely around once it goes into the wood as far as the distance between the threads. This distance is called the *pitch* of the screw.

1. A screw has a pitch of $\frac{1}{8}$ of an inch. How far is it driven into the wood by being turned 12 times? $1\frac{1}{2}''$

2. A screw has a pitch of $\frac{3}{32}$ of an inch. How far is it driven into the wood by being turned around 16 times? $1\frac{1}{2}''$

3. If the average weight of a business letter is $\frac{3}{5}$ of an ounce, how many ounces will 1000 such letters weigh? How many pounds is this? 600 oz.; $37\frac{1}{2}$ lb.

4. A Sunday newspaper weighs 13 ounces ($\frac{13}{16}$ of a pound). How many pounds are there in an edition of 120,000 such papers? How many tons is this? 97500 lbs.; $48\frac{3}{4}$ tons

5. A certain steel plate is $\frac{5}{32}$ of an inch thick. How high is a pile of 150 such plates? $23\frac{7}{16}''$

6. A text book weighs 18 ounces ($1\frac{2}{16}$ or $1\frac{1}{8}$ of a pound) when wrapped for shipment. One day a publishing house sent out 846 of these books by mail. What was the total weight of the shipment? $951\frac{3}{4}$ lb.

7. A certain family consumes on an average $1\frac{5}{8}$ pounds of meat per day. How many pounds is this for a month of 30 days? What is the cost of the meat if the average price is 24 cents a pound? $48\frac{3}{4}$ lb.; \$11.70

8. Two cards for mounting photographs are $6\frac{1}{4}'' \times 8\frac{1}{2}''$, and $4\frac{3}{4}'' \times 8\frac{5}{8}''$. What is the difference in the lengths of these cards? What is the difference in their widths? $\frac{1}{8}''$; $1\frac{1}{2}''$

In canning fruit the following amounts of sugar are used for one quart:

peaches	4 ounces	($\frac{4}{16}$, or $\frac{1}{4}$ of a pound)
blackberries	6 ounces	($\frac{6}{16}$, or $\frac{3}{8}$ of a pound)
strawberries	8 ounces	($\frac{8}{16}$, or $\frac{1}{2}$ of a pound)
quinces	10 ounces	($\frac{10}{16}$, or $\frac{5}{8}$ of a pound)

WRITTEN WORK

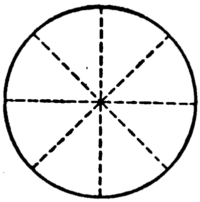
1. How much sugar is used in putting up 40 quarts of peaches? 10 lb.
2. How much sugar is used in putting up 24 quarts of strawberries? 12 lb.
3. How much sugar is used in putting up 30 quarts of blackberries? $11\frac{1}{4}$ lb.
4. How much sugar is used in putting up 12 quarts of quinces? $7\frac{1}{2}$ lb.
5. A hotel keeper figures that for a turkey dinner he needs on an average 10 ounces ($\frac{5}{8}$ pounds) of turkey for each guest. How many pounds of turkey does he buy if he expects 175 guests? $109\frac{3}{8}$ lb.
6. The basement of a house is 48 feet long and 38 feet wide. How much does it cost to lay a cement floor in this basement at \$1.00 a square yard? \$202$\frac{2}{3}$ or \$202.67

This problem may be solved by first finding the number of square feet and then dividing by 9, or by first reducing the dimensions of the basement to yards and then finding the area in square yards. Solve the problem both ways and compare results. How many yards are there in 48 feet? in 38 feet?

7. At \$2.00 a square yard, what is the cost of laying a walk 120 feet long and 5 feet wide? \$133$\frac{1}{3}$ or \$133.33
8. A rug is 13 feet long and $9\frac{3}{8}$ feet wide. Find the number of square feet in the rug. $121\frac{7}{8}$ sq. ft.

48. Product of Two Fractions. The exercises on the next three pages lead to a rule for multiplying fractions.

ORAL EXERCISES

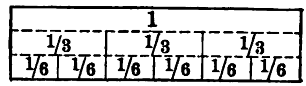


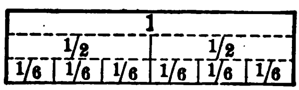
1. Point out $\frac{1}{2}$ of the circle, $\frac{1}{4}$ of it, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$, $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, $\frac{7}{8}$, $\frac{8}{8}$.

2. Read and supply the missing numbers in the following and find each result in the circle: $\frac{1}{2}$ of $\frac{1}{2}=?$, $\frac{1}{2}$ of $\frac{1}{4}=?$, $\frac{1}{2}$ of $\frac{3}{4}=?$

3. In the next figure point out $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$, $\frac{6}{6}$.

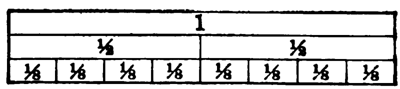
4. Supply the missing numbers in $\frac{1}{2}$ of $\frac{1}{3}=?$ and find the result in the figure.

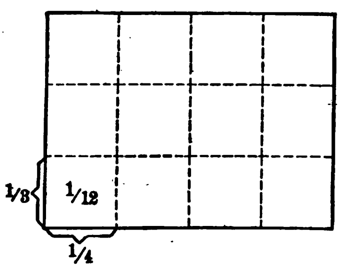




5. Supply the missing numbers in the following and find each result in the figure: $\frac{1}{3}$ of $\frac{1}{2}=?$ $\frac{2}{3}$ of $\frac{1}{2}=?$ Compare $\frac{1}{3}$ of $\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{1}{3}$.

6. Supply the missing numbers in the following and find each result in the figure: $\frac{1}{4}$ of $\frac{1}{2}=?$ $\frac{3}{4}$ of $\frac{1}{2}=?$





7. Show $\frac{1}{3}$ of this rectangle. Also show $\frac{1}{4}$ of the rectangle. Show $\frac{1}{3}$ of $\frac{1}{4}$. What fraction of the whole rectangle is this? Show $\frac{1}{4}$ of $\frac{1}{3}$. What fraction of the rectangle is this? Supply the missing numbers in $\frac{1}{3}$ of $\frac{1}{4}=?$ $\frac{1}{4}$ of $\frac{1}{3}=?$

ORAL EXERCISES

1. Find $\frac{1}{2}$ of $\frac{2}{3}$, $\frac{1}{2}$ of $\frac{4}{5}$, $\frac{1}{2}$ of $\frac{2}{7}$.

2. Supply the missing numbers in the following, and draw a figure to show each result:

   $\frac{1}{2}$ of $\frac{1}{5} = ?$ $\frac{1}{2}$ of $\frac{1}{6} = ?$ $\frac{1}{2}$ of $\frac{1}{8} = ?$ $\frac{1}{8}$ of $\frac{1}{2} = ?$

3. Draw a figure to show the result in each of the following:

   $\frac{1}{3}$ of $\frac{1}{3} = ?$ $\frac{1}{3}$ of $\frac{1}{4} = ?$ $\frac{1}{4}$ of $\frac{1}{3} = ?$ $\frac{1}{3}$ of $\frac{1}{5} = ?$

4. Draw a figure to show the result in each of the following:

   $\frac{1}{4}$ of $\frac{1}{4} = ?$ $\frac{1}{4}$ of $\frac{1}{5} = ?$ $\frac{1}{5}$ of $\frac{1}{4} = ?$ $\frac{1}{4}$ of $\frac{1}{6} = ?$

Notice that to take $\frac{1}{2}$ of a fraction we divide its numerator by 2 or multiply its denominator by 2. That is, we take one-half of the parts or else make each part only one-half as large as it was before.

Similarly, to take $\frac{1}{3}$ of a fraction we divide the numerator by 3 or multiply the denominator by 3. Why do we do this?

5. How do we take $\frac{1}{4}$ of a fraction?

6. How do we take $\frac{1}{5}$ of a fraction? $\frac{1}{6}$ of it? $\frac{1}{7}$ of it?

When we take $\frac{1}{2}$ of a fraction we say we *multiply it by* $\frac{1}{2}$. Hence, instead of $\frac{1}{2}$ of $\frac{1}{2}$ we write $\frac{1}{2} \times \frac{1}{2}$. Similarly, $\frac{1}{3}$ of $\frac{1}{2}$ is written $\frac{1}{3} \times \frac{1}{2}$, and we say we *multiply* $\frac{1}{2}$ by $\frac{1}{3}$.

7. Read the following and supply the missing numbers:

$\frac{1}{5} \times \frac{3}{4} = ?$	$\frac{1}{4} \times \frac{7}{12} = ?$	$\frac{1}{3} \times \frac{5}{11} = ?$
$\frac{1}{6} \times \frac{5}{7} = ?$	$\frac{1}{3} \times \frac{7}{11} = ?$	$\frac{1}{4} \times \frac{5}{12} = ?$
$\frac{1}{3} \times \frac{7}{12} = ?$	$\frac{1}{9} \times \frac{7}{12} = ?$	$\frac{1}{12} \times \frac{5}{11} = ?$

Drill in Fundamentals. Play game No. 3, page 30. Use examples adding fractions for the first event, and examples in multiplying fractions for the second event.

We have already seen that to multiply a fraction by $\frac{1}{2}$ we multiply its denominator by 2; to multiply a fraction by $\frac{1}{3}$ we multiply its denominator by 3, etc.

Thus, $\frac{1}{3}\times\frac{2}{5}=\frac{2}{3\times5}=\frac{2}{15}$, and $\frac{1}{7}\times\frac{2}{3}=\frac{2}{7\times3}=\frac{2}{21}$

Many such examples may be simplified by cancellation.

Example. Multiply $\frac{4}{7}$ by $\frac{1}{6}$.

$$\frac{1}{6}\times\frac{4}{7}=\frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{6}}\times7}=\frac{2}{21}$$

After indicating the product cancel 2, and then multiply 7 by 3.

$$\frac{1}{\underset{3}{\cancel{6}}}\times\frac{\overset{2}{\cancel{4}}}{7}=\frac{2}{21}$$

In practice indicate the multiplication, and cancel as in the second form given here.

WRITTEN EXERCISES

Multiply the following, using the second form:

1. $\frac{1}{5}\times\frac{25}{22}$ $\quad\frac{5}{22}$	5. $\frac{1}{3}\times\frac{6}{7}$ $\quad\frac{2}{7}$	9. $\frac{1}{7}\times\frac{35}{48}$ $\quad\frac{5}{48}$
2. $\frac{1}{7}\times\frac{14}{32}$ $\quad\frac{1}{16}$	6. $\frac{1}{8}\times\frac{16}{17}$ $\quad\frac{2}{17}$	10. $\frac{1}{9}\times\frac{27}{32}$ $\quad\frac{3}{32}$
3. $\frac{1}{4}\times\frac{8}{9}$ $\quad\frac{2}{9}$	7. $\frac{1}{4}\times\frac{12}{13}$ $\quad\frac{3}{13}$	11. $\frac{1}{3}\times\frac{15}{16}$ $\quad\frac{5}{16}$
4. $\frac{1}{2}\times\frac{4}{5}$ $\quad\frac{2}{5}$	8. $\frac{1}{3}\times\frac{9}{32}$ $\quad\frac{3}{32}$	12. $\frac{1}{5}\times\frac{15}{64}$ $\quad\frac{3}{64}$

Since $\frac{2}{3}=2\times\frac{1}{3}$, to multiply by $\frac{2}{3}$ we multiply first by $\frac{1}{3}$ and then by 2.

Example. Multiply $\frac{2}{3}\times\frac{4}{5}$.

Solution: First take $\frac{1}{3}\times\frac{4}{5}$ and then multiply the product by 2. $\frac{1}{3}\times\frac{4}{5}=\frac{4}{15}$, $\quad 2\times\frac{4}{15}=\frac{8}{15}$.

The numerator 8 of the product equals the product of the numerators of the two given fractions (2×4), and the denominator, 15, of the product equals the product of the denominators of the two given fractions (3×5). From this we get the general rule for the multiplication of fractions. (See next page.)

49. **Rule:** *To find the product of two fractions multiply the numerators together for a new numerator, and multiply the denominators together for a new denominator, cancelling when possible.*

If you have any doubt as to the correctness of the rule, check results obtained by it, by finding the product as in the last example on page 66.

ORAL EXERCISES

Find the following products by the use of the above rule:

1. $\frac{2}{3}\times\frac{4}{5}$ $\frac{8}{15}$	5. $\frac{3}{4}\times\frac{3}{7}$ $\frac{9}{28}$	9. $\frac{3}{4}\times\frac{3}{5}$ $\frac{9}{20}$	13. $\frac{5}{6}\times\frac{1}{7}$ $\frac{5}{42}$
2. $\frac{3}{5}\times\frac{4}{7}$ $\frac{12}{35}$	6. $\frac{4}{5}\times\frac{2}{3}$ $\frac{8}{15}$	10. $\frac{2}{5}\times\frac{3}{5}$ $\frac{6}{25}$	14. $\frac{4}{5}\times\frac{6}{7}$ $\frac{24}{35}$
3. $\frac{3}{7}\times\frac{1}{5}$ $\frac{3}{35}$	7. $\frac{1}{8}\times\frac{3}{4}$ $\frac{3}{32}$	11. $\frac{1}{2}\times\frac{3}{7}$ $\frac{3}{14}$	15. $\frac{3}{8}\times\frac{5}{6}$ $\frac{15}{48}$
4. $\frac{6}{7}\times\frac{3}{5}$ $\frac{18}{35}$	8. $\frac{3}{5}\times\frac{6}{7}$ $\frac{18}{35}$	12. $\frac{2}{3}\times\frac{7}{9}$ $\frac{14}{27}$	16. $\frac{2}{7}\times\frac{5}{8}$ $\frac{10}{56}$

The last two examples can be reduced to lower terms by cancellation. In practice we should cancel *before* multiplying the numerator and denominator together.

Example. Find the product of $\frac{2}{3}\times\frac{6}{5}$.

Solution:

$$\frac{2}{\cancel{3}}\times\frac{\overset{2}{\cancel{6}}}{5}=\frac{4}{5}$$

$$\boxed{\frac{\overset{1}{3}}{\underset{2}{4}}\times\frac{\overset{1}{2}}{\underset{3}{9}}=\frac{1}{6}}$$

In multiplying fractions always look carefully for factors common to a numerator and a denominator. Also remember that a number divided by itself gives 1. Notice the example solved here.

WRITTEN EXERCISES

Find the following products in this manner:

1. $\frac{2}{3}\times\frac{5}{6}$ $\frac{5}{9}$	5. $\frac{2}{5}\times\frac{10}{11}$ $\frac{4}{11}$	9. $\frac{4}{5}\times\frac{7}{12}$ $\frac{7}{15}$	13. $\frac{3}{8}\times\frac{5}{9}$ $\frac{5}{24}$
2. $\frac{2}{3}\times\frac{6}{7}$ $\frac{4}{7}$	6. $\frac{2}{5}\times\frac{3}{4}$ $\frac{3}{10}$	10. $\frac{3}{8}\times\frac{4}{5}$ $\frac{3}{10}$	14. $\frac{8}{9}\times\frac{7}{16}$ $\frac{7}{18}$
3. $\frac{3}{4}\times\frac{5}{6}$ $\frac{5}{8}$	7. $\frac{5}{8}\times\frac{6}{7}$ $\frac{15}{28}$	11. $\frac{7}{8}\times\frac{5}{21}$ $\frac{5}{24}$	15. $\frac{3}{7}\times\frac{4}{9}$ $\frac{4}{21}$
4. $\frac{3}{5}\times\frac{5}{18}$ $\frac{1}{6}$	8. $\frac{7}{10}\times\frac{5}{7}$ $\frac{1}{2}$	12. $\frac{3}{4}\times\frac{2}{9}$ $\frac{1}{6}$	16. $\frac{5}{12}\times\frac{7}{16}$ $\frac{35}{192}$

We are frequently required to multiply a mixed number by a whole number. The following form is convenient:

Multiply $3\frac{2}{5}$ by 6. *Solution:* $6\times 3=18$. Hence the product is $20\frac{2}{5}$.

$$6\times\tfrac{2}{5}=\tfrac{12}{5}=\underline{\;2\tfrac{2}{5}\;}$$
$$20\tfrac{2}{5}$$

WRITTEN EXERCISES

In this manner find the products of the following:

1. $5\times 6\frac{3}{4}$ $33\frac{3}{4}$	6. $9\times 3\frac{5}{8}$ $32\frac{5}{8}$	11. $7\times 12\frac{7}{8}$ $90\frac{1}{8}$
2. $7\times 5\frac{2}{3}$ $39\frac{2}{3}$	7. $8\times 7\frac{9}{10}$ $63\frac{1}{5}$	12. $9\times 9\frac{11}{12}$ $89\frac{1}{4}$
3. $9\times 7\frac{3}{5}$ $68\frac{2}{5}$	8. $9\times 11\frac{3}{8}$ $102\frac{3}{8}$	13. $8\times 12\frac{13}{16}$ $102\frac{1}{2}$
4. $8\times 6\frac{4}{9}$ $51\frac{5}{9}$	9. $7\times 12\frac{7}{16}$ $87\frac{1}{16}$	14. $10\times 9\frac{7}{16}$ $94\frac{3}{8}$
5. $7\times 8\frac{6}{7}$ 62	10. $8\times 9\frac{4}{7}$ $76\frac{4}{7}$	15. $12\times 12\frac{7}{8}$ $154\frac{1}{2}$

16. A beam is made by nailing together 3 planks each $1\frac{3}{4}$ inches thick. How thick is the beam? $5\frac{1}{4}''$

17. Each volume of an encyclopedia is $2\frac{1}{8}$ inches thick. How many inches long must a shelf be to hold 24 such volumes? $51''$

18. A man steps on an average $2\frac{4}{5}$ feet. How far does he go if he takes 2480 steps? Is this more or less than one mile? $6944'$; more

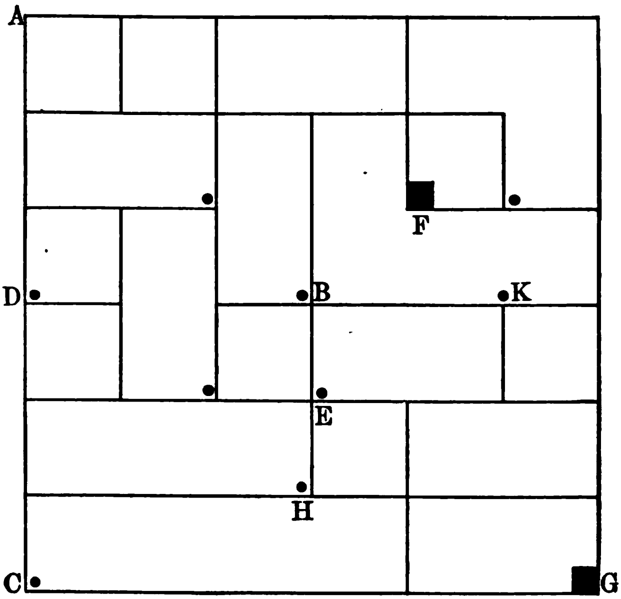
19. If you step $2\frac{2}{5}$ feet, how far will you go if you take 2400 steps? Is this more or less than a mile? how much? $5760'$; more by $480'$

20. A wagon wheel goes $17\frac{1}{3}$ feet in making one complete revolution. How far does this wheel go in making 300 revolutions? Is this more or less than one mile? how much? $5200'$; less by $80'$

21. In a recent year the average yield of wheat in the United States was $15\frac{7}{8}$ bushels per acre. At this rate, how many bushels are raised on a field of 65 acres? $1031\frac{7}{8}$ bu.

22. A man farming on shares gets $\frac{2}{3}$ of the crops. What is his share of a wheat crop of 1145 bushels? $763\frac{1}{3}$ bu.

A MAP



1. With a ruler measure the distances between the following points: A and C, A and D, A and F, and C and F. In each case disregard fractions of an inch less than $\frac{1}{8}$. **$3''$; $1\frac{1}{2}''$; $2\frac{1}{4}''$; $2\frac{3}{4}''$**

2. If one inch on this map represents 2 miles of country, find the actual distances from A to C, from A to D, and from C to F. Notice that these are the distances in straight lines across the fields. **6 miles; 3 miles; $5\frac{1}{2}$ miles**

3. On this same map find the distances between A and G, A and B, A and H, using the same scale as in problem 2. **$8\frac{1}{2}$ miles; $4\frac{1}{2}$ miles; $5\frac{3}{4}$ miles**

4. On a map of your state find how many miles are represented by one inch. Measure the distance on the map between two places you know. Then figure out how many miles there are between these places.

50. Product of a Mixed Number and a Fraction. The following example shows the process:

Example. Multiply $2\frac{1}{3}$ by $\frac{4}{5}$.

$$\begin{aligned}\tfrac{4}{5}\times 2 &= \tfrac{8}{5} = 1\tfrac{3}{5} = 1\tfrac{9}{15}\\ \tfrac{4}{5}\times\tfrac{1}{3} &= \tfrac{4}{15} = \underline{\tfrac{4}{15}}\\ &\ \ 1\tfrac{13}{15}\end{aligned}$$

First multipy 2 by $\frac{4}{5}$, and then multiply $\frac{1}{3}$ by $\frac{4}{5}$. Add the products.

WRITTEN EXERCISES

Multiply the following:

1. $1\frac{3}{4}\times\frac{2}{3}$ $1\frac{1}{6}$	4. $9\frac{1}{3}\times\frac{2}{5}$ $3\frac{11}{15}$	7. $6\frac{2}{3}\times\frac{1}{2}$ $3\frac{1}{3}$	10. $8\frac{3}{7}\times\frac{3}{8}$ $3\frac{9}{56}$
2. $4\frac{1}{4}\times\frac{3}{4}$ $3\frac{3}{16}$	5. $6\frac{1}{2}\times\frac{3}{4}$ $4\frac{7}{8}$	8. $9\frac{1}{4}\times\frac{3}{5}$ $5\frac{11}{20}$	11. $9\frac{3}{4}\times\frac{2}{5}$ $3\frac{9}{10}$
3. $2\frac{4}{5}\times\frac{3}{7}$ $1\frac{1}{5}$	6. $7\frac{1}{3}\times\frac{2}{3}$ $4\frac{8}{9}$	9. $7\frac{3}{4}\times\frac{5}{6}$ $6\frac{11}{24}$	12. $7\frac{7}{8}\times\frac{4}{5}$ $6\frac{3}{10}$

51. Product of two Mixed Numbers. To multiply one mixed number by another, it is usually best to reduce both to improper fractions. If the numbers are large it is best, however, to use the four-step method. (See example 2 below.)

Example 1. Multiply $2\frac{3}{4}$ by $3\frac{1}{3}$.

$$2\tfrac{3}{4} = \tfrac{11}{4}$$
$$3\tfrac{1}{3} = \tfrac{10}{3}$$
$$\tfrac{11}{4}\times\tfrac{10}{3} = \tfrac{55}{6} = 9\tfrac{1}{6}.$$

Reduce $2\frac{3}{4}$ and $3\frac{1}{3}$ to improper fractions, and then multiply.

Example 2. Multiply $32\frac{1}{2}$ by $8\frac{2}{3}$.

$$\begin{aligned} &32\tfrac{1}{2}\\ &\underline{\ \ 8\tfrac{2}{3}}\\ 8\times 32 &= 256\\ \tfrac{2}{3}\times 32 &= \ \ 21\tfrac{1}{3}\\ 8\times\tfrac{1}{2} &= \ \ \ \ 4\\ \tfrac{2}{3}\times\tfrac{1}{2} &= \ \ \ \ \underline{\tfrac{1}{3}}\\ &\ \ 281\tfrac{2}{3}\end{aligned}$$

(Four-step process)

(1) Multiply the integers ($8\times 32=256$)

(2) Multiply 32 by $\frac{2}{3}$ ($\frac{2}{3}\times 32=21\frac{1}{3}$)

(3) Multiply $\frac{1}{2}$ by 8 ($8\times\frac{1}{2}=4$)

(4) Multiply $\frac{1}{2}$ by $\frac{2}{3}$ ($\frac{2}{3}\times\frac{1}{2}=\frac{1}{3}$)

Add the partial products.

Also solve this problem by reducing to improper fractions and compare results.

WRITTEN EXERCISES

Using either of the processes on page 70, find the products of the following:

1. $1\frac{1}{4} \times 3\frac{2}{3}$ $4\frac{7}{12}$	5. $8\frac{3}{5} \times 6\frac{4}{9}$ $55\frac{19}{45}$	9. $8\frac{3}{4} \times 6\frac{1}{3}$ $55\frac{5}{12}$
2. $6\frac{3}{4} \times 3\frac{2}{3}$ $24\frac{3}{4}$	6. $4\frac{1}{3} \times 5\frac{1}{5}$ $22\frac{8}{15}$	10. $2\frac{1}{4} \times 5\frac{1}{3}$ 12
3. $9\frac{6}{7} \times 7\frac{3}{8}$ $72\frac{39}{56}$	7. $6\frac{1}{2} \times 7\frac{1}{3}$ $47\frac{2}{3}$	11. $7\frac{2}{3} \times 4\frac{1}{5}$ $32\frac{1}{5}$
4. $4\frac{1}{3} \times 2\frac{3}{5}$ $11\frac{4}{15}$	8. $3\frac{2}{3} \times 8\frac{1}{4}$ $30\frac{1}{4}$	12. $10\frac{4}{9} \times 6\frac{5}{8}$ $69\frac{7}{36}$

13. At \$6.00 a week how much does a servant get for a month of 30 days? (30 days is $4\frac{2}{7}$ weeks.) How much does she get for a month of 31 days? **\$$25\frac{5}{7}$; \$$26\frac{4}{7}$; or \$25.71; \$26.57**

14. How many minutes are there in one hour? 30 minutes is what fraction of an hour? 15 minutes? 20 minutes? 10 minutes? 40 minutes? 45 minutes? **60; $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{3}$; $\frac{1}{6}$; $\frac{2}{3}$; $\frac{3}{4}$**

15. Express each of the following as hours and fractions of hours: 75 minutes, 90 minutes, 105 minutes. **$1\frac{1}{4}$; $1\frac{1}{2}$; $1\frac{3}{4}$**

16. If a child spends 45 minutes each school day on arithmetic, how many hours does the child spend on arithmetic in one school year of 200 days? (First reduce 45 minutes to a fraction of an hour.) **150 hours**

17. If a child spends 90 minutes of each school day on reading and language, how many hours does the child spend on reading and language in one school year of 200 days? (First reduce 90 minutes to one hour and a fraction of an hour.) **300 hours**

18. From your school program find how many minutes you spend each day on each of your subjects. Then find how many hours you spend on each subject in a school year. (First find how many days you go to school in one year.)

Drill in Fundamentals. Play game No. 4 on page 30. Use examples in multiplication of whole numbers.

52. **Dividing Numbers of the Same Kind.** To divide two numbers of the same kind we simply divide one of the numbers by the other, paying no attention to the kind of number, as shown in the following examples:

Example 1. A farmer hauls 50 bushels of wheat in one load. How many loads are there in his whole crop of 4350 bushels?

Solution:

$$50\overline{)4350} = 87$$

We divide 4350 by 50, getting a quotient of 87, which is the number of loads.

Example 2. Divide $\frac{7}{20}$ by $\frac{2}{20}$.

$\frac{7}{20} \div \frac{2}{20} = 7 \div 2 = \frac{7}{2} = 3\frac{1}{2}$. Since $\frac{7}{20}$ and $\frac{2}{20}$ are numbers of the same kind (twentieths) we simply divide 7 by 2.

53. **Dividing Numbers of Different Kinds.** If the dividend and divisor are different kinds of numbers, they must be changed to numbers of the same kind before dividing, as in the following examples:

Example 1. How many badges, each 7 inches long, may be cut from 12 yards of ribbon?

12 yards $= 36 \times 12$ inches, or 432 inches.

$7\overline{)432}$ = 61, remainder 5.

In this case we must reduce the 12 yards to inches before dividing. That is, the dividend and divisor must be reduced to numbers of the same kind. We then find by dividing that 61 badges may be cut, leaving a piece 5 inches long.

Example 2. Divide $\frac{3}{4}$ by $\frac{2}{5}$.

$\frac{3}{4} \div \frac{2}{5} = \frac{15}{20} \div \frac{8}{20}$.
$15 \div 8 = 1\frac{7}{8}$.

Reducing $\frac{3}{4}$ and $\frac{2}{5}$ to a common denominator we have $\frac{15}{20} \div \frac{8}{20}$. Since these numbers are of the same kind, we divide 15 by 8.

WRITTEN EXERCISES

Using the method of the last example on page 72, find the quotients of the following:

1. $\frac{2}{3}\div\frac{3}{4}$ $\frac{8}{9}$	3. $\frac{1}{4}\div\frac{1}{3}$ $\frac{3}{4}$	5. $\frac{1}{4}\div\frac{2}{3}$ $\frac{3}{8}$	7. $\frac{1}{2}\div\frac{3}{5}$ $\frac{5}{6}$
2. $\frac{1}{3}\div\frac{1}{4}$ $1\frac{1}{3}$	4. $\frac{1}{2}\div\frac{1}{3}$ $1\frac{1}{2}$	6. $\frac{3}{5}\div\frac{1}{2}$ $1\frac{1}{5}$	8. $\frac{3}{8}\div\frac{3}{4}$ $\frac{1}{2}$

54. Dividing by a Fraction. A convenient method for dividing by a fraction is given in the following rule:

> *To divide by a fraction, invert the terms of the divisor and multiply.*

Thus, in the first example in the exercises above, instead of reducing $\frac{2}{3}\div\frac{3}{4}$ to $\frac{8}{12}\div\frac{9}{12}$ and then dividing 8 by 9, getting $\frac{8}{9}$, we may invert the terms of $\frac{3}{4}$ and multiply. That is, $\frac{2}{3}\div\frac{3}{4}=\frac{2}{3}\times\frac{4}{3}=\frac{8}{9}$.

Solve all of the above exercises in these two ways to see that they give the same result.

The same rule may also be used to divide by a whole number:

Thus, $\frac{3}{5}\div 2=\frac{3}{5}\times\frac{1}{2}=\frac{3}{10}$, and $\frac{4}{7}\div 2=\frac{4}{7}\times\frac{1}{2}=\frac{2}{7}$.

In practice, however, the last example would be solved by dividing the numerator 4 by 2 directly, writing $\frac{4}{7}\div 2=\frac{2}{7}$.

WRITTEN EXERCISES

Divide:

1. $\frac{3}{4}\div\frac{3}{5}$ $1\frac{1}{4}$	7. $\frac{6}{7}\div\frac{2}{5}$ $2\frac{1}{7}$	13. $\frac{9}{16}\div\frac{5}{8}$ $\frac{9}{10}$	19. $\frac{5}{8}\div\frac{16}{9}$ $\frac{45}{128}$
2. $\frac{4}{5}\div\frac{3}{4}$ $1\frac{1}{15}$	8. $\frac{4}{9}\div\frac{2}{3}$ $\frac{2}{3}$	14. $\frac{7}{12}\div\frac{5}{18}$ $2\frac{1}{10}$	20. $\frac{5}{16}\div\frac{3}{8}$ $\frac{5}{6}$
3. $\frac{2}{3}\div\frac{3}{7}$ $1\frac{5}{9}$	9. $\frac{3}{8}\div\frac{5}{16}$ $1\frac{1}{5}$	15. $\frac{11}{15}\div\frac{3}{5}$ $1\frac{2}{9}$	21. $\frac{5}{18}\div\frac{7}{12}$ $\frac{10}{21}$
4. $\frac{2}{5}\div\frac{1}{3}$ $1\frac{1}{5}$	10. $\frac{5}{9}\div\frac{3}{7}$ $1\frac{8}{27}$	16. $\frac{7}{16}\div\frac{3}{8}$ $1\frac{1}{6}$	22. $\frac{3}{5}\div\frac{11}{15}$ $\frac{9}{11}$
5. $\frac{3}{7}\div\frac{2}{3}$ $\frac{9}{14}$	11. $\frac{7}{8}\div\frac{9}{13}$ $1\frac{19}{72}$	17. $\frac{3}{32}\div\frac{5}{16}$ $\frac{3}{10}$	23. $\frac{3}{8}\div\frac{7}{16}$ $\frac{6}{7}$
6. $\frac{3}{8}\div\frac{1}{4}$ $1\frac{1}{2}$	12. $\frac{5}{16}\div\frac{3}{32}$ $3\frac{1}{3}$	18. $\frac{9}{13}\div\frac{7}{8}$ $\frac{72}{91}$	24. $\frac{5}{9}\div\frac{6}{7}$ $\frac{35}{54}$

55. Dividing a Fraction by an Integer. In some cases it is simpler to use a direct method for dividing a fraction by an integer.

From $\frac{4}{7} \div 2 = \frac{2}{7}$, and $\frac{3}{7} \div 2 = \frac{3}{14}$, try to formulate a rule for dividing a fraction by an integer.

ORAL EXERCISES

Find the quotients of the following in the simplest way:

1. $\frac{4}{5} \div 2$	3. $\frac{6}{7} \div 3$	5. $\frac{5}{8} \div 2$	7. $\frac{7}{8} \div 3$
2. $\frac{3}{7} \div 4$	4. $\frac{5}{7} \div 3$	6. $\frac{9}{16} \div 6$	8. $\frac{8}{15} \div 4$

9. Instead of the divisors $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{3}{7}$, $\frac{9}{10}$, $\frac{7}{16}$ what multipliers may be used?

ORAL AND WRITTEN EXERCISES

Solve as many as possible orally:

1. $27 \div \frac{3}{5}$	45	12. $72 \div \frac{3}{5}$	120	23. $\frac{3}{8} \div \frac{5}{16}$	$1\frac{1}{5}$	34. $\frac{9}{16} \div \frac{1}{16}$	9
2. $15 \div \frac{3}{4}$	20	13. $72 \div \frac{9}{11}$	88	24. $\frac{4}{5} \div \frac{1}{2}$	$1\frac{3}{5}$	35. $\frac{13}{16} \div \frac{1}{15}$	$12\frac{3}{16}$
3. $8 \div \frac{3}{5}$	$13\frac{1}{3}$	14. $84 \div \frac{7}{10}$	120	25. $\frac{3}{8} \div \frac{1}{3}$	$1\frac{1}{8}$	36. $\frac{7}{8} \div \frac{1}{8}$	7
4. $7 \div \frac{4}{5}$	$8\frac{3}{4}$	15. $20 \div \frac{2}{5}$	50	26. $\frac{7}{10} \div \frac{1}{9}$	$6\frac{3}{10}$	37. $\frac{2}{3} \div \frac{4}{5}$	$\frac{5}{6}$
5. $63 \div \frac{3}{7}$	147	16. $24 \div \frac{5}{6}$	$28\frac{4}{5}$	27. $\frac{5}{16} \div \frac{1}{12}$	$3\frac{3}{4}$	38. $\frac{3}{7} \div \frac{2}{9}$	$1\frac{13}{14}$
6. $54 \div \frac{5}{6}$	$64\frac{4}{5}$	17. $30 \div \frac{3}{7}$	70	28. $\frac{3}{16} \div \frac{9}{10}$	$\frac{5}{24}$	39. $\frac{7}{8} \div \frac{7}{16}$	2
7. $27 \div \frac{9}{10}$	30	18. $45 \div \frac{9}{10}$	50	29. $\frac{4}{16} \div \frac{15}{32}$	$\frac{8}{15}$	40. $\frac{13}{16} \div \frac{5}{32}$	$5\frac{1}{5}$
8. $9 \div \frac{3}{4}$	12	19. $64 \div \frac{4}{7}$	112	30. $\frac{4}{5} \div \frac{1}{3}$	$2\frac{2}{5}$	41. $\frac{5}{7} \div \frac{3}{8}$	$1\frac{19}{21}$
9. $12 \div \frac{4}{5}$	15	20. $54 \div \frac{9}{10}$	60	31. $\frac{5}{8} \div \frac{3}{7}$	$1\frac{11}{24}$	42. $\frac{3}{8} \div \frac{5}{7}$	$\frac{21}{40}$
10. $12 \div \frac{5}{6}$	$14\frac{2}{5}$	21. $48 \div \frac{6}{7}$	56	32. $\frac{7}{8} \div \frac{3}{32}$	$9\frac{1}{3}$	43. $\frac{6}{7} \div \frac{3}{5}$	$1\frac{3}{7}$
11. $16 \div \frac{3}{4}$	$21\frac{1}{3}$	22. $\frac{2}{3} \div \frac{3}{4}$	$\frac{8}{9}$	33. $\frac{7}{32} \div \frac{1}{11}$	$2\frac{13}{32}$	44. $\frac{9}{16} \div \frac{4}{5}$	$\frac{45}{64}$

Drill in Fundamentals. Play game No. 2, page 30. Use examples in division of fractions.

56. Dividing by a Mixed Number. If the divisor is a mixed number, reduce it to an improper fraction and then divide.

Example 1. Divide 34 by $3\frac{4}{5}$.

$3\frac{4}{5}=\frac{19}{5}$

$34\div\frac{19}{5}=34\times\frac{5}{19}=\frac{170}{19}=8\frac{18}{19}$

Reduce $3\frac{4}{5}$ to $\frac{19}{5}$. Then $34\div 3\frac{4}{5}=34\div\frac{19}{5}$.

Example 2. Divide $18\frac{3}{4}$ by $2\frac{1}{3}$. $2\frac{1}{3}=\frac{7}{3}$. Then $18\frac{3}{4}\div 2\frac{1}{3}=18\frac{3}{4}\times\frac{3}{7}$.

$\frac{3}{7}\times 18=\frac{54}{7}=7\frac{5}{7}$

$\frac{3}{7}\times\frac{3}{4}=\quad\frac{9}{28}$

$7\frac{29}{28}=8\frac{1}{28}$

Reduce $2\frac{1}{3}$ to $\frac{7}{3}$. Then we need to divide by $\frac{7}{3}$ or multiply by $\frac{3}{7}$. To multiply $18\frac{3}{4}$ by $\frac{3}{7}$ use the method of the first example on page 70.

WRITTEN EXERCISES

In this manner obtain the results in the following:

1. $7\frac{1}{3}\div\frac{3}{5}$ $12\frac{2}{9}$
2. $9\frac{9}{10}\div 3\frac{2}{3}$ $2\frac{7}{10}$
3. $8\frac{1}{3}\div 6\frac{1}{4}$ $1\frac{1}{3}$
4. $11\frac{2}{3}\div 7\frac{1}{2}$ $1\frac{5}{9}$
5. $12\frac{1}{2}\div\frac{5}{8}$ 20
6. $8\frac{2}{5}\div\frac{3}{7}$ $19\frac{3}{5}$
7. $9\frac{4}{5}\div\frac{7}{10}$ 14
8. $11\frac{4}{7}\div 4\frac{1}{2}$ $2\frac{4}{7}$
9. $6\frac{2}{3}\div 4\frac{1}{7}$ $1\frac{53}{87}$

57. Dividing a Mixed Number by an Integer. If the divisor is an integer we may proceed as in the following:

Example Divide $27\frac{2}{3}$ by 4.

$27\div 4=6$, remainder 3.

$3\frac{2}{3}\div 4=\frac{11}{3}\div 4=\frac{11}{12}$.

Hence, $27\frac{2}{3}\div 4=6\frac{11}{12}$.

4 is contained 6 times in 27, with remainder 3. Then divide $3\frac{2}{3}$ by 4.

In this manner obtain the results in the following:

1. $36\frac{2}{3}\div 5$ $7\frac{1}{3}$
2. $67\frac{1}{2}\div 9$ $7\frac{1}{2}$
3. $76\frac{2}{3}\div 8$ $9\frac{7}{12}$
4. $84\frac{3}{7}\div 9$ $9\frac{8}{21}$
5. $28\frac{2}{3}\div 6$ $4\frac{7}{9}$
6. $43\frac{5}{6}\div 5$ $8\frac{23}{30}$
7. $71\frac{1}{4}\div 6$ $11\frac{7}{8}$
8. $35\frac{3}{5}\div 8$ $4\frac{9}{20}$
9. $56\frac{4}{5}\div 6$ $9\frac{7}{15}$
10. $71\frac{3}{16}\div 8$ $8\frac{115}{128}$
11. $45\frac{7}{8}\div 9$ $5\frac{7}{72}$
12. $59\frac{5}{8}\div 7$ $8\frac{29}{56}$

58. Division of Fractions Related to Multiplication. By the definition of division, dividing 12 by 4 is the same as finding the missing number in $4 \times ? = 12$.

Similarly, to divide 6 by $\frac{1}{3}$ is the same as to find the missing number in $\frac{1}{3}$ of $? = 6$, or $\frac{1}{3} \times ? = 6$.

The problem in $\frac{1}{3} \times ? = 6$ is this: Find a number such that $\frac{1}{3}$ of (or times) it equals 6.

To divide 12 by $\frac{2}{3}$ is the same as to find the missing number in $\frac{2}{3} \times ? = 12$.

The problem stated in words is: "Find a number such that $\frac{2}{3}$ of (or times) it equals 12."

This problem may also be stated: "12 is $\frac{2}{3}$ of what number?"

To divide $\frac{2}{3}$ by $\frac{3}{4}$ is the same as finding the missing number in $\frac{3}{4} \times ? = \frac{2}{3}$.

The problem stated in words is: "Find a number such that $\frac{3}{4}$ of (or times) it equals $\frac{2}{3}$."

This problem may also be stated: "$\frac{2}{3}$ is $\frac{3}{4}$ of what number?"

WRITTEN EXERCISES

Find the missing numbers in the following, and state each problem in words:

1. $\frac{1}{3} \times ? = \frac{5}{6}$ $2\frac{1}{2}$
2. $\frac{3}{4} \times ? = \frac{3}{7}$ $\frac{4}{7}$
3. $\frac{1}{5} \times ? = \frac{2}{3}$ $3\frac{1}{3}$
4. $\frac{3}{4} \times ? = \frac{7}{8}$ $1\frac{1}{6}$
5. $\frac{3}{7} \times ? = 12$ 28
6. $\frac{5}{8} \times ? = 10$ 16
7. $\frac{5}{6} \times ? = \frac{5}{8}$ $\frac{3}{4}$
8. $\frac{2}{7} \times ? = \frac{3}{5}$ $2\frac{1}{10}$
9. $\frac{5}{8} \times ? = \frac{7}{8}$ $1\frac{2}{5}$
10. $\frac{3}{5} \times ? = 3\frac{1}{2}$ $5\frac{5}{6}$
11. $1\frac{1}{2} \times ? = 3\frac{3}{4}$ $2\frac{1}{2}$
12. $4\frac{3}{4} \times ? = 7\frac{1}{2}$ $1\frac{11}{19}$

Drill in Fundamentals. Play game No. 3, page 30. Use examples in multiplication of integers for the first event, and examples in long division for the second event.

59. **The Three Problems in Fractions.** By leaving out in succession each of the numbers in $3\times 4=12$, we obtain three distinct problems which are solved by supplying the missing numbers in the following:

(1) $3\times 4=?$ (2) $3\times ?=12$ (3) $?\times 4=12$

Precisely similar problems occur in fractions.

Thus, from $\frac{2}{3}\times\frac{5}{7}=\frac{10}{21}$ we obtain three problems which are solved by finding the missing number in

(1) $\frac{2}{3}\times\frac{5}{7}=?$ (2) $\frac{2}{3}\times ?=\frac{10}{21}$ (3) $?\times\frac{5}{7}=\frac{10}{21}$

It is of fundamental importance that we understand the simplicity of these problems.

$3\times ?=12$	(I) $\frac{2}{3}\times ?=\frac{10}{21}$
Therefore $?=12\div 3=4$	Therefore $?=\frac{10}{21}\div\frac{2}{3}=\frac{5}{7}$
$?\times 4=12$	(II) $?\times\frac{5}{7}=\frac{10}{21}$
Therefore $?=12\div 4=3$	Therefore $?=\frac{10}{21}\div\frac{5}{7}=\frac{2}{3}$

We now recall that $\frac{2}{3}\times\frac{5}{7}$ means the same as $\frac{2}{3}$ of $\frac{5}{7}$. Hence problems I and II may be read as follows:

(I) $\frac{2}{3}$ of what number equals $\frac{10}{21}$? or $\frac{10}{21}$ is $\frac{2}{3}$ of what number?

(II) What fraction (or part) of $\frac{5}{7}$ equals $\frac{10}{21}$? or $\frac{10}{21}$ is what fraction (or part) of $\frac{5}{7}$?

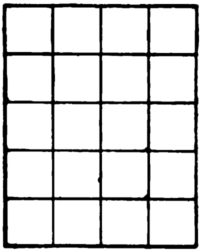
Sometimes the form of the statement of one of these problems as it turns up in practice may be confusing. A careful study of the statements given above will be of great help.

If you know how to add, subtract, multiply, and divide fractions, and if you understand thoroughly pages 76 and 77 of this book, you will seldom meet with a problem in fractions that you can not solve.

WRITTEN EXERCISES

Make examples like (I) and (II) above and solve them.

ORAL EXERCISES



1. Point out $\frac{1}{4}$ of this figure, $\frac{1}{5}$ of it, $\frac{1}{2}$ of it, $\frac{3}{4}$ of it, $\frac{2}{5}$ of it, $\frac{3}{5}$ of it, $\frac{4}{5}$ of it, $\frac{1}{10}$ of it, $\frac{7}{10}$ of it, $\frac{9}{10}$ of it. How many 20ths of the figure are there in each?

2. Point out the sum of $\frac{1}{4}$ and $\frac{1}{5}$. This sum is what fractional part of the figure?

3. Point out the sum of $\frac{1}{2}$ and $\frac{1}{5}$. This sum is what fractional part of the figure?

Read the following and give the sum of each:

4. $\frac{1}{2}+\frac{1}{4}$
5. $\frac{1}{2}+\frac{3}{4}$
6. $\frac{1}{2}+\frac{1}{8}$
7. $\frac{1}{2}+\frac{3}{8}$
8. $\frac{1}{2}+\frac{5}{8}$
9. $\frac{1}{2}+\frac{7}{8}$
10. $\frac{1}{4}+\frac{1}{8}$
11. $\frac{1}{4}+\frac{3}{8}$
12. $\frac{1}{4}+\frac{5}{8}$
13. $\frac{1}{4}+\frac{7}{8}$
14. $\frac{3}{4}+\frac{1}{8}$
15. $\frac{3}{4}+\frac{3}{8}$
16. $\frac{3}{4}+\frac{5}{8}$
17. $\frac{3}{4}+\frac{7}{8}$
18. $\frac{1}{2}+\frac{1}{16}$
19. $\frac{1}{2}+\frac{3}{16}$
20. $\frac{1}{2}+\frac{5}{16}$
21. $\frac{1}{2}+\frac{7}{16}$
22. $\frac{1}{2}+\frac{9}{16}$
23. $\frac{1}{2}+\frac{11}{16}$
24. $\frac{1}{2}+\frac{13}{16}$
25. $\frac{1}{2}+\frac{15}{16}$
26. $\frac{1}{4}+\frac{1}{16}$
27. $\frac{1}{4}+\frac{3}{16}$
28. $\frac{1}{4}+\frac{5}{16}$
29. $\frac{1}{4}+\frac{7}{16}$
30. $\frac{1}{4}+\frac{9}{16}$
31. $\frac{1}{4}+\frac{11}{16}$
32. $\frac{1}{4}+\frac{13}{16}$
33. $\frac{1}{4}+\frac{15}{16}$
34. $\frac{3}{4}+\frac{1}{16}$
35. $\frac{3}{4}+\frac{3}{16}$
36. $\frac{3}{4}+\frac{5}{16}$
37. $\frac{3}{4}+\frac{7}{16}$
38. $\frac{1}{8}+\frac{1}{16}$
39. $\frac{1}{8}+\frac{3}{16}$
40. $\frac{1}{8}+\frac{5}{16}$
41. $\frac{1}{8}+\frac{7}{16}$
42. $\frac{1}{8}+\frac{9}{16}$
43. $\frac{1}{8}+\frac{11}{16}$
44. $\frac{1}{8}+\frac{13}{16}$
45. $\frac{1}{8}+\frac{15}{16}$
46. $\frac{3}{8}+\frac{1}{16}$
47. $\frac{3}{8}+\frac{3}{16}$
48. $\frac{3}{8}+\frac{5}{16}$
49. $\frac{3}{8}+\frac{7}{16}$
50. $\frac{3}{8}+\frac{9}{16}$
51. $\frac{3}{8}+\frac{11}{16}$
52. $\frac{3}{8}+\frac{13}{16}$
53. $\frac{3}{8}+\frac{15}{16}$
54. $\frac{5}{8}+\frac{1}{16}$
55. $\frac{5}{8}+\frac{3}{16}$
56. $\frac{5}{8}+\frac{5}{16}$
57. $\frac{5}{8}+\frac{7}{16}$
58. $\frac{5}{8}+\frac{9}{16}$
59. $\frac{5}{8}+\frac{11}{16}$
60. $\frac{5}{8}+\frac{13}{16}$
61. $\frac{5}{8}+\frac{15}{16}$
62. $\frac{7}{8}+\frac{3}{16}$
63. $\frac{7}{8}+\frac{5}{16}$

64. State which of the two fractions in each of the above is the smaller, and subtract the smaller from the larger.

WRITTEN EXERCISES

Multiply:

1. $7\frac{3}{4} \times 8\frac{1}{3}$ $64\frac{7}{12}$
2. $9\frac{7}{12} \times \frac{13}{24}$ $5\frac{55}{288}$
3. $12\frac{4}{9} \times \frac{15}{16}$ $11\frac{2}{3}$
4. $11\frac{3}{5} \times 9\frac{4}{9}$ $109\frac{5}{9}$
5. $12\frac{3}{10} \times 6\frac{3}{8}$ $78\frac{33}{80}$
6. $15\frac{3}{4} \times 7\frac{1}{9}$ 112
7. $8\frac{2}{3} \times 1\frac{1}{2}$ 13
8. $12\frac{1}{7} \times 9\frac{4}{5}$ 119
9. $5\frac{3}{8} \times 2\frac{4}{5}$ $15\frac{1}{20}$
10. $7\frac{5}{6} \times 3\frac{3}{5}$ $28\frac{1}{5}$
11. $10\frac{2}{3} \times 6\frac{1}{4}$ $66\frac{2}{3}$
12. $8\frac{5}{8} \times 2\frac{3}{4}$ $23\frac{23}{32}$

Divide:

13. $8\frac{5}{9} \div \frac{3}{11}$ $31\frac{10}{27}$
14. $12\frac{3}{5} \div \frac{9}{10}$ 14
15. $15\frac{5}{6} \div 6\frac{1}{3}$ $2\frac{1}{2}$
16. $9\frac{3}{5} \div 3\frac{1}{5}$ 3
17. $16\frac{2}{3} \div 12\frac{1}{2}$ $1\frac{1}{3}$
18. $9\frac{3}{8} \div 6\frac{1}{4}$ $1\frac{1}{2}$
19. $10\frac{2}{7} \div 3\frac{3}{7}$ 3
20. $8\frac{5}{9} \div 5\frac{1}{2}$ $1\frac{5}{9}$
21. $21\frac{1}{2} \div 3\frac{3}{4}$ $5\frac{11}{15}$
22. $32\frac{2}{3} \div 6\frac{4}{5}$ $4\frac{41}{51}$
23. $52\frac{1}{2} \div 4\frac{1}{4}$ $12\frac{6}{17}$
24. $17\frac{3}{8} \div 3\frac{1}{3}$ $5\frac{17}{80}$

Find the results of the following:

25. $9\frac{3}{4} \times 7\frac{2}{13}$ $69\frac{3}{4}$
26. $20\frac{1}{2} \div 5\frac{6}{7}$ $3\frac{1}{2}$
27. $10\frac{5}{6} \div 6\frac{1}{2}$ $1\frac{2}{3}$
28. $16\frac{2}{3} \times 4\frac{1}{5}$ 70
29. $15\frac{3}{4} \div 4\frac{1}{2}$ $3\frac{1}{2}$
30. $12\frac{3}{5} \times 5\frac{5}{9}$ 70
31. $25\frac{3}{4} \times 12\frac{1}{2}$ $321\frac{7}{8}$
32. $21\frac{3}{5} \times 4\frac{4}{9}$ 96
33. $17\frac{3}{5} \div 5\frac{1}{2}$ $3\frac{1}{5}$
34. $20\frac{1}{4} \div 3\frac{1}{9}$ $6\frac{57}{112}$
35. $18\frac{4}{5} \div 12\frac{1}{2}$ $1\frac{63}{125}$
36. $20\frac{1}{4} \div 27$ $\frac{3}{4}$
37. $17\frac{3}{8} \times 3\frac{1}{3}$ $57\frac{11}{12}$
38. $12\frac{1}{7} \div 9\frac{4}{5}$ $1\frac{82}{343}$
39. $21\frac{3}{5} \div 4\frac{4}{9}$ $4\frac{43}{50}$
40. $20\frac{1}{2} \times 5\frac{6}{7}$ $120\frac{1}{14}$
41. $27\frac{1}{3} \div 3\frac{1}{9}$ $8\frac{11}{14}$
42. $81\frac{1}{2} \div 8\frac{2}{3}$ $9\frac{21}{52}$

Since a division may be indicated by a fraction, as in $7 \div 8 = \frac{7}{8}$, we have $1\frac{1}{3} \div 2\frac{1}{2} = \dfrac{1\frac{1}{3}}{2\frac{1}{2}}$

Find the value of each of the following:

43. $\dfrac{4\frac{3}{4}}{2\frac{3}{8}}$ 2
44. $\dfrac{6\frac{1}{2}}{4\frac{1}{4}}$ $1\frac{9}{17}$
45. $\dfrac{4\frac{3}{8}}{9\frac{1}{4}}$ $\frac{35}{74}$
46. $\dfrac{24}{6\frac{1}{3}}$ $3\frac{15}{19}$
47. $\dfrac{4\frac{1}{6}}{7\frac{1}{4}}$ $\frac{50}{87}$
48. $\dfrac{8\frac{3}{16}}{2\frac{1}{8}}$ $3\frac{29}{34}$
49. $\dfrac{7\frac{2}{3}}{4\frac{1}{4}}$ $1\frac{41}{51}$
50. $\dfrac{16\frac{3}{5}}{5\frac{1}{2}}$ $3\frac{1}{55}$

Review pages 28, 29. Walking at an average rate of $3\frac{1}{4}$ miles an hour, how far does a man go in 4 hours?

If you know the average speed and the time, how do you find the distance passed over?

If you know the distance an object has gone and the time it required, how do you find the average speed? State these rules carefully.

WRITTEN PROBLEMS

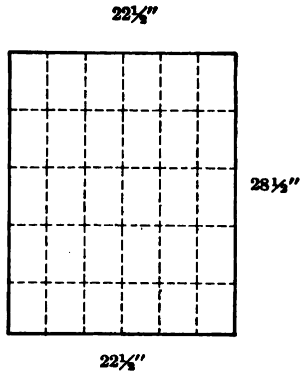
1. At an average speed of $46\frac{1}{2}$ miles an hour, how far will a train go in 3 hours and 45 minutes? (45 minutes is what fractional part of an hour?) $174\frac{3}{8}$ miles

2. In July 1904 a train on the Great Western Railway (England) travelled 1 hour and 24 minutes at the rate $84\frac{3}{5}$ miles per hour. How far did it go? (This is the highest speed at which a train has ever gone over 100 miles.) $118\frac{11}{25}$

Following are some fast runs of passenger trains for long distances. Find each speed in miles per hour. First reduce the time to hours and fractions of an hour.

Railroad	Terminals	Distance in miles	Time in hours and minutes
3. London & Northwestern	London Aberdeen	540	$63\frac{9}{32}$ 8h. 32m.
4. New York Central	New York Buffalo	$436\frac{1}{2}$	$65\frac{385}{397}$ 6h. 37m.
5. Lake Shore & Michigan Southern	Buffalo Chicago	525	$67\frac{1}{47}$ 7h. 50m.
6. Pennsylvania	Altoona Philadelphia	235	$67\frac{97}{209}$ 3h. 29m.

WRITTEN WORK



1. Souvenir postcards are cut from stock 22½" by 28½". If cards are cut from this stock as indicated in the figure, what will be the dimensions of the cards? **3¾"×5$\frac{7}{10}$"**

2. How many cubic feet are there in an excavation 24 feet wide, 36 feet long, and 5 feet deep? How many cubic yards are there in this excavation? **4320 cu. ft.; 160 cu. yds.**

3. Find the number of cubic yards in the excavation in example 2 by first reducing each dimension to yards. 5 feet are how many yards? **160 cu. yds.**

4. If an excavation contains 480 cubic yards, how many loads will it contain at 36 cubic feet to the load? (Note that 36 cubic feet = 1⅓ cubic yards.) **360 loads**

5. An excavation is 30 feet wide, 54 feet long, and 7 feet deep. How many cubic yards does it contain? **420 cu. yds.**

6. A box is 18 inches long, 16 inches wide, and 8 inches deep. How many cubic feet does it hold? **1⅓ cu. ft.**

Suggestion: First reduce 18 inches, 16 inches, and 8 inches to feet and fractions of feet.

7. On a certain screw (bolt) there are 5½ threads to the inch. What is the pitch of this thread? *Suggestion:* Divide 1 by 5½. See page 62. **$\frac{2}{11}$"**

8. If the pitch of a certain screw is $\frac{1}{18}$", how many times must the screw be turned around to make it enter into the wood its full length, which is 2¾"? **49½ times**

Drill in Fundamentals. Play game No. 3, page 30. Use examples in adding integers for each of two events.

CHAPTER II

60. Quotients to the Nearest Integer. Sometimes it is desired to get a quotient to the *nearest* integer, and to omit any reference to the remainder.

Example 1. Find the quotient of $388 \div 24$ to the nearest integer.

$$\begin{array}{r} 16 \\ 24\overline{)388} \\ \underline{24} \\ 148 \\ \underline{144} \\ 4 \end{array}$$

The exact quotient is $16\frac{4}{24} = 16\frac{1}{6}$. Since the fractional part of the quotient is less than $\frac{1}{2}$, the quotient is nearer 16 than 17. Hence 16 is the quotient to the nearest integer.

Example 2. Find the quotient of $574 \div 17$ to the nearest integer.

$$\begin{array}{r} 4 \\ 3\cancel{3} \\ 17\overline{)574} \\ \underline{51} \\ 64 \\ \underline{51} \\ 13 \end{array}$$

The exact quotient is $33\frac{13}{17}$. Since the fractional part of the quotient is greater than $\frac{1}{2}$, the quotient is nearer 34 than 33. Hence 34 is the quotient to the nearest integer

To find the quotient to the nearest integer, the following rule is used:

If the remainder is less than one-half the divisor, it is disregarded. If the remainder is equal to or greater than one-half the divisor one is added to the quotient.

WRITTEN EXERCISES

Find quotients to the nearest integer:

1. $346 \div 23$ 15	6. $378 \div 8$ 47	11. $948 \div 27$ 35
2. $742 \div 18$ 41	7. $946 \div 31$ 31	12. $894 \div 62$ 14
3. $498 \div 17$ 29	8. $478 \div 62$ 8	13. $798 \div 94$ 8
4. $642 \div 13$ 49	9. $482 \div 24$ 20	14. $327 \div 71$ 5
5. $798 \div 14$ 57	10. $678 \div 35$ 19	15. $698 \div 15$ 47

WRITTEN EXERCISES

Divide, finding each quotient to the nearest integer:

1. $8945 \div 76$ 118	6. $17806 \div 124$ 144	11. $24906 \div 203$ 123
2. $7896 \div 67$ 118	7. $18745 \div 102$ 184	12. $24089 \div 235$ 102
3. $9104 \div 83$ 110	8. $19608 \div 163$ 120	13. $36071 \div 287$ 126
4. $8759 \div 58$ 151	9. $16576 \div 178$ 93	14. $38047 \div 314$ 121
5. $7068 \div 69$ 102	10. $16075 \div 106$ 152	15. $81051 \div 291$ 279

The following rule is often useful in shortening division:

A common factor in the dividend and divisor may be cancelled without changing the quotient.

Example. Find the quotient of $6540000 \div 34000$ to the nearest integer.

$$\begin{array}{r}
192 \\
34\cancel{0}\cancel{0}\cancel{0})\overline{6540\cancel{0}\cancel{0}\cancel{0}} \\
\underline{34} \\
314 \\
\underline{306} \\
80 \\
\underline{68} \\
12
\end{array}$$

First divide both dividend and divisor by 1000 by striking off three zeros from each. Then divide 6540 by 34.

WRITTEN EXERCISES

Find quotients to the nearest integer:

1. $4890000 \div 371000$ 13	7. $98400000 \div 6420000$ 15
2. $5190000 \div 298000$ 17	8. $89700000 \div 5960000$ 15
3. $6780000 \div 427000$ 16	9. $79800000 \div 6350000$ 13
4. $5670000 \div 386000$ 15	10. $97800000 \div 7810000$ 13
5. $7610000 \div 597000$ 13	11. $87500000 \div 6940000$ 13
6. $8180000 \div 386000$ 21	12. $97800000 \div 5980000$ 16

61. Examples Involving Cancellation. Any problem in division may be expressed as a fraction. The fraction may then be simplified by cancellation. This is especially useful when the dividend or divisor or both are expressed as products.

Example 1. Divide 4×6 by 2.

Solution:

$$\frac{\overset{2}{\cancel{4}}\times 6}{\cancel{2}}=12$$

Example 2. Divide 3×4×8 by 18.

Solution:

$$\frac{\cancel{3}\times\overset{2}{\cancel{4}}\times\cancel{8}}{\underset{3}{\cancel{\cancel{18}}}}=\frac{2\times 8}{3}=\frac{16}{3}=5\tfrac{1}{3}$$

We first divide both terms of the fraction by 3 and then by 2.

Notice that dividing one factor by a number divides the whole product by that number.

Thus, in 3×4×8, dividing 3 by 3 divides the whole product by 3.

WRITTEN EXERCISES

Solve the following like Examples 1 and 2 above:

1. $\frac{5\times6\times7}{6}$ 35	6. $\frac{16\times3\times5}{8}$ 30	11. $\frac{6\times14\times18}{36}$ 42
2. $\frac{7\times8\times5}{4}$ 70	7. $\frac{5\times18\times6}{9}$ 60	12. $\frac{9\times10\times8}{54}$ $13\frac{1}{3}$
3. $\frac{3\times4\times10}{5}$ 24	8. $\frac{3\times27\times5}{9}$ 45	13. $\frac{3\times4\times6}{18}$ 4
4. $\frac{3\times5\times14}{7}$ 30	9. $\frac{32\times5\times4}{8}$ 80	14. $\frac{14\times16\times6}{42}$ 32
5. $\frac{5\times3\times21}{7}$ 45	10. $\frac{4\times12\times10}{60}$ 8	15. $\frac{4\times9\times12\times28}{140}$ $86\frac{2}{5}$

Example. Divide $24\times35\times11$ by $6\times7\times8$.

$$\frac{\overset{4}{\cancel{24}}\times\overset{5}{\cancel{35}}\times11}{\cancel{6}\times\cancel{7}\times\underset{2}{\cancel{8}}}=\frac{5\times11}{2}=\frac{55}{2}=27\tfrac{1}{2}$$

First cancel the common factor 6 in 24 and 6. Then cancel 4 in 8 and 4, and finally cancel 7 in 35 and 7. This leaves 5×11 in the dividend and 2 in the divisor.

Also solve without cancelling. Which method is shorter?

WRITTEN EXERCISES

In this manner obtain the results of the following by cancelling:

1. $\dfrac{32\times58\times70}{10\times14\times8}$ 116
2. $\dfrac{42\times39\times60}{13\times7\times10}$ 108
3. $\dfrac{36\times30\times54}{5\times12\times9}$ 108
4. $\dfrac{54\times36\times75}{12\times6\times16}$ $126\frac{9}{16}$
5. $\dfrac{72\times64\times85}{5\times24\times16}$ 204
6. $\dfrac{81\times75\times90}{20\times9\times15}$ $202\frac{1}{2}$
7. $\dfrac{25\times18\times12\times3}{27\times15\times9}$ $4\frac{4}{9}$
8. $\dfrac{28\times16\times20\times7}{34\times14\times5\times21}$ $1\frac{13}{51}$
9. $\dfrac{24\times20\times14\times4}{5\times12\times7\times16}$ 4
10. $\dfrac{48\times25\times14\times9}{15\times7\times24\times6}$ 10
11. $\dfrac{54\times22\times36\times8}{84\times18\times11\times12}$ $1\frac{5}{7}$
12. $\dfrac{42\times63\times24\times11}{33\times14\times27\times8}$ 7
13. $\dfrac{4\times7\times19\times36}{14\times38}$ 36
14. $\dfrac{5\times9\times13\times42}{26\times21}$ 45
15. $\dfrac{15\times6\times9\times39}{30\times52\times18}$ $1\frac{1}{8}$
16. $\dfrac{9\times8\times23\times15}{45\times46\times18\times16}$ $\frac{1}{24}$
17. $\dfrac{13\times18\times40\times9}{80\times52\times45\times27}$ $\frac{1}{60}$
18. $\dfrac{17\times19\times30\times11}{57\times85\times44\times16}$ $\frac{1}{32}$
19. $\dfrac{8\times12\times18}{16\times24\times6}$ $\frac{3}{4}$
20. $\dfrac{32\times26\times17}{48\times39\times14}$ $\frac{34}{63}$
21. $\dfrac{14\times18\times20}{35\times24\times8}$ $\frac{3}{4}$

Drill in Fundamentals. Play game No. 3, page 30. Use examples in subtraction of integers for the first event, examples in multiplication of integers for the second, and examples in long division for the third.

Problem 1. How many cubic yards are there in an excavation 36 feet long, 32 feet wide and 6 feet deep?

You have solved this kind of problem before by multiplying 36 by 32, this product by 6, and then dividing by 27. We will now solve the same problem by cancellation.

We are to divide $36\times32\times6$ by 27.

$$\frac{\overset{4}{\cancel{36}}\times32\times\overset{2}{\cancel{6}}}{\underset{\cancel{3}}{\cancel{27}}}=4\times32\times2=256,$$ which is the required number of cubic yards.

Problem 2. How many square feet are there in a rectangle 64 inches long and 45 inches wide?

The number of square inches, 64×45, is divided by the number of square inches in a square foot, or 12×12.

That is, $\dfrac{64\times45}{12\times12}$ is the required number of square feet.

Problem 3. How many cubic feet are there in a block of ice 15 inches thick, 54 inches wide, and 64 inches long.

Solution: We divide the number of cubic inches, $15\times54\times64$, in the block by the number of cubic inches in one cubic foot, or $12\times12\times12$.

That is, $\dfrac{15\times54\times64}{12\times12\times12}$ is the required number of cubic feet.

Complete the work in these two problems.

WRITTEN EXERCISES

1. Find the number of square feet in rectangles having the following dimensions:

28″×46″ $8\frac{17}{18}$ 34″×56″ $13\frac{2}{9}$ 18″×78″ $9\frac{3}{4}$ 21″×32″ $4\frac{2}{3}$

2. Find the number of cubic feet in rectangular solids having the following dimensions:

16″×18″×48″ 8 8″×15″×30″ $2\frac{1}{12}$ 15″×21″×27″ $4\frac{59}{64}$

If in $2\times3\times4=24$ we erase the 4 and write $2\times3\times?=24$ we have a problem in division. That is, we divide 24 by 2×3, or 6, and thus find the missing number.

If we erase the 3 and write $2\times?\times4=24$, we again have a problem in division. That is, we divide 24 by 2×4, or 8. Similarly, if we erase the 2 and write $?\times3\times4=24$ we also have a problem in division. That is, we divide 24 by 3×4, or 12.

Example 1. Find the missing number in $3\times8\times?=768$.

Solution: The missing number is $\dfrac{\overset{\overset{32}{\cancel{96}}}{\cancel{768}}}{\cancel{3}\times\cancel{8}}=32.$

Example 2. Find the missing number in $7\times13\times?=2870$.

Solution: The missing number$=\dfrac{\overset{410}{\cancel{2870}}}{\cancel{7}\times13}=\dfrac{410}{13}=31\frac{7}{13}$

In this case we can cancel the 7, but not the the 13. So we have to divide 410 by 13.

Example 3. Find the missing number in $13\times17\times?=27490$.
Here we can not cancel at all, so we must multiply 17 by 13, and then divide 27490 by the product.

WRITTEN EXERCISES

Find the missing number in each of the following, using cancellation where possible:

1. $3\times9\times?=378$ 14	6. $25\times6\times?=7980$ $53\frac{1}{5}$	11. $?\times26\times2=890$ $17\frac{3}{26}$
2. $?\times5\times7=230$ $6\frac{4}{7}$	7. $8\times4\times?=860$ $26\frac{7}{8}$	12. $?\times32\times15=1560$ $3\frac{1}{4}$
3. $12\times5\times?=924$ $15\frac{2}{5}$	8. $12\times9\times?=1240$ $11\frac{13}{27}$	13. $9\times?\times12=1728$ 16
4. $7\times9\times?=462$ $7\frac{1}{3}$	9. $18\times24\times?=1840$ $4\frac{7}{27}$	14. $6\times?\times16=986$ $10\frac{13}{48}$
5. $6\times?\times9=4536$ 84	10. $16\times?\times18=2460$ $8\frac{13}{24}$	15. $8\times6\times?=2860$ $59\frac{7}{12}$

ORAL EXERCISES

1. How many inch cubes can you place in a box 5 inches long, 3 inches wide and 4 inches deep?

2. If you know the length, width, and depth in inches of a rectangular box, how do you find out how many inch cubes can be placed in it?

The number of cubic inches which can be placed in a box is called its *cubic contents*, or *volume, measured in cubic inches.*

3. If you know the length, width, and height of a room in feet how do you find how many cubic feet there are in it?

4. If you know the length, width and height of a room in yards, how do you find how many cubic yards there are in it?

We now understand the meaning of the statement:

$$\textit{width} \times \textit{length} \times \textit{height} = \textit{volume}.$$

If the volume is not known we have

$$(1)\quad \textit{length} \times \textit{width} \times \textit{height} = ?$$

which is a problem in multiplication.

If the length and width are known and also the volume, but not the height, then

$$(2)\quad \textit{length} \times \textit{width} \times ? = \textit{volume}.$$

This gives a problem in division like those on page 87.

When the width or the length is unknown we have

$$(3)\quad \textit{length} \times ? \times \textit{height} = \textit{volume}.$$

$$(4)\quad ? \times \textit{width} \times \textit{height} = \textit{volume}.$$

The missing numbers are found by division, as on page 87.

We must make it clear to ourselves that the volume of a rectangular box is the *product* of the length, width, and depth. But if a product is given and one of the factors, then the other factor is found by dividing. Hence, if the volume of a box is 180 cubic inches, and if the length and width are 10 inches and 6 inches, then 60 times the depth is 180, and the depth is $180 \div 60 = 3$.

ORAL AND WRITTEN EXERCISES

(Solve orally as many as possible.)

1. State a rule for finding the volume of a rectangular box when its length, width, and depth are known.

2. How many cubic inches does a box contain if it is 10 inches long, 6 inches wide, and 2 inches high? 120

3. How many cubic inches does a box contain if it is 10 inches long, 4 inches wide, and 3 inches deep? 120

4. Select the dimensions of a box in different ways so that its volume shall be 120 cubic inches.

5. If you know how long and how wide a box is and also how many cubic inches it holds, how can you find out how deep the box is?

6. A box is 16 inches long and 8 inches wide. How deep must it be to contain one cubic foot?

Suggestion: $16 \times 8 \times \text{depth} = 12 \times 12 \times 12$ cubic inches.

$$\text{Hence the depth} = \frac{12 \times 12 \times 12}{16 \times 8}$$ $13\frac{1}{2}$ in.

7. A piece of timber is 36 inches long and 8 inches wide. How thick is it if it contains 864 cubic inches? 3 in.

8. How high must a wheat bin be to hold 720 cubic feet if it is 10 feet long and 8 feet wide? 9 ft.

Compare this with the problems on areas on page 24. Also see the examples on cancellation on page 84. Try to understand that the principle in all these problems is the same. A good student learns to understand how the same principle runs through many different looking problems. The poor student sees no such connection, and therefore needs a great many separate rules. These he is sure to forget, and then he is entirely at a loss. The student who understands how problems are related has fewer rules to remember, and so gets on more easily and more rapidly.

Problem 1. At 62 cents a bushel, what is the value of a load of corn weighing 1876 pounds, one bushel of corn weighing 56 pounds?

Operations:

$$\begin{array}{l} 67\ \cancel{469}\ \ 31 \\ \cancel{1876} \\ \overline{}\times\cancel{62} \\ \cancel{56} \\ \cancel{14} \\ \cancel{7} \end{array}$$

$=67\times31=2077$

Hence value is 2077 cents, or $20.77

Analysis:

Weight of load divided by weight of one bushel equals number of bushels.

$$\frac{1876}{56}=\text{number of bushels.}$$

Number of bushels multiplied by price per bushel equals value of load.

$$\frac{1876}{56}\times62=\text{value of load in cents.}$$

Problem 2. At $12.40 a ton, what is the value of a load of hay weighing 2460 pounds?

To avoid difficulty in cancellation we write the value per ton in cents. Thus:

$\frac{2460}{2000}$ represents the number of tons, and $\frac{2460}{2000}\times1240=$ its value in *cents*.

Cancel and complete the solution.

At the University experimental station at Urbana, Illinois, numerous experiments have been made on the effectiveness of various kinds of feeds used for farm animals. On the next page are given accounts of the feeding of four lots of horses by different methods. The accounts are copied from one of their regular bulletins. Find the missing numbers as in the problems on this page. The problems to be solved are exactly those which the makers of the bulletin had to solve.

1 bushel of corn (shelled) weighs 56 pounds 1 bushel of oats weighs 32 pounds

WRITTEN EXERCISES

Lot I. Expenditure per horse.

1912 lbs. corn (shelled)	at $0.83 a bushel	$28.34
30 lbs. oil meal	at $27.00 a ton	.41
2075 lbs. clover hay	at $13.00 a ton	13.49
	Total cost of feed	$42.24

Lot II. Expenditure per horse.

1600 lbs. corn	at $0 83 a bushel	$23.71
533 lbs. oats	at $0.45 a bushel	7.50
35 lbs. oil meal	at $27.00 a ton	.47
2162 lbs. clover hay	at $13.00 a ton	14.05
	Total cost of feed	$45.73

Lot III. Expenditure per horse.

1077 lbs. corn	at $0.83 a bushel	$15.96
1077 lbs. oats	at $0.45 a bushel	15.15
34 lbs. oil meal	at $27.00 a ton	.46
2194 lbs. clover hay	at $13.00 a ton	14.26
	Total cost of feed	$45.83

Lot IV. Expenditure per horse.

1808 lbs. corn	at $0.83 a bushel	$26.80
352 lbs. bran	at $20.00 a ton	3.52
35 lbs. oil meal	at $27.00 a ton	.47
2081 lbs. clover hay	at $13.00 a ton	13.53
	Total cost of feed	$44.32

Drill in Fundamentals. The teacher will read simple combinations, and you will write down the answers.

62. Items of a Bill. A bill is a statement rendered by a seller to the purchaser of goods. A bill should contain the following items:

1. Name and location of seller.
2. Name and location of buyer.
3. Date of sale and price per unit of each item.
4. Total cost of each item.
5. Terms of sale, such as cash, credit for 30 days, etc.

There are also other items which usually go on bills, such as extra charges for freight, cartage deductions in the way of discounts, etc.

A BILL

Chicago, Ill., May 28, 1917.

Mr. James Wolf,
346 55th Street, Chicago.

Bought of ALBERT K. JOHNSON & CO.,

Terms: Cash the first of each month.

10 bu. potatoes	$1.10	$11	00		
5 bbl. flour	8.30	41	50		
60 doz. oranges	0.30	18	00		
35 bu. apples	0.75	26	25	$96	75

The amounts given in the first column is the cost of one unit of each article, such as one bushel of potatoes, or one barrel of flour. Sometimes the letter @ is written before these amounts to show that they indicate the price per unit.

The amounts given in the next column is the cost of the total amount of each article. The single number given in the third column is the sum of the second column, and shows the cost of all the articles on the bill.

63. Extending and Footing Bills. Multiplying the price by the number of articles of each kind to find the cost is called *extending the bill.* Adding the cost of the items to find the total is called *footing the bill.*

WRITTEN EXERCISES

1. Copy the bill on page 92, extend the items, and foot the bill.

For each of the following make out a bill, extending the items and footing the bill:

2. May 23, 1917. Brown & Co., of New York, sold to Albert Walker of Princeton, New Jersey, 42 chests of green tea at \$31.60, 20 chests of black tea at \$24.60, 10 chests of uncolored Japan tea at \$51.60, 20 boxes of lemons at \$5.30, 30 boxes of oranges at \$3.40. Terms: Cash. \$2543.20

3. January 7, 1918. J. M. Lucy & Sons of Janesville, Wisconsin, sold to Geo. S. Parker of Janesville, Wisconsin, 12 dining chairs at \$22.50, one dining table at \$85.50, 4 sections of bookcases at \$5.10, one library table at \$110, 4 sitting room chairs at \$50.00. Terms: Cash first of the month. \$685.90

4. June 10, 1918. Solomender Optical Co., of Cleveland, sold to East Technical High School, 40 Federal Ruling pens at \$.45, 30 Federal Bow pencils at \$.72, 25 Federal Bow pens at \$.76, 30 compasses at \$.65, 15 compasses at \$1.60. Terms: Cash in 30 days. \$102.10

5. October 8, 1917. J. T. Black & Sons of Buffalo, N. Y., sold to S. Williams of Buffalo.

6 boxes shredded wheat	260 lb.	at	\$0.02
6 boxes tea	1360 lb.	at	0.37
8 bbls. sugar	1740 lb.	at	$0.08\frac{1}{2}$
10 bbls. flour	1840 lb.	at	$0.04\frac{1}{8}$

\$736.03

6. Walter Beck & Co. of Chicago, sold to John S. Smith of Decatur, Ill., 35 suits of men's clothes at \$15.50, 45 suits at \$17.50, 60 suits at \$20.00, 25 suits at \$25.00, and 20 suits at \$30.00. \$3755.00

7. Get some real bills, and extend and foot them. Also make up some bills, and extend and foot them.

ORAL EXERCISES

1. What is a fraction? Name a fraction and give its numerator and also its denominator.

2. What does the denominator of a fraction tell? What does its numerator tell?

3. What are the terms of a fraction?

4. How may a fraction be changed without changing its value?

5. What is a proper fraction? an improper fraction? a mixed number? Give examples of each.

6. What are like fractions? How may like fractions be added and subtracted?

7. In adding unlike fractions what is the first operation to be performed on them?

8. What is a factor of a number? Give a number and one of its factors.

9. What is a common factor of two numbers? Give two numbers having a common factor and also the common factor.

10. What is a multiple of a number? Give a number and several of its multiples.

11. What is the least common multiple of two numbers? Give two numbers and also their L. C. M.

12. How may a common denominator of two fractions be found?

13. What is cancellation? Name a fraction which may be simplified by cancellation.

14. State the rule for multiplying two fractions. Name two fractions and give their product.

15. State the rule for dividing by a fraction. Does this rule hold whether the dividend is a fraction or an integer?

PROBLEMS WITHOUT NUMBERS

1. If the product of two numbers is given, and also one of the numbers, how may the other number be found?

2. If the speed of a train is known, and also the time of running, how may the distance be found?

3. If the time and the distance are known, how may the speed be found?

4. If the speed and the distance are known, how may the time be found?

5. If the numerator of a fraction is multiplied by a whole number, how is the value of the fraction affected?

6. If the denominator of a fraction is multiplied by a whole number, how is the value of the fraction affected?

7. If the numerator of a fraction is increased, is the value of the fraction increased or decreased?

8. If the denominator of a fraction is increased, is the value of the fraction increased or decreased?

9. If you know the sum of two fractions, and also one of the fractions, how do you find the other fraction?

10. If a number is multiplied by a proper fraction, is the product greater or less than the number itself?

11. If a number is divided by a proper fraction, is the quotient greater or less than the number itself?

12. If any number (fraction or integer) is multiplied by a number greater than one, is the product greater or less than the given number?

13. If a number is divided by an improper fraction, is the quotient greater or less than the given number?

Review.

ORAL EXERCISES

Read the following and supply the missing numbers:

1.	$\frac{1}{2}=\frac{?}{6}$	$\frac{1}{2}=\frac{?}{10}$	$\frac{1}{2}=\frac{?}{8}$	$\frac{1}{2}=\frac{?}{16}$	$\frac{3}{4}=\frac{?}{24}$
2.	$\frac{1}{3}=\frac{?}{6}$	$\frac{1}{3}=\frac{?}{9}$	$\frac{1}{3}=\frac{?}{15}$	$\frac{1}{3}=\frac{?}{27}$	$\frac{5}{6}=\frac{?}{30}$
3.	$\frac{2}{3}=\frac{?}{6}$	$\frac{2}{3}=\frac{?}{9}$	$\frac{2}{3}=\frac{?}{15}$	$\frac{2}{3}=\frac{?}{27}$	$\frac{3}{7}=\frac{?}{21}$
4.	$\frac{3}{5}=\frac{?}{15}$	$\frac{3}{5}=\frac{?}{30}$	$\frac{3}{5}=\frac{?}{45}$	$\frac{3}{5}=\frac{?}{25}$	$\frac{4}{5}=\frac{24}{?}$
5.	$\frac{5}{6}=\frac{?}{18}$	$\frac{5}{6}=\frac{15}{?}$	$\frac{5}{6}=\frac{35}{?}$	$\frac{5}{6}=\frac{45}{?}$	$\frac{3}{7}=\frac{12}{?}$
6.	$\frac{5}{8}=\frac{?}{32}$	$\frac{5}{8}=\frac{15}{?}$	$\frac{5}{8}=\frac{35}{?}$	$\frac{5}{8}=\frac{50}{?}$	$\frac{2}{3}=\frac{12}{?}$
7.	$\frac{7}{16}=\frac{14}{?}$	$\frac{7}{16}=\frac{35}{?}$	$\frac{7}{16}=\frac{21}{?}$	$\frac{7}{16}=\frac{49}{?}$	$\frac{3}{8}=\frac{18}{?}$
8.	$\frac{15}{32}=\frac{45}{?}$	$\frac{15}{16}=\frac{75}{?}$	$\frac{15}{16}=\frac{90}{?}$	$\frac{15}{16}=\frac{60}{?}$	$\frac{5}{9}=\frac{15}{?}$

Example. Reduce $3\frac{2}{3}$ to an improper fraction.
Solution: $3\frac{2}{3}=\frac{9}{3}+\frac{2}{3}=\frac{11}{3}$.

Such examples should always be solved without using pencil and paper.

Reduce each of the following to an improper fraction:

9.	$1\frac{1}{2}$	$2\frac{1}{3}$	$1\frac{2}{3}$	$2\frac{1}{4}$	$3\frac{3}{4}$	$6\frac{1}{2}$	$6\frac{1}{3}$	$5\frac{3}{7}$
10.	$3\frac{4}{5}$	$2\frac{5}{6}$	$1\frac{1}{8}$	$2\frac{3}{8}$	$3\frac{3}{8}$	$4\frac{4}{8}$	$5\frac{5}{8}$	$7\frac{7}{8}$
11.	$3\frac{5}{8}$	$3\frac{3}{6}$	$8\frac{3}{5}$	$6\frac{5}{6}$	$5\frac{3}{8}$	$7\frac{2}{3}$	$8\frac{4}{5}$	$6\frac{3}{16}$

12. Reduce each of the following numbers to 12ths: 2, 3, 5, 6, 7.

13. Reduce each of the following numbers to 16ths: 1, 2, 3, 4, 5.

Reduce each of the following to a mixed number:

14.	$\frac{7}{2}$	$\frac{9}{4}$	$\frac{13}{5}$	$\frac{15}{8}$	$\frac{18}{4}$	$\frac{17}{16}$	$\frac{24}{7}$	$\frac{35}{8}$
15.	$\frac{15}{7}$	$\frac{25}{4}$	$\frac{12}{5}$	$\frac{29}{9}$	$\frac{34}{7}$	$\frac{31}{8}$	$\frac{38}{5}$	$\frac{49}{16}$

Drill in Fundamentals. The teacher will read combinations, and you will write the answers.

ORAL EXERCISES

Give each of the following indicated sums:

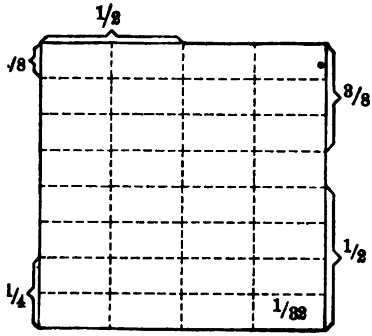
1.	$\frac{1}{3}+\frac{1}{3}$	$\frac{2}{3}+\frac{2}{3}$	$\frac{1}{2}+\frac{1}{4}$	$\frac{1}{2}+\frac{1}{8}$	$\frac{1}{2}+\frac{3}{8}$
2.	$\frac{1}{2}+\frac{5}{8}$	$\frac{1}{2}+\frac{7}{8}$	$\frac{1}{2}+\frac{3}{4}$	$\frac{1}{2}+\frac{1}{16}$	$\frac{1}{2}+\frac{3}{16}$
3.	$\frac{1}{2}+\frac{5}{16}$	$\frac{1}{2}+\frac{7}{16}$	$\frac{1}{2}+\frac{9}{16}$	$\frac{1}{2}+\frac{11}{16}$	$\frac{1}{2}+\frac{13}{16}$
4.	$\frac{1}{2}+\frac{15}{16}$	$\frac{1}{2}+\frac{1}{32}$	$\frac{1}{4}+\frac{1}{4}$	$\frac{1}{4}+\frac{1}{3}$	$\frac{1}{4}+\frac{1}{8}$
5.	$\frac{3}{4}+\frac{1}{3}$	$\frac{3}{4}+\frac{2}{3}$	$\frac{1}{4}+\frac{3}{8}$	$\frac{1}{4}+\frac{5}{8}$	$\frac{1}{4}+\frac{7}{8}$
6.	$\frac{3}{4}+\frac{1}{8}$	$\frac{3}{4}+\frac{3}{8}$	$\frac{3}{4}+\frac{5}{8}$	$\frac{3}{4}+\frac{7}{8}$	$\frac{1}{8}+\frac{3}{8}$
7.	$\frac{1}{4}+\frac{1}{16}$	$\frac{1}{4}+\frac{3}{16}$	$\frac{1}{4}+\frac{5}{16}$	$\frac{1}{4}+\frac{7}{16}$	$\frac{1}{4}+\frac{9}{16}$
8.	$\frac{1}{4}+\frac{11}{16}$	$\frac{1}{4}+\frac{13}{16}$	$\frac{1}{4}+\frac{15}{16}$	$\frac{3}{4}+\frac{1}{16}$	$\frac{3}{4}+\frac{3}{16}$
9.	$\frac{3}{4}+\frac{5}{16}$	$\frac{3}{4}+\frac{7}{16}$	$\frac{3}{4}+\frac{9}{16}$	$\frac{3}{4}+\frac{11}{16}$	$\frac{3}{4}+\frac{13}{16}$
10.	$\frac{3}{4}+\frac{15}{16}$	$\frac{3}{8}+\frac{1}{16}$	$\frac{1}{8}+\frac{1}{16}$	$\frac{1}{8}+\frac{3}{16}$	$\frac{3}{8}+\frac{3}{16}$
11.	$\frac{3}{8}+\frac{5}{16}$	$\frac{1}{8}+\frac{5}{16}$	$\frac{3}{8}+\frac{7}{16}$	$\frac{1}{8}+\frac{7}{16}$	$\frac{1}{8}+\frac{9}{16}$
12.	$\frac{3}{8}+\frac{9}{16}$	$\frac{3}{8}+\frac{11}{16}$	$\frac{1}{8}+\frac{11}{16}$	$\frac{5}{8}+\frac{3}{16}$	$\frac{7}{8}+\frac{5}{16}$
13.	$\frac{1}{3}+\frac{1}{4}$	$\frac{2}{3}+\frac{1}{4}$	$\frac{1}{3}+\frac{3}{4}$	$\frac{2}{3}+\frac{3}{4}$	$\frac{1}{3}+\frac{3}{8}$

Give each of the following indicated differences:

14.	$\frac{1}{2}-\frac{1}{4}$	$\frac{1}{2}-\frac{1}{8}$	$\frac{1}{2}-\frac{3}{8}$	$\frac{1}{2}-\frac{1}{16}$	$\frac{1}{2}-\frac{3}{16}$
15.	$\frac{1}{2}-\frac{5}{16}$	$\frac{1}{2}-\frac{7}{16}$	$\frac{1}{2}-\frac{1}{32}$	$\frac{1}{2}-\frac{3}{32}$	$\frac{1}{2}-\frac{5}{32}$
16.	$\frac{1}{2}-\frac{7}{32}$	$\frac{1}{2}-\frac{9}{32}$	$\frac{1}{2}-\frac{11}{32}$	$\frac{1}{2}-\frac{13}{32}$	$\frac{1}{2}-\frac{15}{32}$
17.	$\frac{1}{4}-\frac{1}{8}$	$\frac{1}{4}-\frac{1}{16}$	$\frac{1}{4}-\frac{3}{16}$	$\frac{3}{4}-\frac{3}{8}$	$\frac{3}{4}-\frac{5}{8}$
18.	$\frac{3}{4}-\frac{1}{16}$	$\frac{3}{4}-\frac{3}{16}$	$\frac{3}{4}-\frac{5}{16}$	$\frac{3}{4}-\frac{7}{16}$	$\frac{3}{4}-\frac{9}{16}$
19.	$\frac{3}{4}-\frac{11}{16}$	$\frac{5}{8}-\frac{1}{16}$	$\frac{5}{8}-\frac{5}{16}$	$\frac{7}{8}-\frac{13}{16}$	$\frac{7}{8}-\frac{11}{16}$

Drill in Fundamentals. Play game No. 4, page 30. Use examples in adding fractions.

ORAL EXERCISES



1. Point out $\frac{1}{2}$ of this figure, $\frac{1}{4}$ of it, $\frac{3}{4}$ of it, $\frac{1}{8}$ of it, $\frac{3}{8}$ of it, $\frac{5}{8}$ of it, $\frac{7}{8}$ of it, $\frac{1}{16}$ of it, $\frac{5}{16}$ of it, $\frac{9}{16}$ of it, $\frac{15}{16}$ of it. How many 32ds of the figure are there in each of these?

2. Point out $\frac{1}{2}$ of $\frac{1}{4}$, or $\frac{1}{2}\times\frac{1}{4}$. What is the product?

3. Point out $\frac{1}{2}$ of $\frac{1}{8}$. What is the product?

4. Point out $\frac{1}{2}$ of $\frac{1}{16}$. What is the product?

5. Point out $\frac{1}{2}$ of $\frac{3}{4}$. What is the product?

6. Point out $\frac{1}{2}$ of $\frac{1}{2}$. What is the product?

7. Point out $\frac{1}{2}$ of $\frac{3}{8}$. What is the product?

8. Point out $\frac{1}{4}$ of $\frac{1}{2}$. What is the product?

Read the following and give the product in each case, remembering that $\frac{1}{4}\times\frac{1}{2}$ means the same as $\frac{1}{4}$ of $\frac{1}{2}$:

9. $\frac{1}{2}\times\frac{1}{2}$	16. $\frac{3}{4}\times\frac{1}{4}$	23. $\frac{1}{2}\times\frac{3}{8}$	30. $\frac{2}{3}\times\frac{1}{4}$
10. $\frac{1}{2}\times\frac{1}{4}$	17. $\frac{3}{4}\times\frac{5}{8}$	24. $\frac{1}{2}\times\frac{5}{16}$	31. $\frac{1}{3}\times\frac{3}{4}$
11. $\frac{1}{2}\times\frac{1}{8}$	18. $\frac{3}{4}\times\frac{7}{8}$	25. $\frac{1}{2}\times\frac{13}{16}$	32. $\frac{2}{3}\times\frac{3}{4}$
12. $\frac{1}{2}\times\frac{1}{16}$	19. $\frac{1}{2}\times\frac{3}{4}$	26. $\frac{1}{2}\times\frac{15}{16}$	33. $\frac{1}{4}\times\frac{1}{3}$
13. $\frac{1}{4}\times\frac{1}{2}$	20. $\frac{1}{2}\times\frac{3}{8}$	27. $\frac{1}{2}\times\frac{1}{3}$	34. $\frac{3}{4}\times\frac{1}{2}$
14. $\frac{1}{4}\times\frac{1}{4}$	21. $\frac{1}{2}\times\frac{5}{8}$	28. $\frac{1}{3}\times\frac{2}{3}$	35. $\frac{3}{8}$ of $\frac{3}{4}$
15. $\frac{1}{4}\times\frac{1}{8}$	22. $\frac{1}{2}\times\frac{7}{8}$	29. $\frac{1}{3}\times\frac{1}{4}$	36. $\frac{5}{8}$ of $\frac{1}{4}$

ORAL AND WRITTEN EXERCISES

On this page solve all the examples you can without the aid of pencil and paper.

1.	$\frac{3}{4}\times\frac{2}{5}$ $\frac{3}{10}$	$\frac{2}{5}\times\frac{5}{8}$ $\frac{1}{4}$	$\frac{9}{16}\times\frac{10}{21}$ $\frac{15}{56}$	$\frac{8}{9}\times\frac{7}{12}$ $\frac{14}{27}$
2.	$\frac{7}{8}\times\frac{4}{5}$ $\frac{7}{10}$	$\frac{5}{12}\times\frac{3}{20}$ $\frac{1}{16}$	$\frac{3}{4}\times\frac{1}{6}$ $\frac{1}{8}$	$\frac{8}{15}\times\frac{3}{4}$ $\frac{2}{5}$
3.	$\frac{5}{16}\times\frac{3}{5}$ $\frac{3}{16}$	$\frac{8}{9}\times\frac{3}{16}$ $\frac{1}{6}$	$\frac{7}{8}\times\frac{3}{4}$ $\frac{21}{32}$	$\frac{9}{16}\times\frac{3}{8}$ $\frac{27}{128}$

Give the product of each of the following:

4.	$2\times\frac{2}{3}$ $1\frac{1}{3}$	$1\times\frac{1}{4}$ $\frac{1}{4}$	$2\times\frac{3}{4}$ $1\frac{1}{2}$	$2\times\frac{2}{5}$ $\frac{4}{5}$
5.	$3\times\frac{1}{3}$ 1	$3\times\frac{2}{3}$ 2	$3\times\frac{3}{4}$ $2\frac{1}{4}$	$3\times\frac{3}{8}$ $1\frac{1}{8}$
6.	$4\times\frac{1}{3}$ $1\frac{1}{3}$	$4\times\frac{2}{3}$ $2\frac{2}{3}$	$4\times\frac{1}{4}$ 1	$4\times\frac{3}{4}$ 3
7.	$4\times\frac{7}{8}$ $3\frac{1}{2}$	$4\times\frac{3}{8}$ $1\frac{1}{2}$	$4\times\frac{1}{8}$ $\frac{1}{2}$	$4\times\frac{5}{16}$ $1\frac{1}{4}$

Multiply each of the following without first reducing the multiplicand to an improper fraction:

8.	$2\times1\frac{1}{3}$ $2\frac{2}{3}$	$2\times2\frac{1}{4}$ $4\frac{1}{2}$	$2\times2\frac{3}{4}$ $5\frac{1}{2}$	$2\times2\frac{1}{8}$ $4\frac{1}{4}$
9.	$3\times2\frac{1}{2}$ $7\frac{1}{2}$	$3\times2\frac{1}{4}$ $6\frac{3}{4}$	$3\times2\frac{3}{4}$ $8\frac{1}{4}$	$3\times3\frac{1}{2}$ $10\frac{1}{2}$
10.	$4\times1\frac{1}{2}$ 6	$4\times2\frac{1}{3}$ $9\frac{1}{3}$	$4\times3\frac{1}{4}$ 13	$4\times2\frac{3}{4}$ 11
11.	$8\times3\frac{2}{3}$ $29\frac{1}{3}$	$7\times5\frac{4}{5}$ $40\frac{3}{5}$	$9\times4\frac{7}{8}$ $43\frac{7}{8}$	$8\times4\frac{3}{5}$ $36\frac{4}{5}$

In making the following multiplications remember that the multiplier and multiplicand may be interchanged without changing the product. If one of these is a whole number, always use that one as the multiplier.

12.	$1\frac{1}{2}\times3$ $4\frac{1}{2}$	$2\frac{1}{2}\times3$ $7\frac{1}{2}$	$2\frac{3}{8}\times3$ $7\frac{1}{8}$	$5\frac{1}{4}\times3$ $15\frac{3}{4}$
13.	$2\frac{1}{3}\times6$ 14	$8\frac{1}{2}\times6$ 51	$6\frac{3}{4}\times6$ $40\frac{1}{2}$	$5\frac{3}{8}\times6$ $32\frac{1}{4}$
14.	$3\frac{9}{16}\times8$ $28\frac{1}{2}$	$1\frac{5}{8}\times10$ $16\frac{1}{4}$	$3\frac{3}{4}\times7$ $26\frac{1}{4}$	$\frac{7}{16}\times9$ $3\frac{15}{16}$
15.	$9\times1\frac{3}{8}$ $12\frac{3}{8}$	$12\times1\frac{1}{4}$ 15	$1\frac{5}{16}\times10$ $13\frac{1}{8}$	$2\frac{1}{4}\times12$ 27

Drill in Fundamentals. Play game No. 4 on page 30. Use examples in multiplication of fractions.

ORAL AND WRITTEN EXERCISES

(Solve as many as possible without using pencil and paper.)

Give the products of the following:

1. $\frac{3}{4}\times\frac{3}{4}$ $\frac{9}{16}$ $\frac{3}{4}\times\frac{1}{8}$ $\frac{3}{32}$ $\frac{1}{2}\times\frac{9}{16}$ $\frac{9}{32}$ $\frac{3}{5}\times\frac{1}{5}$ $\frac{3}{25}$ $\frac{5}{6}\times\frac{3}{8}$ $\frac{5}{16}$

2. $\frac{3}{4}\times\frac{5}{8}$ $\frac{15}{32}$ $\frac{4}{5}\times\frac{7}{8}$ $\frac{7}{10}$ $\frac{2}{3}\times\frac{3}{8}$ $\frac{1}{4}$ $\frac{2}{3}\times\frac{5}{8}$ $\frac{5}{12}$ $\frac{3}{5}\times\frac{7}{8}$ $\frac{21}{40}$

3. $\frac{3}{4}\times\frac{4}{5}$ $\frac{3}{5}$ $\frac{2}{5}\times\frac{5}{8}$ $\frac{1}{4}$ $\frac{3}{8}\times\frac{4}{5}$ $\frac{3}{10}$ $\frac{3}{16}\times\frac{5}{8}$ $\frac{15}{128}$ $\frac{5}{8}\times\frac{3}{4}$ $\frac{15}{32}$

4. $\frac{5}{7}\times\frac{3}{4}$ $\frac{15}{28}$ $\frac{2}{7}\times\frac{6}{7}$ $\frac{12}{49}$ $\frac{4}{5}\times\frac{15}{16}$ $\frac{3}{4}$ $\frac{7}{8}\times\frac{3}{4}$ $\frac{21}{32}$ $\frac{7}{8}\times\frac{4}{5}$ $\frac{7}{10}$

Before multiplying the following, change both multiplier and multiplicand to improper fractions or use the four-step method (see page 70):

5. $1\frac{1}{2}\times1\frac{1}{2}$ $2\frac{1}{4}$ $1\frac{1}{2}\times2\frac{1}{2}$ $3\frac{3}{4}$ $1\frac{1}{2}\times3\frac{1}{2}$ $5\frac{1}{4}$ $8\frac{1}{2}\times9\frac{2}{3}$ $82\frac{1}{6}$

6. $1\frac{1}{3}\times4\frac{1}{3}$ $5\frac{7}{9}$ $1\frac{1}{3}\times5\frac{1}{3}$ $7\frac{1}{9}$ $2\frac{3}{4}\times3\frac{1}{2}$ $9\frac{5}{8}$ $16\frac{1}{4}\times8\frac{1}{2}$ $138\frac{1}{8}$

7. $6\frac{1}{2}\times8\frac{1}{2}$ $55\frac{1}{4}$ $5\frac{1}{2}\times9\frac{1}{2}$ $52\frac{1}{4}$ $14\frac{1}{2}\times8\frac{1}{2}$ $123\frac{1}{4}$ $27\frac{3}{4}\times19\frac{1}{4}$ $534\frac{3}{16}$

8. $5\frac{1}{2}\times5\frac{1}{2}$ $30\frac{1}{4}$ $12\frac{1}{2}\times14\frac{1}{2}$ $181\frac{1}{4}$ $8\frac{3}{4}\times16\frac{1}{2}$ $144\frac{3}{8}$ $48\frac{1}{2}\times6\frac{3}{4}$ $327\frac{3}{8}$

Give the quotients of the following:

9. $\frac{4}{5}\div2$ $\frac{2}{5}$ $\frac{6}{7}\div3$ $\frac{2}{7}$ $\frac{8}{9}\div4$ $\frac{2}{9}$ $\frac{9}{16}\div3$ $\frac{3}{16}$ $\frac{3}{4}\div3$ $\frac{1}{4}$

10. $\frac{2}{3}\div3$ $\frac{2}{9}$ $\frac{1}{4}\div3$ $\frac{1}{12}$ $\frac{1}{8}\div3$ $\frac{1}{24}$ $\frac{5}{8}\div3$ $\frac{5}{24}$ $\frac{7}{8}\div3$ $\frac{7}{24}$

11. $\frac{3}{4}\div4$ $\frac{3}{16}$ $\frac{5}{8}\div4$ $\frac{5}{32}$ $\frac{7}{8}\div4$ $\frac{7}{32}$ $\frac{5}{16}\div4$ $\frac{5}{64}$ $\frac{9}{16}\div4$ $\frac{9}{64}$

12. $\frac{1}{2}\div\frac{1}{3}$ $1\frac{1}{2}$ $\frac{3}{4}\div\frac{2}{5}$ $1\frac{7}{8}$ $\frac{4}{5}\div\frac{2}{3}$ $1\frac{1}{5}$ $\frac{5}{8}\div\frac{1}{4}$ $2\frac{1}{2}$ $\frac{7}{8}\div\frac{1}{2}$ $1\frac{3}{4}$

In the following do not change the dividend to an improper fraction before dividing:

13. $4\frac{1}{2}\div2$ $2\frac{1}{4}$ $6\frac{1}{2}\div2$ $3\frac{1}{4}$ $8\frac{3}{4}\div2$ $4\frac{3}{8}$ $10\frac{1}{2}\div2$ $5\frac{1}{4}$ $12\frac{1}{3}\div2$ $6\frac{1}{6}$

14. $6\frac{1}{3}\div3$ $2\frac{1}{9}$ $9\frac{2}{3}\div3$ $3\frac{2}{9}$ $12\frac{1}{4}\div3$ $4\frac{1}{12}$ $15\frac{3}{4}\div3$ $5\frac{1}{4}$ $18\frac{5}{8}\div3$ $6\frac{5}{24}$

15. $4\frac{1}{2}\div3$ $1\frac{1}{2}$ $10\frac{2}{3}\div3$ $3\frac{5}{9}$ $14\frac{1}{2}\div3$ $4\frac{5}{6}$ $16\frac{1}{2}\div3$ $5\frac{1}{2}$ $27\frac{1}{4}\div4$ $6\frac{13}{16}$

16. $5\frac{3}{4}\div4$ $1\frac{7}{16}$ $7\frac{5}{8}\div4$ $1\frac{29}{32}$ $10\frac{7}{8}\div4$ $2\frac{23}{32}$ $15\frac{1}{16}\div4$ $3\frac{49}{64}$ $42\frac{3}{5}\div6$ $7\frac{1}{10}$

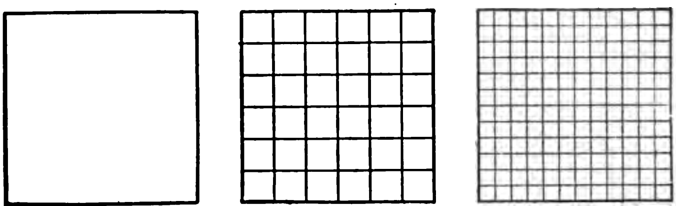
ORAL AND WRITTEN EXERCISES

Get each result in the examples below in the shortest way. See how long it takes you to do the first 21. Then see if you can do the next column in less time.

1.	$8\times\frac{4}{7}$	$4\frac{4}{7}$	22.	$12\times15\times1\frac{1}{4}$	225	43.	$4\frac{2}{3}\times3\frac{1}{5}\times6$	$89\frac{3}{5}$
2.	$6\times\frac{4}{9}$	$2\frac{2}{3}$	23.	$6\times4\times3\frac{2}{3}$	88	44.	$\frac{7}{8}\div\frac{3}{4}$	$1\frac{1}{6}$
3.	$12\times1\frac{1}{2}$	18	24.	$12\times8\times5\frac{3}{4}$	552	45.	$\frac{6}{7}\div\frac{2}{3}$	$1\frac{2}{7}$
4.	$8\times3\frac{4}{5}$	$30\frac{2}{5}$	25.	$13\times\frac{3}{5}$	$7\frac{4}{5}$	46.	$4\frac{3}{4}\div\frac{4}{5}$	$5\frac{15}{16}$
5.	$9\div5\frac{5}{6}$	$1\frac{19}{35}$	26.	$\frac{2}{3}\times\frac{6}{7}$	$\frac{4}{7}$	47.	$5\frac{1}{3}\div2\frac{2}{3}$	2
6.	$12\div8\frac{3}{4}$	$1\frac{13}{35}$	27.	$\frac{4}{5}\div\frac{3}{8}$	$2\frac{2}{15}$	48.	$7\frac{5}{6}\div3\frac{2}{3}$	$2\frac{3}{22}$
7.	$16\times5\frac{7}{8}$	94	28.	$\frac{7}{9}\div\frac{12}{21}$	$1\frac{13}{36}$	49.	$4\frac{2}{3}\times2\frac{4}{5}$	$13\frac{1}{15}$
8.	$15\div6\frac{2}{3}$	$2\frac{1}{4}$	29.	$\frac{5}{9}\div\frac{6}{25}$	$2\frac{17}{54}$	50.	$18\frac{1}{2}\times6\frac{1}{4}$	$115\frac{5}{8}$
9.	$2\frac{1}{3}\div6$	$\frac{7}{18}$	30.	$\frac{8}{15}\div\frac{9}{16}$	$\frac{128}{135}$	51.	$51\frac{1}{2}\div6\frac{1}{2}$	$7\frac{12}{13}$
10.	$5\frac{2}{3}\times9$	51	31.	$\frac{7}{32}\div\frac{8}{21}$	$\frac{147}{256}$	52.	$46\frac{1}{4}\times16\frac{1}{2}$	$763\frac{1}{8}$
11.	$8\frac{4}{5}\times15$	132	32.	$\frac{5}{9}\div\frac{3}{8}$	$1\frac{13}{27}$	53.	$27\frac{1}{3}\div8\frac{1}{2}$	$3\frac{11}{51}$
12.	$24\frac{2}{3}\times3$	74	33.	$\frac{7}{12}\div\frac{15}{16}$	$\frac{28}{45}$	54.	$46\frac{1}{4}\div8\frac{1}{3}$	$5\frac{11}{20}$
13.	$19\frac{3}{5}\times10$	196	34.	$\frac{4}{7}\div\frac{7}{8}$	$\frac{32}{49}$	55.	$41\frac{1}{8}\times81\frac{1}{4}$	$3341\frac{13}{32}$
14.	$17\frac{3}{4}\times8$	142	35.	$3\frac{1}{2}\div2\frac{1}{2}$	$1\frac{2}{5}$	56.	$59\frac{1}{3}\div8\frac{2}{3}$	$6\frac{11}{13}$
15.	$34\frac{5}{6}\times12$	418	36.	$1\frac{3}{4}\times2\frac{1}{3}$	$4\frac{1}{12}$	57.	$108\frac{1}{4}\times12\frac{1}{2}$	$1353\frac{1}{8}$
16.	$106\frac{2}{3}\div24$	$4\frac{4}{9}$	37.	$4\frac{1}{2}\times2\frac{2}{3}$	12	58.	$42\frac{3}{7}\div16\frac{1}{2}$	$2\frac{4}{7}$
17.	$2\times4\times6\frac{1}{2}$	52	38.	$6\times3\frac{1}{2}\times1\frac{1}{2}$	$31\frac{1}{2}$	59.	$27\frac{5}{8}\div19\frac{2}{3}$	$1\frac{191}{472}$
18.	$6\times8\times2\frac{1}{3}$	112	39.	$5\times2\frac{2}{3}\times1\frac{2}{3}$	$22\frac{2}{9}$	60.	$580\frac{1}{4}\div7$	$82\frac{25}{28}$
19.	$24\times5\times1\frac{2}{3}$	200	40.	$1\frac{1}{2}\times2\frac{1}{3}\times3\frac{2}{3}$	$12\frac{5}{6}$	61.	$364\frac{5}{8}\div14$	$26\frac{5}{112}$
20.	$36\times8\times4\frac{1}{3}$	1248	41.	$5\times4\frac{2}{3}\times3\frac{2}{5}$	$79\frac{1}{3}$	62.	$63\frac{1}{2}\times41\frac{1}{3}$	$2624\frac{2}{3}$
21.	$35\times8\times4\frac{1}{3}$	$1213\frac{1}{3}$	42.	$6\frac{1}{2}\times3\frac{3}{4}\times2\frac{1}{5}$	$53\frac{5}{8}$	63.	$81\frac{3}{8}\times31\frac{1}{4}$	$2542\frac{31}{32}$

64. Drawing to Scale. In making drawings of large objects the drawings must be smaller than the objects themselves.

If the drawing is half as long and half as wide as the thing it represents it is said to be "to the scale 1: 2" or "$\frac{1}{2}$." If one inch in the drawing represents 10 feet in the object the scale is said to be "1 inch to 10 feet" or $\frac{1}{120}$.



Scale $\frac{1}{1}$ Scale $\frac{1}{6}$ Scale $\frac{1}{12}$

In the figure the square to the left represents one square inch, and the scale is 1: 1 or $\frac{1}{1}$. The next square represents a square 6 inches on the side, and the scale is 1 : 6 or $\frac{1}{6}$. The last square represents a square 12 inches on the side, and the scale is 1:12 or $\frac{1}{12}$. If this last square is made to represent a square 12 feet on the side the scale would be 1 inch to 12 feet or 1 : 144 ($\frac{1}{144}$).

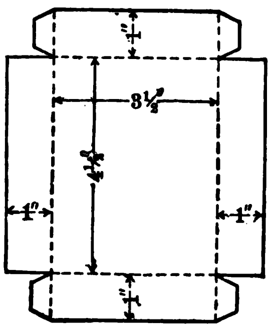
In order to make a drawing to scale of an object we must first measure it, and then decide the scale to be used. The larger the object to be represented the smaller the scale should be.

1. What scale would you use in representing each of the following:
   (a) The top of a desk 30″ x 36″.
   (b) A blackboard 3′ x 18′.
   (c) A schoolroom floor 30′ x 36′.
   (d) A schoolyard 160′ x 180′.
   (e) A city block 300′ x 480′.
   (f) A farm 160 rods by 240 rods.

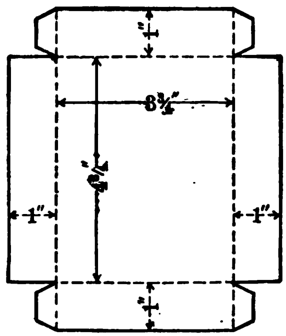
2. Make a drawing to scale of each of the things named under Example 1.

65. **Working Drawings.** People who build houses, make furniture, machinery, and many other things first make careful drawings. These drawings show the sizes of all the parts, and by following the drawings carefully the parts will be of the right size, and will fit together properly.

The following are drawings for a small candy box which some fifth-grade children made. The cardboard is bent up along the dotted lines and the corners fastened.



Box



Cover

ORAL AND WRITTEN EXERCISES

1. How long and how wide is the bottom of the box?
2. How deep will the box be?
3. How long and how wide will the cover be? Why is the cover made larger than the box?
4. On cardboard make drawings of this box and also of the cover, using the scale 1: 1. That is, make the drawing just the size of the box. Then cut out to make the box and the cover. After finishing the box measure its dimensions with care to see whether or not your drawings were right.
5. Measure the length and width of a piece of street and draw to scale.

Scale: 1 inch represents 1000 miles.

1. On this map what distance is represented by 2 inches? by 3 inches? by $2\frac{3}{4}$ inches? by $1\frac{7}{8}$ inches? by $2\frac{7}{16}$ inches? 2000; 3000; 2750: 1875; $2437\frac{1}{2}$ miles

2. Measure accurately within $\frac{1}{16}$ of an inch the distance on this map between Boston and Chicago. According to this measurement, what is the straight-line distance between Boston and Chicago? First decide exactly where each city is located. 875 miles

3. In the same manner find the straight-line distances from the City of New York to Chicago, from New York to St. Louis, to Denver, to San Francisco, to Seattle. 750; 1000; 1750; 2625; 2500

4. Find the distances between Chicago and New Orleans, Minneapolis and Seattle, Denver and San Francisco, Boston and St. Louis. 875; 1500; 1000; 1125

5. Locate your own home as nearly as you can on this map. Find the distances from your home to New York, to Chicago, to Buffalo, to Cleveland, to Denver, to San Francisco.

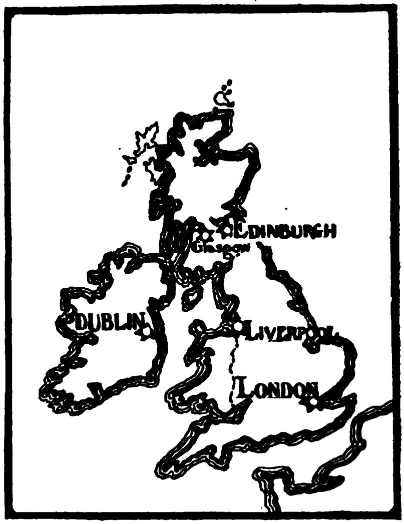
6. On a large wall map of the United States measure these same distances and find the actual distances.

ORAL AND WRITTEN EXERCISES

Answer as many as you can without using pencil and paper.

1. On this map what is represented by 1 inch? $1\frac{5}{16}$ inches? $2\frac{7}{8}$ inches? 300; $393\frac{3}{4}$; $862\frac{1}{2}$

2. By means of a ruler measure accurately within $\frac{1}{16}$ of an inch the distance on the map between London and Liverpool. What is the distance between London and Liverpool? $168\frac{3}{4}$



Scale: 1 inch represents 300 miles.

3. Find the distances from London to each of the following cities: Glasgow, Dublin, Edinburgh. $318\frac{3}{4}$; $281\frac{1}{4}$; 300

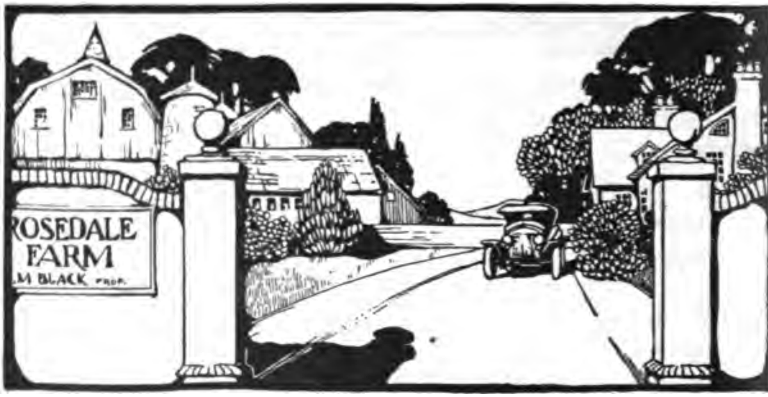
4. What is the straight-line distance from the most southerly point in England to the most northerly point in Scotland? $562\frac{1}{2}$

5. What is the distance from the most westerly point of Ireland to the most easterly point of England? $481\frac{1}{4}$

6. What is the extreme length of Ireland? $262\frac{1}{2}$

7. Get a map of your county, and find to what scale it is drawn. Then find the distances from the county seat to several places in the county. Also find the distances between other important points.

8. Get a map of your own State, and find to what scale it is drawn. Then find the distances from your home to several cities or other interesting places in the State.



1. At 14$\frac{5}{8}$ cents a pound, what is the value of a bale of cotton weighing 496 pounds? $72.54

2. At 1$\frac{4}{5}$ tons of hay to the acre, how many tons will 17$\frac{1}{2}$ acres yield? 31$\frac{1}{2}$ tons

3. If one cow eats 2$\frac{3}{8}$ tons of hay a year, how many tons will 35 cows eat? 83$\frac{1}{8}$ tons

4. If a meadow yields on an average 2$\frac{1}{4}$ tons of hay to the acre, how many acres are required to furnish 83 tons of hay? 36$\frac{8}{9}$

5. At $17.50 a ton, what is the value of a load of hay weighing 2380 pounds? $20.83

6. At 95 cents a bushel, what is the value of a load of corn weighing 2420 pounds? (One bushel of corn on the cob weighs 70 pounds.) $32.84

7. A field containing 42$\frac{3}{4}$ acres yields 2790 bushels of corn. What is the yield per acre? 65$\frac{5}{19}$

8. At $23.50 per ton, what is the cost of 1355 pounds of bran? $15.92

9. At $.75 a bushel what is the value of a load of potatoes weighing 3150 lbs.? (One bushel of potatoes weighs 60 lbs.) $39.38

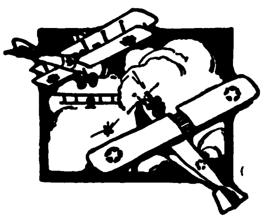
1. A military step is $2\frac{1}{2}$ feet. How many such steps are there in a mile? 2112

2. If a regiment marches at the rate of $3\frac{3}{4}$ miles per hour, how long will it take to march $17\frac{1}{2}$ miles? $4\frac{2}{3}$

3. On a forced march a regiment marched $21\frac{3}{8}$ miles in $5\frac{1}{4}$ hours. How many miles per hour did they march? $4\frac{1}{14}$

4. A fast military aeroplane has a speed of 160 miles per hour. What portion of an hour will it take this aeroplane to cover a distance of 48 miles? How many minutes is this? $\frac{3}{10}$ hr.; 18 min.

Gunpowder is made of sulphur, charcoal and saltpetre. While the composition of different powders varies slightly, it is substantially $\frac{1}{10}$ sulphur, $\frac{3}{4}$ saltpetre, and the rest charcoal.



5. What fraction of gunpowder is charcoal? $\frac{3}{20}$

6. How many pounds of sulphur, saltpetre and charcoal are there in 1000 pounds of gunpowder? 100; 750; 150

7. 25 pounds of sulphur is sufficient for making how many pounds of gunpowder? *Suggestion:* 25 is $\frac{1}{10}$ of what number? 250

8. 75 pounds of saltpetre is sufficient for making how many pounds of gunpowder? *Suggestion:* 75 is $\frac{3}{4}$ of what number? See page 77. 100

9. 50 pounds of charcoal is sufficient for making how many pounds of gunpowder? *Suggestion:* 50 is $\frac{3}{20}$ of what number? $333\frac{1}{3}$

10. Make and solve other problems on the composition of gunpowder. See who can make the most interesting problems.

United States Coins.

Following are the combinations of metal used for making coins in the United States:

Coin:	Weight:	Composition:
Gold coin	$25\frac{4}{5}$ grains per dollar	$\frac{9}{10}$ gold, $\frac{1}{10}$ copper
Silver dollar	$412\frac{1}{2}$ grains	$\frac{9}{10}$ silver, $\frac{1}{10}$ copper
Half-dollar	$192\frac{9}{10}$ grains	$\frac{9}{10}$ silver, $\frac{1}{10}$ copper
5-cent piece	$77\frac{4}{35}$ grains	$\frac{3}{4}$ copper, $\frac{1}{4}$ nickel
1-cent piece	48 grains	$\frac{19}{20}$ copper, $\frac{1}{20}$ tin and zinc

Note that *one pound = 7000 grains.*

1. How much do two half-dollars weigh? Compare this with the weight of one silver dollar. What is the difference? **$385\frac{4}{5}$; $26\frac{7}{10}$**

2. Quarters and dimes are made of the same composition of metals as the dollar and half-dollar, and weigh $\frac{1}{2}$ and $\frac{1}{5}$ as much as a half-dollar respectively. What is the weight of a quarter? of a dime? **$96\frac{9}{20}$ gr.; $38\frac{29}{50}$ gr.**

3. How many grains of copper are there in one 5-cent piece? **$57\frac{117}{140}$**

4. What is the weight in pounds of \$1000 in gold coin? **$3\frac{24}{35}$ lb.**

5. What is the weight of the pure gold in \$5000 in gold coin? **$16\frac{41}{70}$ lb.**

6. How many grains of gold coin can be made from one pound of pure gold? *Suggestion:* 7000 is $\frac{9}{10}$ of what number? **$7{,}777\frac{7}{9}$ gr.**

7. How many dollars of gold coin can be made from one pound of pure gold? **$301\frac{1}{2}$ nearly**

LIGHTING A BUILDING

A standard amount of window space for a living-room is $\frac{1}{6}$ the floor space, for a bedroom $\frac{1}{10}$ the floor space, and for a schoolroom $\frac{1}{5}$ the floor space.

1. A living-room 13 feet wide and 17 feet long has 2 windows each $3\frac{1}{2}$ feet wide and $5\frac{3}{4}$ feet high. Is this window space sufficient according to the above standard? *Suggestion:* Divide the total area of the windows by the floor space area. Is the quotient greater or less than $\frac{1}{6}$? **$\frac{161}{884}$; sufficient**

2. A schoolroom 27 feet wide and 33 feet long has 7 windows each 3 feet wide and $8\frac{3}{4}$ feet high. Is this window space sufficient according to the above standard? **$\frac{245}{1188}$; sufficient**

3. A bedroom 12 feet wide and 14 feet long has one window $3\frac{1}{4}$ feet wide and $5\frac{3}{4}$ feet high. Is this window space sufficient according to the above standards? **$\frac{299}{2688}$; yes**

4. Measure the dimensions of your schoolroom and also the windows. Is your room sufficiently lighted according to the above standard?

5. Measure the dimensions of your bedroom and also the windows in it. Is the room sufficiently lighted according to the standard given above?

1. The sum of two numbers is $84\frac{5}{6}$, and one of the numbers is $36\frac{1}{4}$. What is the other number? $48\frac{7}{12}$

2. The product of two numbers is $4\frac{3}{4}$, and one of the numbers is $3\frac{1}{2}$. What is the other number? $1\frac{5}{14}$

3. A contractor is to dig 1 mile of sewer. He averages $4\frac{5}{8}$ rods a day. How many days will it take him to dig the sewer? $69\frac{7}{37}$

4. A screw $1\frac{7}{8}$ inches long goes into the wood $\frac{3}{16}$ of an inch each time it is turned around. How many times must this screw be turned around to go into the wood its whole length? 10

5. A man finds that his automobile averages $12\frac{1}{2}$ miles on a gallon of gasoline. How many miles can he go on $17\frac{3}{4}$ gallons? $221\frac{7}{8}$

6. A steel rod will stand a pull of 40,000 pounds per square inch of cross-section. How much of a pull will a rod stand which is $1\frac{1}{8}$ inches thick and $1\frac{3}{4}$ inches wide? 78,750 lb.

7. Good building brick can stand a pressure of 4500 pounds to the square inch. At this rate, how much pressure can a brick stand which is $8\frac{1}{4}$ inches long and $4\frac{1}{8}$ inches wide? $153140\frac{5}{8}$ lb.

8. The circumference of a certain wheel is $16\frac{3}{8}$ feet. How many times will this wheel revolve in going 1 mile? $322\frac{58}{131}$

9. It is estimated that $\frac{2}{5}$ of the total population of a certain city are registered voters. According to this, what is the population of a city which has 12,680 registered voters? 31,700

10. A boy who lives $\frac{7}{8}$ of a mile from school walks this distance four times each school day. How far does he walk in one school month of 20 days? 70 miles

11. A boy buys papers for $1\frac{3}{8}$ cents apiece, and sells them at 2 cents apiece. How much does he make in one week if he sells 346 papers? $2.16

66. Decimal Fractions. In \$4.75 the 4 stands for 4 dollars, the 7 stands for 7 dimes or $\frac{7}{10}$ of a dollar, and the 5 stands for 5 cents or $\frac{5}{100}$ of a dollar. Twenty-five cents is $\frac{25}{100}$ of a dollar, and is written \$.25; 5 cents is $\frac{5}{100}$ of a dollar, and is written \$.05. This way of writing fractions of a dollar is an example of a general way of writing certain fractions, which we now proceed to study.

Fractions whose denominators are 10, 100, 1000, etc., are called *decimal fractions*, and are usually written in a form different from that of other fractions. Thus, $\frac{7}{10}$ is written .7, and $\frac{3}{100}$ is written .03. In each case only the numerator is written, and the denominator is indicated by the location of the period, which is called the *decimal point*.

An integer in the first place to the right of the decimal point represents tenths, an integer in the second place represents hundredths, and so on.

$$\frac{34}{100} \text{ is written } .34. \text{ That is, } .34 = \frac{3}{10} + \frac{4}{100}.$$

The integer in the third place represents thousandths.

$$\text{Thus, } .007 = \frac{7}{1000}, \text{ and } .049 = \frac{4}{100} + \frac{9}{1000}, \text{ or } \frac{49}{1000}$$

67. The Decimal Form. Fractions written in the form .7, .34, .049 are said to be in the decimal form. Mixed numbers, such as 3.8, 76.089, are also said to be in the decimal form. Numbers in the decimal form are usually referred to simply as decimals.

68. One-Place, Two-Place, Three-Place Decimals. The decimal .7 is a one-place decimal, .34 and .70 are two-place decimals, .080, .487, .094 are three-place decimals, and so on.

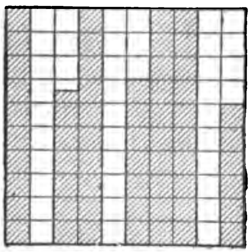
A zero is often placed to the left of the decimal point to make certain that the decimal point will be noticed. Thus, .7 may be written 0.7.

While decimal fractions are exactly the same kind of fractions as those we have studied before, we shall find that when written in the decimal form they are like whole numbers in many respects.

ORAL EXERCISES

1. If the large square represents 1, then one of the small squares represents what?

2. How many hundredths does the shaded column to the left represent? how many tenths?



3. How many hundredths are represented by the shaded part to the right?

4. How many hundredths are represented by the second shaded part from the right? This part shows that .2+.07 =.27.

5. Show that the next shaded part represents .165. Note that one-half of .01 is .005.

6. How many hundredths are represented by each of the unshaded areas?

7. Draw a figure to represent each of the following: .6, .08, .21, .64.

8. Read the numbers 400, 40, 4, .4, .04, .004.

The number 444.444 is a mixed decimal consisting of the whole number 444 and the decimal fraction .444.

9. In the number 444.444 what is represented by each figure?

69. **Place Value in Decimals.** We now notice that we can say of a mixed decimal what we have said of a whole number, namely: *Any figure represents just ten times as much as is represented by the same figure in the next place to the right of it.*

This is a very important fact, since it enables us to treat decimals in much the same way that we treat ordinary whole numbers.

Complete this statement: In a decimal any figure represents as much as is represented by the same figure in the next place to the left of it.

On the cyclometer dial shown here how many miles are represented by the figure 1? by the 4? by the 5? by the 3? by the 7?

01453·7

70. Reading Decimals. The fraction .064 represents thousandths, and is read "Sixty-four thousandths." The number .1846 represents ten-thousandths, and is read, "One thousand eight hundred forty-six ten-thousandths."

In reading a decimal the word "*and*" is used between the integral and the fractional parts. Thus, 347.981 is read, "Three hundred forty-seven *and* nine hundred eighty-one thousandths."

A one-place decimal is read *tenths*, a two-place decimal is read *hundredths*, a three-place decimal is read *thousandths*, a four-place decimal is read *ten thousandths*, and so on.

ORAL EXERCISES

Read the following numbers:

59.93, 491.076, 187.407, 81.034, 0.076, 1.0564, 0.064, 9.1560, 428.91, 20.048, 0.009, 182.046.

Persons who do a great deal of computing read a number like 3641.0047: thirty-six, forty-one, *point*, double 0, forty-seven. 4007.2901 is read: four, double 0, seven, *point*, twenty-nine, 0, one.

In this manner read the following numbers:

4217.2156, 1800.2406, 5060.3060, 7000.0028, 4900.3008, 4021.4026.

WRITTEN EXERCISES

Write the following numbers, in the decimal form:

1. One hundred seventy-five and twenty-four hundredths. 175.24
2. Seventy-six and four hundred thirteen thousandths. 76.413
3. Two and sixty-nine hundredths. 2.69
4. Sixty-three and two hundred forty-one thousandths. 63.241
5. Five hundred forty-seven and ninety-eight thousandths. 547.098
6. Five thousand four hundred seventy-nine and eight tenths. 5479.8

71. Addition of Decimals. Since $\frac{8}{10}=\frac{80}{100}=\frac{800}{1000}$, we see that we can annex as many zeros as we wish to the right of a decimal fraction without changing its value.

Example: Add, 43.096, 2.864, 517.04, 210.8, 784.915.

Solution: First write the numbers so that the decimal points stand in a straight column.

43.096	It is customary to annex zeros so there will be the same number of decimal places in all the numbers.	
2.864		
517.040	*Thousandths:*	Adding we get 15. Write 5, carry 1 (hundredth).
210.800	*Hundredths:*	Adding we get 21. Write 1, carry 2 (tenths).
784.915	*Tenths:*	Adding we get 27. Write 7, carry 2 (ones).
1558.715	*Ones:*	Adding we get 18. Write 8, carry 1 (ten).
		The remaining columns are added as usual.

Put a decimal point in the sum directly below the decimal points in the numbers added.

In practice we simply add the first column, getting 15. Then we say "Write 5, carry 1," and so on, as if there were no decimal point.

From this example we see that decimal numbers are added exactly like whole numbers. This is one of the great advantages of decimal fractions over other fractions.

WRITTEN EXERCISES

Copy and add the following:

1. 2194.0491+73.81+2.091+8,468+.0412. 10737.9913

2. 43.805+125.1704+3104.1509+1.0478. 3274.1741

3. .00478+35.419+749.041+875.0149. 1659.47968

4. 3146.1508+.04912+89.17+7.46+8.05. 3250.87992

5. 142.823+92.764+39.87+810.094+38.971. 1124.522

Drill in fundamentals. Play game No. 3 on page 30. Add common fractions for the first event, multiply fractions for the second event, and divide fractions for the third.

We have already written a number like 25 dollars and 75 cents in the form $25.75. This is a real decimal.

WRITTEN EXERCISES

Add the following:

1. $81.09	2. $124.36	3. $240.23
243.74	48.19	75.14
61.35	74.06	89.12
24.76	135.24	195.35
146.51	73.41	47.56
$557.45	$455.26	$647.40
4. $720.14	5. $583.21	6. $29.41
86.03	96.15	36.23
94.25	83.52	147.28
147.50	146.31	633.91
89.40	284.19	339.17
$1137.32	$1193.38	$1186.00

Example: From 49.06 subtract 25.324.

49.060
25.324
23.736

Write the numbers so that the decimal points stand in a column If necessary annex zeros in the minuend until it has as many decimal places as the subtrahend. Subtract exactly as if the numbers were whole numbers. Put a decimal point in the remainder directly under the decimal points in the subtrahend and minuend.

Subtract the following.

7. 24.49	8. 123.01	9. 239.12
17.28	84.30	184.40
7.21	38.71	54.72
10. 196.035	11. 384.067	12. 186.07
83.12	168.07	116.181
112.915	215.997	69.889

13. Following are readings of an automobile cyclometer. Find how far the machine went each day.

Monday morning	1489.4	Wednesday night	1947.8
Monday night	1643.8	Thursday night	2102.0
Tuesday night	1793.1	Friday night	2284.6

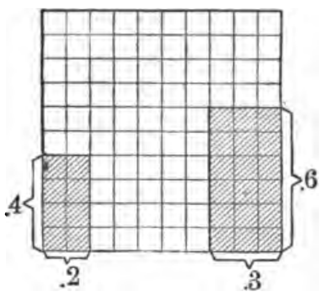
154.4; 149.3; 154.7; 154.2; 182.6

72. Multiplication of Decimals. Notice that in the large square below one vertical column represents .1 of the whole figure.

ORAL EXERCISES

1. The shaded area on the left represents .4 of .2. How many hundredths of the whole figure is this?

2. The shaded area on the right represents .6 of .3. How many hundredths of the whole figure is this?



3. In this same figure point out .7 of .6. How many hundredths of the whole figure is this:

4. Draw a figure like this to represent .7 of .2. How many hundredths is this?

5. In this same figure show .1 of .3. How many hundredths is this?

6. In this figure point out .4 of .8. How many hundredths of the whole figure is this?

Since .4 of .2 is the same as .4×.2, and .6 of .3 is the same as 6×.3, we have just found these products.

7. From the examples already solved find the following products: .7×.2, .1×.3, .7×.6, .4×.8.

8. Give each of the following products, finding as many as you can without drawing a figure:

.3×.7	.5×.8	.1×.5	.3×.3	.4×.4
.5×.5	.6×.6	.7×.7	.8×.8	.9×.9
.4×.8	.3×.5	.5×.3	.6×.3	.7×.3
.7×.6	.3×.9	.5×.2	.6×.9	.7×.5
.8×.9	.4×.7	.5×.7	.6×.7	.7×.8
.4×.8	.4×.2	.5×.9	.6×.8	.7×.9

We will now study the multiplication of decimals systematically and without using the figure.

Example: Multiply .3 by .4.

Since .3 and .4 may be written $.3 = \frac{3}{10}$ and $.4 = \frac{4}{10}$, we simply need to find the product of $\frac{3}{10} \times \frac{4}{10} = \frac{12}{100} = .12$. Hence $3. \times .4 = .12$. Similarly $.24 \times .8 = \frac{24}{100} \times \frac{8}{10} = \frac{192}{1000} = .192$.

WRITTEN EXERCISES

In this manner find the following indicated products:

1. $.15 \times .3$ .045
2. $.41 \times .5$ .205
3. $.03 \times .12$ .0036
4. $.04 \times .73$ .0292
5. $.14 \times .21$ .0294
6. $.64 \times .02$ .0128

73. **Placing the Decimal Point in the Product.** In each of these the product could be obtained more easily by multiplying as if the numbers were whole numbers, and then properly placing the decimal point in the product.

The examples $.5 \times .3 = .15$, $.41 \times .5 = .205$, and $.03 \times .12 = .0036$ illustrate the following rule:

> *To find the product of two numbers, one or both of which are decimals, multiply as if they were whole numbers, and then point off as many decimal places in the product as there are decimal places in both factors together.*

ORAL EXERCISES

According to this rule, what is the number of decimal places in the products of the following:

1. $.04 \times .6$	5. $.01 \times .04$	9. $.25 \times .75$
2. $.26 \times .04$	6. $.391 \times .214$	10. $.24 \times .04$
3. $.39 \times .6$	7. $.702 \times .4$	11. $.62 \times .081$
4. $.35 \times .6$	8. $.104 \times .12$	12. $.81 \times .142$

Find the products in these examples, and make sure you put the decimal point in the right place.

ORAL EXERCISES

Find the products of the following, using the rule on page 117.

1. $.08 \times .07$	4. $.09 \times .05$	7. $.9 \times .08$
2. $.09 \times .04$	5. $.90 \times .8$	8. $.8 \times .02$
3. $.70 \times .4$	6. $.07 \times .6$	9. $.06 \times .04$

Example. Multiply 3.4 by 6.

$$\begin{array}{r} 3.4 \\ 6 \\ \hline 20.4 \end{array}$$

By the rule on page 117 we get 20.4 as the product. Test this by writing $3.4 = 3\frac{4}{10}$ and then taking the product, $6 \times 3\frac{4}{10}$.

WRITTEN EXERCISES

Multiply:

1. $54 \times .9$	48.6	7. $16 \times .28$	4.48	13. $4 \times .024$	.096
2. $8 \times .42$	3.36	8. $5 \times .162$	.810	14. $8 \times .08$	.64
3. $9 \times .092$	.828	9. $7 \times .25$	1.75	15. $9 \times .422$	3.798
4. $3 \times .64$	1.92	10. $41 \times .64$	26.24	16. $5 \times .62$	3.10
5. 5×6.3	31.5	11. 8×12.5	100	17. 6×48.4	290.4
6. 9×4.5	40.5	12. 7×24.3	170.1	18. 4×38.45	153.8

Example. Multiply 2.463 by 45.

$$\begin{array}{r} 2.463 \\ 45 \\ \hline 12315 \\ 9852 \\ \hline 110.835 \end{array}$$

Multiply exactly as if the numbers were integers, and then place the decimal point in the product according to the rule on page 117. The decimal points are omitted in the partial products.

In this manner multiply the following:

19. 64×14.36 919.04	22. 29×197.013 5713.377	25. 17×96.048 1632.816
20. 78×85.019 6631.482	23 124×69.103 8568.772	26. 59×19.340 1141.06
21. 9×1566.207 14095.863	24. 245×61.01 14947.45	27. 132×243.01 32077.32

Example. Multiply 3.7 by 2.4.

3.7 2.4 148 74 8.88	Again multiply exactly as if the numbers were ordinary integers, and then place the decimal point in accordance with the rule on page 117.

To verify that this gives the correct result, we write 3.7 and 2.4 in forms $3\frac{7}{10}$ and $2\frac{4}{10}$, and then multiply them as ordinary mixed numbers. In which form are these numbers multiplied more easily?

74. Products to the Nearest Tenth, Nearest Hundredth, etc. The method of finding products to the nearest tenth, etc., is shown in the following examples:

Examples. Find the product of 8.7×19.7 to the nearest tenth, and find the product of .78×39.42 to the nearest hundredth.

19.7 8.7 1379 1576 171.39	The product is nearer 171.4 than 171.3. Hence, 171.4 is the required product.	39.42 .78 31536 27594 30.7476	The product is nearer 30.75 than 30.74. Hence 30.75 is the required product. The product to the nearest thousandth is 30.748.

In case the exact product is, for instance, 17.85, it is customary to take 17.9 as the nearest tenth, though 17.8 is equally near the exact result.

WRITTEN EXERCISES

Find the exact product of the first five, the products of the next five to the nearest hundredth, and of the last five to the nearest thousandth.

1.	3.9×6.2	24.18	6.	4.8×1.64	7.87	11.	3.14×.86	2.700
2.	1.3×5.9	7.67	7.	3.52×5.72	20.13	12.	4.93×3.61	17.797
3.	82.5×2.4	198.	8.	5.23×7.52	39.33	13.	7.46×9.23	68.856
4.	41.6×3.8	158.08	9.	39.8×26.7	1062.66	14.	4.9×2.785	13.647
5.	1.36×45.2	61.472	10.	41×4.79	196.39	15.	8.6×1.426	12.264

Gold coins weigh $25\frac{4}{5}$, or 25.8 grains per dollar. The silver dollar weighs $412\frac{1}{2}$ or 412.5 grains. The half-dollar weighs $192\frac{9}{10}$, or 192.9 grains. The metal in the gold and silver coins is $\frac{9}{10}$, or .9 pure, the remaining .1 being copper. (See page 108.)

WRITTEN EXERCISES

In the following problems use the decimal form of the fractions:

1. How many grains of pure gold per dollar are there in a gold coin? how many grains of copper? 23.22; 2.58

2. How many grains of pure silver are there in a silver dollar? how many grains of copper? 371.25; 41.25

3. How many grains of pure silver are there in 200 half-dollars? how many grains of copper? 34722: 3858

4. How many grains of pure silver are there in 100 silver dollars? how many grains of copper? 37125; 4125

On page 108 similar problems were solved, using the common fractions. Which are more convenient, the common fractions or the decimal fractions?

The composition of gunpowder (see page 107) may be expressed in decimals: $\frac{1}{10}$ or .1 is sulphur, $\frac{3}{4}$ or $\frac{75}{100}$ or .75, is saltpetre, and $\frac{3}{20}$ or $\frac{15}{100}$ or .15, is charcoal.

5. How many pounds of each of these ingredients are needed to make 1000 pounds of gunpowder? 100; 750; 150

The usual measure of distance used at sea is the knot, or nautical mile. This is 6080 feet.

6. How much does a nautical mile differ from 1.151 statute miles? 2.72 ft.

Suggestion: First multiply 1.151 by 5280.

7. On her trial trip the Mauretania (the fastest merchant steamship afloat) made 27.3 knots per hour. How many ordinary miles is this, counting 1.151 miles as one knot. 31.42

1. The fastest journey ever made across the Atlantic Ocean was made at an average speed of 26.02 knots per hour. How many miles per hour was this (1 knot = 1.151 miles)? **29.95**

2. One rod is $5\frac{1}{2}$ (5.5) yards. How many square yards are there in one square rod? How many square yards are there in 160 square rods, or one acre? **30.25; 4840.**

3. One rod is $16\frac{1}{2}$ (16.5) feet. How many square feet are there in one square rod? in one acre? **272.25; 43,560**

4. One bushel contains exactly 2150.42 cubic inches. Multiply this number of cubic inches by .8. By how much does the result differ from the number of cubic inches in one cubic foot? (1728 cu. in. = cu. ft.) **7.664**

5. Assuming that one cubic foot is .8 of a bushel, how many bushels are there in 10 cubic feet? in 100 cubic feet? **8; 80**

6. A grain bin is $8\frac{1}{2}$ (8.5) feet long and $6\frac{3}{4}$ (6.75) feet wide. How many bushels of grain does it hold if the grain is 5 feet deep in the bin? Regard one cubic foot as .8 of a bushel. **229.5**

7. One cubic foot of water weighs about 62.5 pounds. Ice weighs .92 times as much as water. What is the weight of one cubic foot of ice? **57.5**

8. By how much does 35 cubic feet of ice differ in weight from one ton? **12.5 lb**

9. The weight of cork is .24 that of water. What is the weight of one cubic foot of cork? By how much does this differ from the weight of one cubic foot of water? **15 lb.; 47.5 lb.**

10. The weight of the water in the Dead Sea is 1.24 times that of ordinary fresh water. What is the weight of one cubic foot of water from the Dead Sea? **77.5**

11. Pure gold weighs 19.36 times as much as water. What is the weight of one cubic foot of gold? (See example 7.) **1210 lb.**

75. Multiplying by 10, 100, etc. To multiply a decimal by 10 move the decimal point one place to the right, to multiply by 100 move the decimal point two places to the right, and to multiply by 1000 move the decimal point three places to the right. If necessary annex zeros.

Thus, $100 \times .4 = 40.$ and $100 \times 4. = 400.$

This follows directly from the place value in the decimal notation.

ORAL EXERCISES

Give the products in the following, annexing zeros if necessary:

1. $100 \times .3$	5. $100 \times .03$	9. $100 \times .003$
2. 100×2.4	6. 100×24.01	10. $100 \times .247$
3. $100 \times 4.$	7. $100 \times 70.$	11. $100 \times .510$
4. $1000 \times .04$	8. $1000 \times .3202$	12. 1000×5.2

13. How may a number be multiplied by 10,000?

76. Dividing by 10, 100, 1000, etc. To divide by 10, 100, 1000 move the decimal point one, two, or three places to the left.

This may be seen directly from the definition of division.

Thus, $10 \times 4.151 = 41.51$, *therefore* $41.51 \div 10 = 4.151.$
$100 \times 4.151 = 415.1$, *therefore* $415.1 \div 100 = 4.151.$
$1000 \times 4.151 = 4151$, *therefore* $4151 \div 1000 = 4.151.$

ORAL EXERCISES

Perform the following indicated operations, if necessary prefixing zeros before placing the decimal point:

1. $.21 \div 10$	6. $84.21 \div 10$	11. $.08 \div 100$
2. $.5 \div 100$	7. $6.93 \div 100$	12. $.001 \div 1000$
3. $.28 \div 1000$	8. $421. \div 1000$	13. $.096 \div 100$
4. $17. \div 100$	9. $27.5 \div 10000$	14. $.02 \div 1000$
5. $.24 \div 100$	10. $.03 \div 10$	15. $0.71 \div 1000$

Example. Divide 871.490 by 37.

```
         4
    23.55~~3~~
37)871.490
   74
   131
   111
    204
    185
     199
     185
      140
      111
       29
```

Divide exactly as if there were no decimal point in the dividend. The first number in the quotient is 2 (tens), which is placed directly above the tens in the dividend. Place a decimal point in the quotient directly above the decimal point in the dividend.

To test, multiply the quotient by 37 and add the remainder.

If the last remainder is less than half the divisor it is disregarded. If it is equal to or greater than half the divisor, 1 is added to the last figure in the quotient. In the above example the quotient is 23.554, which is correct to the nearest thousandth.

```
  23.553
      37
  164871
 70659
 871.461
      29
 871.490
```

WRITTEN EXERCISES

In each of the following find the quotient to the nearest thousandth.

1.	29.374 ÷ 19	1.546	13.	46.94 ÷ 29	1.619
2.	134.048 ÷ 87	1.541	14.	789.0013 ÷ 497	1.588
3.	218.309 ÷ 122	1.789	15.	849.0147 ÷ 830	1.023
4.	197.016 ÷ 88	2.239	16.	973.0504 ÷ 97	10.031
5.	246.009 ÷ 129	1.907	17.	841.9062 ÷ 196	4.295
6.	213.017 ÷ 301	.708	18.	9470.841 ÷ 856	11.064
7.	715.290 ÷ 285	2.510	19.	3746.240 ÷ 563	6.654
8.	649.290 ÷ 386	1.682	20.	2913.572 ÷ 365	7.982
9.	947.0341 ÷ 278	3.407	21.	1923.275 ÷ 1240	1.551
10.	649.021 ÷ 486	1.335	22.	4936.216 ÷ 365	13.524
11.	785.463 ÷ 594	1.322	23.	746.539 ÷ 416	1.795
12.	973.058 ÷ 786	1.238	24.	1342.741 ÷ 894	1.502

Example 1. Divide 1.2042 by 87.

```
   .0138
87)1.2042
    87
    334
    261
     732
     696
      36
```

The first dividend is 1.20, or 120 hundredths, and the first quotient figure is placed directly above the last figure in the first dividend (in hundredth's place). The decimal point in the quotient is placed directly above the decimal point in the dividend.

WRITTEN EXERCISES

Find quotients to the nearest ten-thousandth.

1. 2.3604 ÷ 198 .0119
2. 3.0421 ÷ 297 .0102
3. 7.0809 ÷ 346 .0205
4. 8.4012 ÷ 427 .0197
5. 9.4708 ÷ 597 .0159
6. 96.058 ÷ 3970 .0242
7. 84.096 ÷ 298 .2822
8. 78.009 ÷ 3090 .0252
9. .0781 ÷ 317 .0002
10. .0091 ÷ 248 .00004
11. 0.4729 ÷ 67 .0071
12. 1.7164 ÷ 24 .0715
13. 46.2091 ÷ 73 .6330
14. 19.0246 ÷ 94 .2024
15. 5.9007 ÷ 31 .1903

Example 2. Divide 34.1415 by 2.36.

```
          7
      14.4~~6~~
236)3414.15
    236
    1054
     944
     1101
      944
      1575
      1416
       159
```

First multiply both divisor and dividend by 100 to get rid of the decimal in the divisor.

Then divide as before. Since the remainder is greater than half the divisor, add 1 to the last figure in the quotient. The result is 14.47, which is correct to the nearest hundredth.

Find quotients to the nearest tenth.

16. 8.937 ÷ 1.45 6.2
17. 9.403 ÷ 2.57 3.7
18. 7.516 ÷ 3.08 2.4
19. 12.046 ÷ 4.03 3.0
20. 16.471 ÷ 5.43 3.0
21. 23.057 ÷ 6.24 3.7
22. 34.079 ÷ 8.03 4.2
23. 25.573 ÷ 9.24 2.8
24. 30.057 ÷ 8.76 3.4
25. 52.91 ÷ 1.41 37.5
26. 7.943 ÷ 2.78 2.9
27. 3.298 ÷ .781 4.2

Example 1. Divide 2479.3 by 376. Find the quotient to the nearest thousandth.

$$\begin{array}{r}
4 \\
6.59\cancel{3} \\
376\overline{)2479.3} \\
2256 \\
\overline{2233} \\
1880 \\
\overline{3530} \\
3384 \\
\overline{1460} \\
1128 \\
\overline{332}
\end{array}$$

To get the quotient beyond tenths, annex zeros to the remainders. This has exactly the same effect as if the dividend were written 2479.300, which we know from page 114 equals 2479.3.

The quotient to the nearest thousandth is 6.594.

Example 2. Divide 164.7 by 2.051.

Multiplying *dividend* and divisor by 1000, we have $2051\overline{)164700.}$ Notice that we must annex zeros in the dividend before moving the decimal point three places to the right.

WRITTEN EXERCISES

Find the quotients to the nearest thousandth:

1. $.947 \div 8.76$ — .108
2. $.89 \div 3.47$ — .256
3. $.975 \div .431$ — 2.262
4. $.202 \div 6.9$ — .029
5. $.7471 \div 9.8$ — .076
6. $.9743 \div 8.7$ — .112
7. $.0891 \div .064$ — 1.392
8. $12.07 \div .089$ — 135.618
9. $.008 \div .075$ — .107
10. $.09 \div .199$ — .452
11. $.04 \div .007$ — 5.714
12. $.00471 \div .81$ — .006
13. $1.1 \div 1.65$ — .667
14. $2.1 \div 1.84$ — 1.141
15. $3.674 \div 2.39$ — 1.537
16. $75.7 \div 4.32$ — 17.523
17. $9.047 \div 4.06$ — 2.228
18. $18.048 \div 5.19$ — 3.477
19. $9.391 \div .462$ — 20.327
20. $3.409 \div .062$ — 54.984
21. $1.16 \div .217$ — 5.346
22. $2.34 \div .214$ — 10.935
23. $5.26 \div .043$ — 122.326
24. $.783 \div .471$ — 1.662
25. $.519 \div .125$ — 4.152
26. $6.509 \div .375$ — 17.357
27. $39.28 \div 5.84$ — 6.726
28. $67.14 \div 51.6$ — 1.301
29. $41.94 \div 65.1$ — .644
30. $1205.4 \div .0142$ — 84887.324

Drill in Fundamentals. Play game No. 1, page 30. Use examples in dividing fractions.

PROBLEMS OF THE DAIRY

1. A certain cow yields 9840 pounds of milk in one year. Of this milk .041 is butter fat. What is the value of the butter fat at 43 cents a pound? Find result to the nearest cent.

(Review table of liquid measure. See page 288.) **$173.48**

2. A farmer feeds on an average 2.16 tons of hay a year to each cow. How much hay does he feed 17 cows? What is the value of this hay at $14.50 a ton? **36.72 tons; $532.44**

3. A quart of milk weighs 2.15 pounds. How many quarts of milk does the cow in problem 1 yield in one year? At an average price of $.042 per quart, what is the value of this milk? Get result to the nearest cent. **4576.7 qt.; $192.22**

4. An empty milk can weighs 16.8 pounds. Filled with milk it weighs 81.4 pounds. How many pounds of milk are there in the can? At 8.6 pounds to the gallon, how many gallons does this can hold? **64.6 lb.; 7.51 gal.**

5. A farmer sells on an average 14.3 gallons of milk each day, during the month of January. If he gets 18.6 cents a gallon, how much does he get for his milk this month? Find result to the nearest cent. **$82.45**

MISCELLANEOUS PROBLEMS

1. A load of coal weighs 7580 pounds after deducting the weight of the wagon. How many tons of coal are there in the load?

Solution: Since one ton weighs 2000 pounds, the number of tons is $\frac{7580}{2000}=\frac{758}{200}=\frac{7.58}{2}=3.79$.

2. An automobile runs 165 miles on 14 gallons of gasoline. How far does it run on one gallon? Get result to the nearest tenth of a mile. **11.8 miles**

3. How many acres are there in a field 24.6 rods wide and 61.3 rods long? **9.425 acres**

In the following problems find all results to the nearest cent:

4. At \$6.75 a ton, what is the cost of a load of coal weighing 6840 pounds?

Solution: The number of tons is $\frac{6840}{2000}$. Hence the cost in dollars is $\frac{6840}{2000}\times 6.75=3.42\times 6.75=23.085$. Hence, \$23.09 is the cost.

5. At \$1.86 a bushel, what is the value of a load of wheat weighing 2360 pounds? (1 bushel of wheat weighs 60 pounds.) **\$73.16**

6. At \$0.94 a bushel, what is the value of a load of corn in the ear weighing 3240 pounds? (1 bushel of corn in the ear weighs 70 pounds.) **\$43.51**

7. Hay is selling at \$18.45 a ton. What is the value of a load weighing 2480 pounds? **\$22.88**

8. At \$0.28 per square foot, what is the value of a lot 22.7 feet wide and 87.3 feet deep? **\$554.88**

9. In the first non-stop aeroplane flight across the Atlantic, made by Alcock and Brown, on June 15th, 1919, the distance covered was 1650 nautical miles. If one nautical mile is 1.151 ordinary miles, find this distance in ordinary miles. **1899.15 miles**

77. Reducing Common Fractions to Decimals. Since decimal fractions are more convenient in many ways than common fractions, the latter are often reduced to decimals.

Example. Reduce $\frac{3}{16}$ to a decimal fraction.

$$\begin{array}{r} .1875 \\ 16\overline{)3.0000} \\ 1\,6 \\ \hline 140 \\ 128 \\ \hline 120 \\ 112 \\ \hline 80 \end{array}$$

Since $\frac{3}{16}$ means $3 \div 16$, we need only to divide 3 by 16. We write a decimal point after the 3 and add as many zeros as we wish. The result of dividing is .1875.

In practice we use short division when the divisor is not too large. Thus, to reduce $\frac{3}{8}$ to a decimal we write

$$8\overline{)3.000}^{\,.375} \quad \text{or simply} \quad 8\overline{)3.}^{\,.375}$$

The great majority of fractions in practical use have 2, 4, 8, 16, 32, or 64 as denominators. We will now reduce such fractions to decimals.

We know at sight that $\frac{1}{2}=.5$, $\frac{1}{4}=.25$. That is, halves may be reduced to 10ths, and 4ths to 100ths. Similarly, $\frac{1}{8}=.125$.

WRITTEN EXERCISES

1. Reduce each of the following to a decimal: $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$. From these we see that 8ths are reducible to 1000ths. .375; .625; .875

2. Reduce $\frac{1}{16}$, $\frac{3}{16}$, $\frac{5}{16}$, $\frac{7}{16}$, $\frac{9}{16}$, $\frac{11}{16}$, $\frac{13}{16}$, $\frac{15}{16}$, to decimals. From these we see that 16ths are reducible to 10000ths.
.0625; .1875; .3125; .4375; .5625; .6875; .8125; .9375

3. Write all proper fractions which are in their lowest terms and whose denominators are 32. Reduce five of these to decimals. What do you conclude about the reducibility of 32ds?
$\frac{1}{32}=.03125$; $\frac{3}{32}=.09375$; $\frac{5}{32}=.15625$; $\frac{7}{32}=.21875$; $\frac{9}{32}=.28125$

4. Write all proper fractions which are in their lowest terms and whose denominators are 64. Reduce five of these to decimals. What do you conclude about the reducibility of 64ths?
$\frac{1}{64}=.015625$; $\frac{3}{64}=.046875$; $\frac{5}{64}=.078125$; $\frac{7}{64}=.109375$; $\frac{9}{64}=.140625$

Example 1. Reduce $\frac{1}{3}$ to a decimal fraction, giving the result correct to the nearest thousandth.

$$3\overline{)1.000}\quad .333$$

Dividing, we obtain a quotient, .333 and a remainder, 1. Since the remainder is less than one-half of 3, 333 is the required quotient.

Example 2. Reduce $\frac{2}{3}$ to a decimal fraction, giving the result to the nearest thousandth.

$$3\overline{)2.000}\quad .667$$

Dividing, we get .666 with a remainder of 2, which is more than $\frac{1}{2}$ of 3. Hence, the result to the nearest thousandth is .667.

WRITTEN EXERCISES

Reduce the following fractions to decimals and give results to the nearest ten-thousandths.

1. $\frac{7}{8}$ .8750	6. $\frac{3}{16}$ .1875	11. $\frac{4}{11}$ .3636	16. $\frac{5}{16}$ .3125	21. $\frac{9}{14}$ .6429
2. $\frac{3}{7}$ .4286	7. $\frac{5}{12}$ .4167	12. $\frac{7}{9}$ .7778	17. $\frac{7}{15}$ .4667	22. $\frac{11}{16}$ .6875
3. $\frac{4}{7}$ .5714	8. $\frac{3}{8}$ .3750	13. $\frac{5}{8}$ .6250	18. $\frac{5}{14}$ .3571	23. $\frac{11}{18}$ .6111
4. $\frac{4}{9}$ .4444	9. $\frac{9}{11}$ .8182	14. $\frac{6}{11}$ .5455	19. $\frac{9}{16}$ .5625	24. $\frac{11}{12}$ .9167
5. $\frac{6}{7}$ .8571	10. $\frac{8}{9}$ .8889	15. $\frac{7}{12}$ .5833	20. $\frac{5}{18}$ .2778	25. $\frac{7}{16}$ .4375

Sometimes a common fraction is reduced to part decimal and part common fraction.

Thus, $\frac{1}{3} = .33\frac{1}{3}$. This is read "$33\frac{1}{3}$ hundredths." Similarly, $\frac{2}{3} = .66\frac{2}{3}$.

WRITTEN EXERCISES

Reduce each of the following to a two-place decimal and a fraction (if necessary):

1. $\frac{5}{6}$ $.83\frac{1}{3}$	6. $\frac{7}{12}$ $.58\frac{1}{3}$	11. $\frac{9}{16}$ $.56\frac{1}{4}$	16. $\frac{7}{20}$ .35
2. $\frac{3}{8}$ $.37\frac{1}{2}$	7. $\frac{11}{12}$ $.91\frac{2}{3}$	12. $\frac{11}{16}$ $.68\frac{3}{4}$	17. $\frac{9}{20}$ .45
3. $\frac{5}{8}$ $.62\frac{1}{2}$	8. $\frac{3}{16}$ $.18\frac{3}{4}$	13. $\frac{13}{16}$ $.81\frac{1}{4}$	18. $\frac{11}{20}$ .55
4. $\frac{7}{8}$ $.87\frac{1}{2}$	9. $\frac{5}{16}$ $.31\frac{1}{4}$	14. $\frac{15}{16}$ $.93\frac{3}{4}$	19. $\frac{13}{20}$ .65
5. $\frac{5}{12}$ $.41\frac{2}{3}$	10. $\frac{7}{16}$ $.43\frac{3}{4}$	15. $\frac{3}{20}$ .15	20. $\frac{17}{20}$ .85

78. Fractions Reducible to Decimals. The fraction $\frac{1}{2}$ can be reduced to 10ths because 10 is a multiple of 2. Likewise $\frac{1}{4}$ can be reduced to 100ths because 100 is a multiple of 4.

On the other hand, $\frac{1}{3}$ or $\frac{2}{3}$ cannot be reduced exactly to a decimal, because none of the numbers, 10, 100, 1000, etc., is a multiple of 3.

Since the numbers 10, 100, 1000, etc., have no prime factor except 2 and 5, it follows that:

> *A fraction may be reduced exactly to a decimal if its denominator has no factor except 2 and 5; otherwise not.*

That is, fractions with denominators, 2, 4, 5, 8, 16, 20, 25, 32, 50, can be reduced exactly to decimals, while fractions whose denominators are 3, 7, 12, 15, 24 cannot be so reduced.

ORAL EXERCISES

State which of the following can be reduced exactly to decimals: $\frac{3}{5}$, $\frac{5}{8}$, $\frac{4}{9}$, $\frac{5}{12}$, $\frac{9}{16}$ $\frac{7}{20}$, $\frac{9}{25}$, $\frac{9}{32}$, $\frac{1}{120}$, $\frac{17}{75}$.

79. Fractions and Decimals to be Memorized. There are certain fractions whose decimal equivalents should be memorized. These are:

$\frac{1}{2}=.50$	$\frac{1}{5}=.20$	$\frac{1}{8}=.12\frac{1}{2}$
$\frac{1}{3}=.33\frac{1}{3}$	$\frac{2}{5}=.40$	$\frac{1}{12}=.08\frac{1}{3}$
$\frac{2}{3}=.66\frac{2}{3}$	$\frac{3}{5}=.60$	$\frac{1}{16}=.06\frac{1}{4}$
$\frac{1}{4}=.25$	$\frac{4}{5}=.80$	$\frac{1}{20}=.05$
$\frac{3}{4}=.75$	$\frac{1}{6}=.16\frac{2}{3}$	$\frac{1}{25}=.04$

For the purpose of comparison, common fractions are sometimes reduced to mixed decimal and common fractions, even when they can be reduced exactly to decimals having a larger number of places.

Thus, the fractions $\frac{3}{8}$, $\frac{2}{5}$, $\frac{7}{20}$, $\frac{5}{14}$ equal respectively $.37\frac{1}{2}$, $.40$, $.35$, $.35\frac{5}{7}$. These may now readily be arranged in order of their magnitude.

1. How may a decimal number be multiplied by 10? by 100? by 1000?

2. How may a decimal number be divided by 10? by 100? by 1000?

3. Tell how to multiply decimals, also how to divide them.

4. How do you find the value of a load of wheat if you know the price per bushel and the weight of the wheat?

5. If you know the number of cubic yards to be removed from an excavation, and also the number which can be removed in one day, how can you tell how long it will take to complete the work?

6. If you know the weight of a load of coal when weighed on the wagon, and also the weight of the empty wagon, how can you tell how many tons there are in the load?

7. If you know the dimensions in feet of a grain bin, and the number of cubic feet in one bushel, how do you find the number of bushels which the bin will hold?

8. If you know the dimensions of a basement in feet, how do you find how many cubic yards it contains?

9. If you know the dimensions of a box in inches, how do you find how many cubic feet it contains?

10. If you know the area of a rectangle and also its length, how do you find its width?

11. If you know the volume of a box and two of its dimensions, how do you find the third dimension?

12. A farmer knows the area of two hay fields and the number of tons of hay in one of them. How can he find the number of tons in the other field if the two fields run about the same number of tons to the acre?

ORAL EXERCISES

1. In a certain class of 39 pupils there are 18 boys. What fractional part of the class are boys? Clearly $18/39$ or $6/13$ of the class are boys. What fractional part of the class are girls?

2. Count the number of boys and girls in your class. What fractional part of the class are girls? What fractional part of the class are boys?

3. The baseball team of the Jones School played 17 games and won 11. What fraction of the games played did they win?

4. The baseball team of the Franklin School played 23 games and won 15. What fraction of the games played did they win?

Example. Which of the two teams in examples 3 and 4 won a larger fraction of the games played?

Solution: Reducing the fractions $\frac{11}{17}$ and $\frac{15}{23}$ to a common denominator, we have $\frac{11}{17}=\frac{253}{391}$ and $\frac{15}{23}=\frac{255}{391}$, which can be compared easily.

It is frequently necessary to arrange in the order of their magnitude a large number of fractions. This is done most easily by reducing them to decimals, and then comparing the decimals.

WRITTEN WORK

1. Reduce to a decimal the fraction won of the games played by each of the following schools:

School	*Games played*	*Won*	*Lost*	*Standing*
Crane..................	17	8	9	.471
Marshall...............	18	10	8	.556
Lake View..............	19	9	10	.474
Carter Harrison........	18	9	9	.500
Wendell Phillips.......	17	10	7	.588
Hyde Park..............	16	7	9	.438
Nicholas Senn..........	19	9	10	.474

WRITTEN WORK

The following were the number of games played and the number of games won and lost by each team of the National and American Leagues during a recent season:

National League				American League			
Club		Games Played	Games Won	Club		Games Played	Games Won
Boston	.431	123	53	Boston	.595	126	75
Brooklyn	.452	126	57	Chicago	.425	134	57
Chicago	.651	129	84	Cleveland	.566	129	73
Cincinnati	.531	128	68	Detroit	.437	126	55
New York	.573	124	71	New York	.488	123	60
Philadelphia	.447	123	55	Philadelphia	.406	128	52
Pittsburgh	.520	125	65	St. Louis	.484	124	60
St. Louis	.395	129	51	Washington	.563	128	72

1. What fraction of the number of games played was won by each team? Reduce each of these fractions to a decimal, correct to the nearest thousandth. Make a list of the teams in each league, showing their standing at the end of the season. Arrange them in the order of their standing.

This is exactly how the standing of these teams is computed from day to day during the whole baseball season.

2. Find the number of games won and lost by some baseball teams in your neighborhood during the last season. What fraction of the games played did they win?

3. Find the number of games lost and won by each team in one of the big leagues during the season just passed, and make a list showing their standing for the season.

4. Find the number of boys and the number of girls in each room in your school building, and arrange them in order so that the one having the largest fractional part of girls shall be first, the one with the next largest fractional part of girls second, and so on. Do the same for the boys. If your school is very small the teacher will suggest numbers of boys and girls for an eight-room school.

80. Percentage. Many fractions are regularly expressed as hundredths. Indeed this is done so frequently that a special name is given to hundredths. Instead of saying so many *hundredths*, we say so many *per cent.* The words *per cent* come from the Latin *per centum* (in the hundred). The symbol % stands for per cent. Thus, .06 = 6%, and .15 = 15%.

Example. How many per cent are there in .048?

Solution: To find how many per cent or hundredths there are in a number multiply it by 100. Hence $100 \times .048 = 4.8$ is the number of per cent in .048.

We can also see this directly. The 4 stands in hundredths' place, and hence represents 4 hundredths, or 4%. The 8 is in thousandths' place, and hence represents 8 tenths of one hundredth, or .8%.

You should make the above perfectly clear to yourself, as it is fundamental in all work involving per cent.

How many per cent are there in each of the following decimals?

1. .043	4. .125	7. .142	10. .392
2. .254	5. .093	8. .286	11. .005
3. .035	6. .085	9. .491	12. .017

Example. Express $\frac{1}{3}$ as per cent, giving result to the nearest tenth of one per cent.

Solution: Reduce $\frac{1}{3}$ to a decimal correct to the nearest thousandth and multiply by 100.

That is, $\frac{1}{3} = .333 = 33.3\%$.

WRITTEN EXERCISES

Reduce each of the following fractions to the nearest thousandth, and read the results as per cent: $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{3}{5}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$.

25%, 75%, 33.3%, 20%, 60%, 16.7%, 83.3%, 12.5%, 37.5%, 62.5%

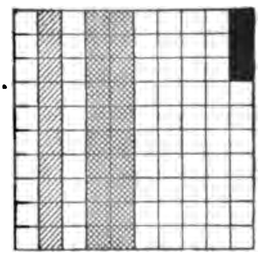
WRITTEN EXERCISES

Reduce each of the fractions given below to thousandths. Write and read each as per cent to the nearest tenth of one per cent.

1. $\frac{1}{2}$ 50%	6. $\frac{5}{8}$ 62.5%	11. $\frac{7}{8}$ 87.5%	16. $\frac{9}{16}$ 56.3%
2. $\frac{1}{4}$ 25%	7. $\frac{11}{12}$ 91.7%	12. $\frac{5}{12}$ 41.7%	17. $\frac{11}{16}$ 68.8%
3. $\frac{3}{4}$ 75%	8. $\frac{1}{5}$ 20%	13. $\frac{3}{16}$ 18.8%	18. $\frac{13}{16}$ 81.3%
4. $\frac{1}{3}$ 33.3%	9. $\frac{3}{7}$ 42.9%	14. $\frac{5}{16}$ 31.3%	19. $\frac{15}{16}$ 93.8%
5. $\frac{2}{3}$ 66.7%	10. $\frac{3}{8}$ 37.5%	15. $\frac{7}{16}$ 43.8%	20. $\frac{11}{12}$ 91.7%

81. Special Study of Per Cent. The use of per cent (hundredths) is so important that we will study such fractions more fully:

ORAL EXERCISES



1. How many small squares are there in this figure?
2. How many per cent of the large square are there in one small square?
3. How many per cent of the large square are there in 5 small squares? in 10 small squares?
4. How many per cent of the large square are black?
5. How many per cent of the large square are shaded so ?
6. How many per cent of it are shaded so ?
7. How many per cent of the large square are there in 25 small squares? in 50 small squares? in 100 small squares?
8. If a number is in the form of a decimal, how do you find the number of per cent to which it is equal? Show by examples.
9. How do you find the number of per cent equal to a given common fraction? Show by an example.

ORAL EXERCISES

1. Find .06 of 200. Find 6% of 200.

2. Find 5% or .05 of each of the following: 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000.

3. Find 10% or .10 of each of the numbers in example 2.

4. Find 20% of each of the following: 200, 400, 600. Notice that 20% is the same as $\frac{1}{5}$.

5. Find 25% of 400, of 600, of 800, of 1000. Notice that 25% is the same as $\frac{1}{4}$.

6. 50% is equal to what fraction? Find 50% of each of the following: 240, 684, 860.

REDUCTION OF 40% ON ALL SUITS AND OVERCOATS

This is from an advertisement announcing a special sale of clothing. It means that the prices are reduced 40% of the usual price. Thus, a suit marked \$25 will be reduced .40×\$25=\$10.00, and hence will sell for \$15.00.

ORAL AND WRITTEN EXERCISES

At a reduction of 40%, what are the prices of these articles? Give as many as you can orally.

1. A suit................marked \$30.00 \$18.00
2. A raincoat............marked \$18.00 10.80
3. An overcoat..........marked \$25.00 15.00
4. A suit...............marked \$40.00 24.00
5. An overcoat...........marked \$36.00 21.60
6. Boy's suit.............marked \$12.00 7.20

Look in your paper to see if there are any advertisements like these. Bring one to school and make problems from it.

82. Discounts. Many articles, such as books, are listed at certain prices in catalogues, and then sold at a *discount*. If a book is listed at \$1.50 and sold at a discount of 20%, this means that the price is reduced by 20% of the list price.

Following is a bill for such a book:

THE WEBSTER FORD COMPANY,
Chicago, Illinois.

August 14, 1918.

Sold to Robert J. Brown,
Des Moines, Iowa.

August 13.	Successful Houses and How to Build Them...	\$1.80
	Less 20%	.36
	Net..	\$1.44

WRITTEN EXERCISES

Make out bills for the following, using the name of a company you know as the seller, and your own name as the buyer:

1. One book at \$2.50, discount 10%. **\$2.25**

2. One book at \$3.50, discount 25%. **\$2.63**

3. A pair of shoes marked \$5.00, discount 30%. **\$3.50**

4. A picture marked \$15, discount 20%. **\$12.00**

Sometimes discount is allowed for immediate cash payments, or for other reasons. Make out bills for the following:

5. One Century Dictionary at \$85, discount 10% for cash. **\$76.50**

6. One set of Encyclopedia Britannica, cloth binding, \$150, discount 5% for cash. **\$142.50**

7. One set of the same Encyclopedia, suède binding, \$225, discount 5% for cash. **\$213.75**

8. One piano \$450, discount 10% for cash. **\$405**

WRITTEN EXERCISES

1. Copy and fill out the following bill. Insert the name of a store you know as the seller and use your own name as the purchaser. Use the date on which you are doing this work.

CRAWFORD & COMPANY,
Cleveland, Ohio.

December 30, 1918.

Sold to Stephen Langmaid,
489 E. 103rd Street, City.

Dec. 4.	7 pairs of curtains at \$3.75 per pair	\$26.25
	1 picture \$15.00	15.00
	1 picture \$7.50	7.50
	1 picture \$10.75	10.75
	6 small pictures at \$1.40 apiece	8.40
	1 clock \$18.00	18.00
		\$85.90
	Less 10%	8.59
	Net	\$77.31

Make a bill for each of the following:

2. 6 dining room chairs at \$4.75 apiece, 3 parlor chairs at \$7.80 apiece, 2 rugs at \$45.00 apiece, 3 small rugs at \$7.50 apiece, 1 center-table \$30, one sideboard \$60, one dining table \$45. Discount, 10%. **\$269.46**

3. One set dishes \$35.00, one fireless cooker \$15.00, 3 pans at 75¢, one bread-box \$1.25, other kitchen utensils \$12.40. Discount, 5%. **\$62.61**

4. 16 sheets at 45¢, 24 pillow-cases at 25¢, 5 dozen towels at \$3.40 per dozen, 8 bath towels at 32¢ apiece, 18 dish-towels at 12¢ apiece. Discount 15%. **\$29.68**

5. One suit \$27.50, one pair of shoes \$6.50, one pair of shoes, \$8.00, one overcoat \$35.00, 6 shirts at \$2.25 apiece, 4 suits of underwear at \$1.75 a suit, one hat \$5.00, 8 pairs of socks at 45c. a pair. Discount 20%. **\$84.88**

CHAPTER III.

THE DECIMAL NUMBER SYSTEM

ORAL REVIEW

1. In the number 22, what does each 2 represent?

2. In the number 4444, what does each 4 represent?

3. In the number 3.3, what does each 3 represent?

4. In the number 55.55, what does each 5 represent?

5. In the number 66,666.6666 what does each 6 represent?

6. Which is *ones'* place? *tens'* place? *hundreds'* place? *thousands'* place?

7. Which is *tenths'* place? *hundredths'* place? *thousandths'* place? *ten thousandths'* place?

83. **Place Value in the Decimal System.** The value of the figures in each place is shown by the following:

billions	hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones	tenths	hundredths	thousandths	ten thousandths
2	8	9	5	3	2	5	6	7	4	.6	4	9	2
Billions	Millions			Thousands			Ones			Decimal Fractions.			

This number is read, two billion, eight hundred ninety-five million, three hundred twenty-five thousand, six hundred seventy-four, and six thousand four hundred ninety-two ten thousandths

A decimal fraction is read as a whole number, and the name of the last figure is given to it. Thus if the last figure is in thousandths' place the fraction is read as thousandths.

84. Place Value in the Decimal Notation. The most important fact about the decimal notation is:

> *A figure in any one place represents ten times as much as the same figure in the next place to the right.*

This gives rise to the rule for multiplying and dividing a number by 10, 100 and so on.

That is, to multiply a number like 6 by 10 we need only to place the 6 in tens' place, which we do by annexing one zero to the right, or by moving the decimal point one place to the right.

ORAL EXERCISES

1. How may a number be multiplied by 10? by 100?

2. If in 74.26 the decimal point is moved one place to the right, how is the value of each figure changed?

Notice that by moving the decimal point one place to the right, each figure is made to represent ten times as much as it did before.

3. How may a number like 34.78 be multiplied by 10? by 100? by 1000?

4. If in 419.74 the decimal point is moved one place to the left, how is the value of each figure changed.

Notice that by moving the decimal point one place to the left each figure is made to represent one-tenth as much as it did before.

5. How may a number like 1983.71 be divided by 10? by 100? by 1000?

6. How may a number like .064 be divided by 10? by 100? by 1000?

It is difficult to overestimate the importance of the decimal notation in our modern life, in which a great deal of computing is required. To convince yourself of this write two numbers like 286 and 1892 in the Roman notation and try to get their products.

85. Adding by Groups. In adding columns greater speed may be made by *grouping* some of the figures.

6
5
4
7
9
2
8
7
3
51

Beginning at the top we notice when we come to 2 that 2 and 8 make 10, and also that 3 and 7 make 10. Adding, we say, 6, 11, 15, 22, 31, 41, 51. If we add upward, we say 10, 20, 29, 36, 40, 45, 51.

Numbers other than those which make 10 are sometimes grouped, but that is more difficult, and much more practice is required to make it worth while. If the sum of two or more numbers is 10, they should always be grouped.

ORAL EXERCISES

In this manner add the following:

1.	2.	3.	4.	5.	6.	7.	8.
4	9	5	5	8	5	4	3
9	1	3	7	9	4	8	9
8	4	9	9	7	3	2	2
6	6	6	8	8	2	4	6
4	7	2	2	2	1	6	8
9	4	7	4	4	8	7	2
1	3	6	3	7	9	3	4
6	8	4	6	6	1	5	3
4	7	8	9	3	5	8	7
51	49	50	53	54	38	47	44

WRITTEN EXERCISES

Copy and add the following:

9.	10.	11.	12.
5973	43006	1234	4331
1209	1487	9876	6675
3400	97500	379	890
9146	2469	340	3943
256	891	8926	86598
47	5643	317	1543
8091	796	3894	246
4237	84	327	9400
3107	651	1650	5920
1376	5390	8728	9347
36842	157917	35671	128893

WRITTEN EXERCISES

See how long it takes you to find the sums of the following:

1.	2.	3.	4.
8540	65200	9428	9839
719	1648	256	4206
39570	924	3847	31936
672	19467	4600	2476
1349	3928	3902	3930
8558	7276	25607	6042
3400	13400	19008	54320
12560	6731	2480	1719
7780	8764	5937	8943
1242	8193	6527	8370
5491	2789	1012	1494
89881	138320	82604	133275

To add decimals write them in a column, so that the decimal points are in a straight column. Then add them exactly the same as whole numbers.

Copy and add the following:

5. 4.08+390.2+568.+.091+8.34+43.67+1.24. 1015.621

6. 268.009+13.031+7.032+2.901+.0084.+8.734+3.19. 302.9054

7. 67480+.2913+4.721+.0216+3.491+1.742+.0291. 67490.296

8. .0917+2.947+4.496+4.246+8.428+.098+.72. 21.0267

9. 1.098+7.854+.0321+.0246+5.40+2.190+5.741. 22.3397

10. 81.09+4.657+1.32+2.624+5.40+2.190+5.741. 103.022

11. 67.8109+12+.736+58.24+16.67+.0219+.192. 155.6708

12. .1804+.129+4.91+2.502+10.84+6.18+.49. 25.2314

13. 2.790+3.094+.076+1.914+.004+0.191+14.68. 22.749

14. .894+76.4+1.426+.094+9.473+864.+29.9. 982.187

15. 0.496+3.98+43.6+743.1+0.084+0.91+1.87. 794.04

Example 1. From 1843 subtract 587.

1843 587 1256	Explain each step in this subtraction.

WRITTEN EXERCISES

Subtract:

1. 5432 2205 3227	2. 1956 1448 508	3. 672 439 233	4. 785 357 428
5. 3491 2378 1113	6. 1205 987 218	7. 2742 1894 848	8. 5034 2870 2164
9. 6007 3442 2565	10. 27450 3719 23731	11. 3700 1394 2306	12. 65391 31628 33763
13. 31620 19374 12246	14. 19300 8234 11066	15. 4302 2397 1905	16. 65283 37809 27474

Example 2. From 192.4 subtract 24.651.

192.400 24.651 167.749	First annex enough zeros in the minuend to make the number of decimal places the same as in the subtrahend. Subtract as if there were no decimal points present.

To subtract one decimal number from another write them so that the decimal points stand in a column.

Subtract the following:

17. 27.09 − 12.38	14.71	24. 1.064 − 0.8704	.1936
18. 5.928 − .064	5.864	25. 81.90 − 35.086	46.814
19. 29.4 − 3.78	25.62	26. 12.1 − 9.872	2.228
20. 326.04 − 237.208	88.832	27. 0.947 − 0.1796	.7674
21. 675 − 279.046	395.954	28. 1.304 − 0.847	.457
22. 56.2 − 13.489	42.711	29. 24.76 − 3.942	20.818
23. 4.98 − 1.094	3.886	30. .0194 − .0083	.0111

	Boys		Girls	
Age	Height	Weight	Height	Weight
2	2.60	25.01	2.56	23.53
3	2.82	28.30	2.78	26.2
4	3.04	31.38	3.00	28.67
5	3.26	34.64	3.21	32.10
6	3.44	38.80	3.33	35.29
7	3.58	40.14	3.50	38.65
8	3.76	43.08	3.68	42.82
9	4.00	49.95	3.92	47.10
10	4.18	55.03	4.08	51.95
11	4.36	59.77	4.26	56.57
12	4.57	67.34	4.41	64.35
13	4.72	75.81	4.60	72.65
14	4.93	88.75	4.76	80.45
15	5.07	96.40	4.92	92.04
16	5.18	107.50	5.02	94.02
17	5.36	116.56	5.10	104.43
18	5.44	127.59	5.13	112.55

The above table gives the average height and weight of boys and girls from 2 to 18 years of age. The heights are given in feet and decimal fractions of feet, and the weights in pounds and decimal fractions of pounds.

WRITTEN EXERCISES

1. Make a table like the above, showing the gain in height of the boys each year.

2. Make a table like the above, showing the gain in weight of the boys each year.

3. Make tables like these for the girls.

A table such as the above can be only a rather rough approximation. It would be different for different nationalities and also for different groups of the same nationality.

86. Multiplying by 50, 25, 125. A number may be multiplied by 50 by multiplying it by 100 and dividing the product by 2. Hence to multiply by 50 annex two zeros and divide by 2.

That is, $50\times786=2\overline{)78600}=39300$.

Such products may be written down at once as $50\times786=39300$.

A number may be multiplied by 25 by multiplying it by 100 and dividing the product by 4. Hence to multiply by 25 annex two zeros and divide by 4.

That is, $25\times826=4\overline{)82600}=20650$.

Since 125 is $\frac{1}{8}$ of 1000, a number may be multiplied by 125 by multiplying by 1000 and then dividing by 8. Hence, to multiply by 125 annex three zeros and divide by 8.

That is, $125\times942=8\overline{)942000}=117750$.

WRITTEN EXERCISES

. Write the products of 50×748, 50×910, 50×3487.
37400; 45500; 174350

2. Write the products of 25×731, 25×398, 25×448, 25×13900, 25×437.
18275; 9950; 11200; 347500; 10925

3. Write the products of 125×640, 125×872, 125×448, 250×384, 500×434, 500×192. 80,000; 109,000; 56,000; 96,000; 217,000; 96,000

Multiply:

4. 183	5. 914	6. 4021	7. 5970
241	286	607	635
44,103	261,404	2,440,747	3,790,950
8. 3491	9. 1205	10. 2700	11. 5034
237	386	189	860
827,367	465,130	510,300	4,329,240
12. 6070	13. 2450	14. 4270	15. 3910
423	371	394	163
2,567,610	908,950	1,682,380	637,330
16. 3021	17. 2390	18. 4302	19. 5283
473	824	267	809
1,428,933	1,969,360	1,148,634	4,273,947

WRITTEN EXERCISES

1. If you did not know how to multiply .1 by .2 show how you could find the product by writing these as common fractions.

2. In the same way show how you could find the products of .03×.08, .09×.7, .13×.10.

3. State the rule for placing the decimal point in the product of two decimal fractions.

Find the products of the following:

4. .097×8.56 .83032	5. 2.91×.0031 .009021	6. 7.42×.421 3.12382
7. 137×.062 8.494	8. 5.34×.571 3.04914	9. .0056×841 4.7096
10. .095×3.47 .32965	11. .026×.079 .002054	12. 1.49×.084 .12516
13. .184×265.1 48.7784	14. 5.9×.0734 .43306	15. 1.024×.0075 .00768

16. A load of coal contains 2.45 tons. How much does it cost at $5.75 a ton? Express the result to the nearest cent. **$14.09**

17. A farmer hauls $3\frac{1}{4}$ (3.25) cords of wood in a load. What is the value of 9 such loads at $4.25 a cord? **$124.31**

18. At $0.85 per thousand cubic feet, what is the bill for 5860 cubic feet of gas? Express 5860 cubic feet as 5.86 thousands. **$4.98**

19. An oriental rug is 6 feet 3 inches (6.25 feet) wide, and 9 feet 9 inches (9.75 feet) long. At $1.60 a square foot, what is its value? Express the result to the nearest cent. **$97.50**

20. A lot in the city of New York was sold at $1.45 a square foot. What was its value if it was 24.37 feet wide and 81.64 feet deep? Express the result to the nearest dollar. **$2885.**

21. At 57.5 pounds to the cubic foot, find the weight in tons of the ice on a pond of 75,000 square feet if it is 1 foot thick. **2156.25**

22. At $8.25 a ton what is the cost of a year's supply of coal consisting of 18.5 tons? **$152.63**

ORAL EXERCISES

Give the steps in long division, and illustrate with an example.

WRITTEN EXERCISES

1. Divide 492.76 by 28. 17.599

Explain how the first figure in the quotient is placed. Where is the decimal point in the quotient placed?

2. Divide 5.9176 by 231. .0256

To place the decimal point in the quotient we must prefix a zero. Complete the division.

3. Divide 1.3467 by 2.145. .628

Explain how the divisor is changed into a whole number.

4. Find quotient in $8462 \div 35$ to the nearest integer.

Explain why it is 242 and not 241.

5. Find quotient in $394.2 \div 58.7$ to nearest hundredth.

Explain why it is 6.72 and not 6.71.

Find the quotients in the following to the nearest integers:

6. $\begin{array}{r}2391\\32\overline{)76500}\end{array}$ 7. $\begin{array}{r}1896\\192\overline{)364000}\end{array}$ 8. $\begin{array}{r}1692\\286\overline{)484000}\end{array}$

9. $\begin{array}{r}765\\81\overline{)62000}\end{array}$ 10. $\begin{array}{r}214\\394\overline{)84300}\end{array}$ 11. $\begin{array}{r}165\\392\overline{)64800}\end{array}$

In the following find the quotients to the nearest tenth:

12. $\begin{array}{r}144.5\\4.7\overline{)679.24}\end{array}$ 13. $\begin{array}{r}2.0\\18.6\overline{)37.94}\end{array}$ 14. $\begin{array}{r}17.9\\49\overline{)876.20}\end{array}$

15. $\begin{array}{r}3.6\\18.9\overline{)68.60}\end{array}$ 16. $\begin{array}{r}258.2\\75.4\overline{)19470.8}\end{array}$ 17. $\begin{array}{r}95.3\\38.6\overline{)3678.93}\end{array}$

Find the quotients in the following to the nearest hundredth:

18. $\begin{array}{r}.77\\63.9\overline{)49.28}\end{array}$ 19. $\begin{array}{r}300.68\\2.91\overline{)874.97}\end{array}$ 20. $\begin{array}{r}.23\\.319\overline{).0749}\end{array}$

21. $\begin{array}{r}35.67\\14.7\overline{)524.36}\end{array}$ 22. $\begin{array}{r}3.58\\13.9\overline{)49.75}\end{array}$ 23. $\begin{array}{r}.09\\4.76\overline{)0.4213}\end{array}$

24. $\begin{array}{r}.37\\67.8\overline{)24.93}\end{array}$ 25. $\begin{array}{r}55.08\\0.87\overline{)47.916}\end{array}$ 26. $\begin{array}{r}.33\\0.94\overline{).3124}\end{array}$

27. $\begin{array}{r}7.66\\42.4\overline{)324.98}\end{array}$ 28. $\begin{array}{r}42.45\\1.46\overline{)61.97}\end{array}$ 29. $\begin{array}{r}332.01\\0.018\overline{)5.9762}\end{array}$

1. State the principle of place value in the decimal notation. Does this principle hold for both integers and decimals?

2. How may a whole number be multiplied by 10? by 100? by 1000? How may a decimal fraction be multiplied by these numbers?

3. How may a whole number or a decimal fraction be divided by 10? by 100? by 1000?

4. Compare addition of whole numbers and of decimal fractions. Are there any differences, and if so, what?

5. Compare subtraction of whole numbers and of decimal fractions. Are there any differences, and if so, what?

6. State the rule for multiplying decimal fractions.

7. State the rule for placing the decimal point in the quotient when the divisor is a whole number.

8. A certain divisor is in the form of a decimal fraction. How may it be changed to a whole number?

9. Give a short method for multiplying by 25, by 125.

10. Give a short method for dividing by 25, by 125.

11. How is a quotient found to the nearest unit? to the nearest tenth? to the nearest hundredth?

12. Give a definition of subtraction which shows how subtraction is related to addition.

13. Give a definition of division which shows how division is related to multiplication.

14. If the age of each pupil in a class is given, how may the average age be found?

15. If the dimensions of a box are measured in inches how may its volume in cubic feet be found?

1. In a certain American city having a population of 567,000 there are 69,600 telephones. How many telephones are there for each 1000 inhabitants? **123**

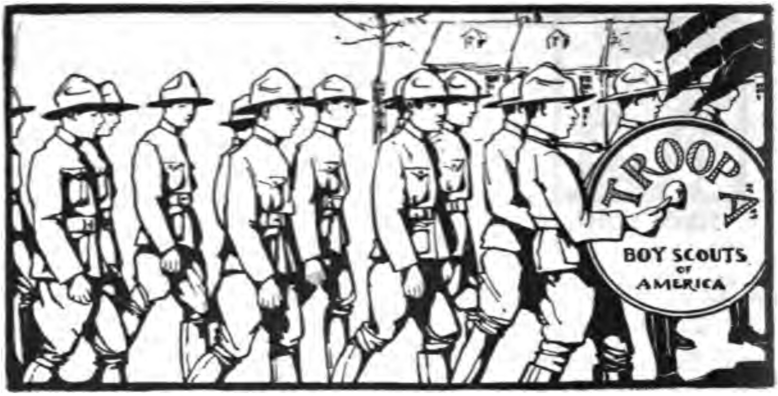
2. In the city of Tokio with a population of 2,170,000 there were 27,800 telephones in a recent year. How many telephones were there for each 1000 inhabitants. Compare with the result in example 1. **13**

3. It has been found that it takes on an average 5.6 pounds of corn to produce one pound of pork when feeding it to hogs. A farmer feeds 4680 pounds of corn to his hogs. How many pounds of pork should this produce? **835.7**

4. During 11 years a certain cattle company averaged feeding 13.3 pounds of grain and 9.4 pounds of hay for each pound of gain in beef cattle. How many tons of grain and hay did they feed a herd which gained 7890 pounds? **52.47 tons grain; 37.08 tons hay**

5. On a certain farm the average yield of hay per acre is $2\frac{3}{5}$ tons. At this rate what is the value of the yield from 27 acres if hay is worth $17.40 per ton? (Reduce $2\frac{3}{5}$ to a decimal.) **$1221.48**

6. The American bushel contains 2150.42 cubic inches, and the English Imperial bushel 2218.192 cubic inches. What is the difference in cubic inches between 1000 American and 1000 English bushels? How many American bushels would this difference amount to? **67,772 cu. in; 31.5 bu.**



In a troop of boy scouts each boy buys an outfit as follows:

Hat	$1.15	Shirt	$1.50	Hatchet	$1.50
Coat	2.15	Gaiters	1.10	Staff	.25
Belt	.60	Breeches	2.50	Knife	1.25

WRITTEN EXERCISES

1. Find the cost of each boy's outfit. **$12.00**

2. If there are 32 boys in the troop, find how much the outfits of all the boys cost. **$384**

3. In the month of July John worked 17 days on the golf links as a caddie. He earned 75 cents a day for 6 days, $1 a day for 7 days, and $1.25 a day for 4 days. How much did he earn? Did he earn enough to pay for his boy scout outfit? How much did he have left after buying the outfit? **$16.50; $4.50**

4. On a camping trip the boy scouts went fishing and swimming. What articles would they bring for these purposes? Find what these articles would cost, and make problems about them.

A troop of 32 boys with three scoutmasters went on a two-weeks' camping trip. They borrowed tents and other necessaries from the U. S. Forestry Service.

PROBLEMS

1. If the camp was 37 miles from the boys' homes, how far did each boy travel going to the camp and returning? 74

2. Food cost each boy $3 a week. How much did he pay for food for the two weeks? $6

3. Beside the regular outfit John bought a fishing rod for $1.75, hooks and flies for $2.25, and a reel for $2.50. How much did these articles cost him? $6.50

4. Harry bought a camera for $5.00, films for 75 cents, and paid $1.25 for developing pictures. How much did he pay for these altogether? $7

5. If transportation cost each boy one cent a mile, how much did the whole troop pay for transportation, including the transportation of the scoutmasters? $25.90

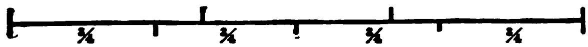
6. At $3 a week for food, how much did the food cost the whole troop, including the scoutmasters, for the two weeks of camping? $210

7. Make up a list of the total expenditure of each boy for outfit, transportation, and food. What are these expenses? What are the total expenses of the troop for all these items, not including the three scoutmasters? $18.74; $599.68

87. Meaning of a Fraction. If a line one inch long is divided into 4 equal parts, and 3 of them are taken, we have a line $\frac{3}{4}$ of an inch long.

Hence, $\frac{3}{4}$ may be regarded as 3 of the 4 equal parts of anything. This illustrates the following statement:

I. *The denominator of a fraction tells into how many equal parts a whole has been divided, and the numerator tells how many of these parts are taken.*



If a line 3 inches long is divided into 4 equal parts, then each part is $\frac{3}{4}$ of an inch long.

Hence, $\frac{3}{4}$ may also be regarded as 3 divided by 4. This illustrates the following statement:

II. *A fraction is an indicated quotient in which the numerator is divided by the denominator.*

These two ways of regarding a fraction should be clearly understood, since they are both in practical use.

ORAL EXERCISES

Give the meaning of each of the following fractions in the two ways just discussed: $\frac{7}{8}$, $\frac{5}{7}$, $\frac{13}{9}$, $\frac{13}{16}$, $\frac{25}{4}$, $\frac{11}{32}$, $\frac{47}{9}$.

88. Fractional Units. Such fractions as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, which have a numerator 1, are called *fractional units*, or *unit fractions*.

The fraction $\frac{2}{3}$ represents 2 of the fractional units $\frac{1}{3}$. In $\frac{4}{5}$ the fractional unit is $\frac{1}{5}$, and $\frac{4}{5}$ represents 4 of these.

ORAL EXERCISES

1. What is the fractional unit in each of the following: $\frac{3}{4}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{5}{9}$, $\frac{13}{16}$, $\frac{7}{16}$, $\frac{7}{12}$, $\frac{3}{10}$? How many of the fractional units of each fraction are represented by each of these?

2. What is the fractional unit in each of the following, and how many of them are represented: $\frac{3}{7}$, $\frac{4}{3}$, $\frac{9}{8}$, $\frac{14}{9}$, $\frac{3}{16}$, $\frac{21}{5}$?

ORAL EXERCISES

1. How many 3ds are there in 1? how many 4ths? how many 10ths? how many 65ths?

2. How many 8ths are there in 1? in 2? in 4? in 7? in 9?

3. How many 6ths are there in 5? in 6? in 8? in 10?

4. How many ones are there in $\frac{3}{3}$? in $\frac{6}{3}$? in $\frac{9}{3}$? in $\frac{15}{3}$? in $\frac{36}{3}$? in $\frac{48}{3}$? in $\frac{60}{3}$?

5. What is a mixed number? Give an example.

6. How many ones are there in $\frac{8}{3}$? How many 3ds are left over?

Reduce the following to mixed numbers:

7. $\frac{9}{4}$, $\frac{15}{4}$, $\frac{12}{5}$, $\frac{17}{5}$, $\frac{17}{4}$, $\frac{21}{8}$, $\frac{27}{8}$, $\frac{33}{8}$.

8. $\frac{21}{4}$, $\frac{49}{8}$, $\frac{78}{4}$, $\frac{34}{3}$, $\frac{47}{5}$, $\frac{89}{9}$, $\frac{94}{12}$, $\frac{73}{8}$.

9. $\frac{25}{6}$, $\frac{43}{7}$, $\frac{58}{9}$, $\frac{67}{8}$, $\frac{35}{4}$, $\frac{59}{7}$, $\frac{84}{9}$, $\frac{53}{6}$.

10. Reduce each of the numbers 4, 3, 5, 6, 7, 8, 2, 1, 9, 12 to 3ds.

11. Reduce each of the numbers, 5, 2, 6, 9, 4, 3, 8, 7, 1 to 7ths.

12. What is a proper fraction? an improper fraction? Give an example of each. Is $\frac{5}{5}$ a proper or an improper fraction?

Reduce the following to improper fractions:

13. $1\frac{1}{2}$, $3\frac{1}{4}$, $5\frac{1}{3}$, $6\frac{2}{3}$, $7\frac{3}{4}$, $9\frac{5}{8}$, $7\frac{7}{8}$, $3\frac{9}{16}$.

14. $2\frac{7}{8}$, $5\frac{3}{16}$, $3\frac{1}{32}$, $1\frac{5}{32}$, $4\frac{3}{16}$, $2\frac{5}{16}$, $3\frac{7}{16}$, $5\frac{9}{16}$.

15. $4\frac{5}{12}$, $8\frac{5}{8}$, $9\frac{3}{8}$, $1\frac{5}{64}$, $1\frac{9}{64}$, $2\frac{11}{32}$, $3\frac{7}{32}$, $2\frac{15}{32}$.

16. $\frac{2}{2}$ of a number is what part of it?

17. $\frac{3}{3}$ of 15 is what? $\frac{9}{9}$ of 18 is what?

18. $\frac{5}{5}$ of a number is what part of the number?

19. What is a fractional unit? What number in a fraction tells the size of the fractional unit? What number tells how many of these units are represented?

ORAL EXERCISES

1. Compare $\frac{1}{2}$ and $\frac{1}{4}$, $\frac{1}{4}$ and $\frac{1}{8}$, $\frac{1}{3}$ and $\frac{1}{15}$, $\frac{1}{5}$ and $\frac{1}{20}$.

2. How is the size of a fractional unit affected by multiplying the denominator by 2? by 3? by 4?

3. If the denominator of a fraction is multiplied by any number, what must be done with the numerator to avoid changing the value of the fraction?

Read and supply the missing numbers:

4. $\frac{3}{4}=\frac{?}{8}$ $\quad\frac{3}{5}=\frac{?}{15}$ $\quad\frac{3}{6}=\frac{?}{12}$ $\quad\frac{?}{3}=\frac{2}{6}$

5. $\frac{3}{7}=\frac{?}{14}=\frac{?}{21}=\frac{?}{28}=\frac{?}{35}=\frac{?}{42}=\frac{?}{49}=\frac{?}{56}$

6. $\frac{5}{8}=\frac{?}{16}=\frac{?}{24}=\frac{?}{32}=\frac{?}{40}=\frac{?}{48}=\frac{?}{56}=\frac{?}{64}$

7. $\frac{7}{9}=\frac{?}{18}=\frac{?}{27}=\frac{?}{36}=\frac{?}{45}=\frac{?}{54}=\frac{?}{63}=\frac{?}{72}$

Give the sums of the following:

8. $\frac{1}{2}+\frac{1}{4}$
9. $\frac{1}{2}+\frac{3}{4}$
10. $\frac{1}{4}+\frac{1}{8}$
11. $\frac{1}{4}+\frac{3}{8}$
12. $\frac{1}{4}+\frac{5}{8}$
13. $\frac{1}{4}+\frac{7}{8}$
14. $\frac{3}{4}+\frac{1}{8}$
15. $\frac{3}{4}+\frac{3}{8}$
16. $\frac{3}{4}+\frac{6}{8}$
17. $\frac{3}{4}+\frac{7}{8}$
18. $\frac{1}{2}+\frac{1}{8}$
19. $\frac{1}{2}+\frac{3}{8}$
20. $\frac{1}{8}+\frac{1}{16}$
21. $\frac{1}{8}+\frac{3}{16}$
22. $\frac{3}{8}+\frac{1}{16}$
23. $\frac{5}{8}+\frac{1}{16}$
24. $\frac{3}{8}+\frac{3}{16}$
25. $\frac{3}{8}+\frac{5}{16}$
26. $\frac{1}{4}+\frac{1}{16}$
27. $\frac{1}{4}+\frac{3}{16}$
28. $\frac{1}{4}+\frac{5}{16}$
29. $\frac{1}{4}+\frac{7}{16}$
30. $\frac{1}{4}+\frac{9}{16}$
31. $\frac{1}{2}+\frac{1}{8}$
32. $\frac{1}{2}+\frac{5}{8}$
33. $\frac{1}{2}+\frac{1}{16}$
34. $\frac{1}{2}+\frac{3}{16}$
35. $\frac{1}{2}+\frac{5}{16}$
36. $\frac{1}{2}+\frac{1}{5}$
37. $\frac{1}{2}+\frac{2}{5}$

38. In each of examples 8–37 subtract the smaller fraction from the larger.

ORAL EXERCISES

1. If $\frac{2}{3}$ of a number is subtracted from it, what fractional part of it remains?

2. Two men own a mill. One man owns $\frac{3}{5}$ of it. What fraction of the mill does the other man own?

3. In a certain school $\frac{4}{7}$ of the pupils are girls. What fraction of the pupils are boys?

4. A farmer plants $\frac{1}{5}$ of his farm in corn, while $\frac{2}{5}$ of it is in pasture and meadow. What fraction of the farm is left for other purposes?

5. In a closely built city $\frac{9}{16}$ of the space is covered with buildings. The remainder is in streets, alleys, and lawns. What fraction of the city is covered by streets, alleys, and lawns?

Give the remainders in the following:

6. $1-\frac{2}{3}$	10. $2-\frac{3}{8}$	14. $4-\frac{11}{16}$	18. $9-\frac{3}{5}$
7. $1-\frac{3}{5}$	11. $2-\frac{7}{16}$	15. $4-\frac{15}{32}$	19. $12-\frac{5}{7}$
8. $1-\frac{3}{7}$	12. $2-\frac{5}{8}$	16. $4-\frac{7}{15}$	20. $6-\frac{9}{32}$
9. $1-\frac{3}{8}$	13. $2-\frac{9}{16}$	17. $8-\frac{3}{8}$	21. $4-\frac{7}{9}$

22. A school baseball team wins 7 games and loses 5. What fraction of the games played do they win? What fraction do they lose?

WRITTEN EXERCISES

In each of the following pairs of fractions which is the greater, and how much?

1. $\frac{3}{4}, \frac{7}{8}$ $\frac{1}{8}$	5. $\frac{5}{8}, \frac{11}{16}$ $\frac{1}{16}$	9. $\frac{7}{9}, \frac{13}{18}$ $\frac{1}{18}$
2. $\frac{3}{8}, \frac{7}{15}$ $\frac{11}{120}$	6. $\frac{7}{8}, \frac{13}{16}$ $\frac{1}{16}$	10. $\frac{7}{8}, \frac{19}{32}$ $\frac{9}{32}$
3. $\frac{2}{3}, \frac{5}{8}$ $\frac{1}{24}$	7. $\frac{5}{12}, \frac{11}{24}$ $\frac{1}{24}$	11. $\frac{5}{12}, \frac{7}{16}$ $\frac{1}{48}$
4. $\frac{3}{4}, \frac{13}{16}$ $\frac{1}{16}$	8. $\frac{7}{12}, \frac{13}{24}$ $\frac{1}{24}$	12. $\frac{9}{16}, \frac{13}{24}$ $\frac{1}{48}$

89. Factors. If a whole number is the product of two other whole numbers, these are called *factors* of the given number.

Thus, 2 and 3 are factors of 6; 4 and 3 are factors of 12; 1 and 6 are also factors of 6.

90. Primes and Composite Numbers. A number which has no factors except 1 and itself is called a *prime* number. Thus, 1, 2, 3, 5, 7, 11 are prime numbers.

Numbers which are not prime are called *composite numbers.*

ORAL EXERCISES

1. Give two factors of each composite number below 30.

The prime factors of 8 are 2, 2, 2, because 2 is a prime number and because $2 \times 2 \times 2 = 8$. Similarly, 2, 2, 3 are the prime factors of 12.

2. Give the prime factors of 9, 16, 18, 20, 24, 27, 30, 32.

3. What is the first step in adding or subtracting fractions not having a common denominator?

WRITTEN EXERCISES

Reduce the following to like fractions. (See pages 42 and 49.)

1. $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}$ $\frac{4}{8}, \frac{6}{8}, \frac{5}{8},$
2. $\frac{1}{6}, \frac{3}{8}, \frac{5}{12}$ $\frac{4}{24}, \frac{9}{24}, \frac{10}{24}$
3. $\frac{4}{5}, \frac{3}{4}, \frac{1}{3}$ $\frac{48}{60}, \frac{45}{60}, \frac{20}{60}$
4. $\frac{3}{16}, \frac{5}{14}, \frac{7}{32}$ $\frac{42}{224}, \frac{80}{224}, \frac{49}{224}$
5. $\frac{3}{8}, \frac{5}{12}, \frac{4}{15}$ $\frac{45}{120}, \frac{50}{120}, \frac{32}{120}$
6. $\frac{1}{7}, \frac{1}{9}, \frac{4}{15}$ $\frac{45}{315}, \frac{35}{315}, \frac{84}{315}$
7. $\frac{3}{8}, \frac{5}{6}, \frac{1}{6}$ $\frac{9}{24}, \frac{20}{24}, \frac{4}{24}$
8. $\frac{3}{16}, \frac{7}{24}, \frac{7}{36}$ $\frac{27}{144}, \frac{42}{144}, \frac{28}{144}$
9. $\frac{5}{12}, \frac{11}{18}, \frac{7}{30}$ $\frac{75}{180}, \frac{110}{180}, \frac{42}{180}$

Perform the following indicated operations:

10. $\frac{1}{4}+48+\frac{5}{12}$ $48\frac{2}{3}$
11. $\frac{5}{6}-\frac{3}{4}$ $\frac{1}{12}$
12. $\frac{11}{16}-\frac{7}{12}$ $\frac{5}{48}$
13. $\frac{7}{12}+\frac{2}{3}+\frac{5}{18}$ $1\frac{19}{36}$
14. $\frac{7}{8}+\frac{3}{5}+\frac{9}{10}$ $2\frac{3}{8}$
15. $\frac{9}{10}+\frac{1}{12}+\frac{1}{60}$ 1
16. $\frac{3}{4}+\frac{5}{6}+\frac{7}{8}$ $2\frac{11}{24}$
17. $\frac{8}{9}+\frac{5}{6}+\frac{3}{4}$ $2\frac{17}{36}$
18. $\frac{4}{5}+\frac{3}{7}+\frac{8}{21}$ $1\frac{64}{105}$
19. $\frac{1}{12}+\frac{1}{15}+\frac{1}{16}$ $\frac{17}{80}$
20. $\frac{5}{6}+\frac{7}{8}+\frac{9}{10}$ $2\frac{73}{120}$
21. $\frac{7}{12}+\frac{5}{9}+\frac{5}{6}$ $1\frac{35}{36}$
22. $\frac{3}{8}+\frac{5}{16}+\frac{7}{24}$ $\frac{47}{48}$
23. $\frac{1}{3}+\frac{1}{12}+\frac{1}{15}$ $\frac{29}{60}$
24. $\frac{4}{9}+\frac{5}{18}+\frac{7}{27}$ $\frac{53}{54}$

ORAL AND WRITTEN EXERCISES

1. How do you multiply a fraction by an integer?

Give the products of the following, being careful to perform all possible cancellations before multiplying:

2. $\frac{3}{4}\times 6$, $\frac{2}{3}\times 7$, $\frac{4}{5}\times 15$, $\frac{7}{10}\times 25$. $4\frac{1}{2}$, $4\frac{2}{3}$, 12, $17\frac{1}{2}$

3. $\frac{5}{6}\times 16$, $\frac{3}{4}\times 18$, $\frac{2}{5}\times 12$, $\frac{3}{7}\times 21$. $13\frac{1}{3}$, $13\frac{1}{2}$, $4\frac{4}{5}$, 9

4. Give the rule for multiplying one fraction by another.

Give the products indicated below. Do as many as you can without using pencil and paper.

5. $\frac{5}{7}\times\frac{4}{5}$, $\frac{3}{5}\times\frac{2}{9}$, $\frac{6}{7}\times\frac{3}{4}$, $\frac{1}{4}\times\frac{2}{5}$. $\frac{4}{7}$, $\frac{2}{15}$, $\frac{9}{14}$, $\frac{1}{10}$

6. $\frac{3}{10}\times\frac{5}{9}$, $\frac{7}{8}\times\frac{3}{4}$, $\frac{7}{12}\times\frac{5}{8}$, $\frac{9}{16}\times\frac{4}{15}$. $\frac{1}{6}$, $\frac{21}{32}$, $\frac{35}{96}$, $\frac{3}{20}$

7. What is the best method for multiplying a mixed number by a whole number, as $2\frac{3}{4}$ by 6?

8. Does it make any difference whether you multiply $2\frac{3}{4}$ by 6 or 6 by $2\frac{3}{4}$?

9. Give the products of $1\frac{1}{2}\times 2$, $2\frac{1}{3}\times 6$, $4\times 1\frac{4}{5}$, $5\frac{2}{3}\times 6$. 3, 14, $7\frac{1}{5}$, 34

10. Find the product of $1\frac{1}{3}\times 2\frac{3}{4}$ by reducing each mixed number to an improper fraction. $3\frac{2}{3}$

11. Find the product of $34\frac{1}{4}\times 18\frac{3}{8}$ by the four-step method. (See page 70.) $629\frac{11}{32}$

The method of example 10 is usually shorter when the numbers are small, and the four-step method when they are large.

Find the products of the following:

12.	$2\frac{1}{4}\times 4\frac{1}{5}$	$9\frac{9}{20}$	13.	$1\frac{1}{2}\times 5\frac{1}{3}$	8	14.	$8\frac{1}{4}\times 3\frac{2}{3}$	$30\frac{1}{4}$
15.	$6\frac{4}{5}\times 5\frac{3}{4}$	$39\frac{1}{10}$	16.	$4\frac{1}{2}\times 5\frac{1}{3}$	24	17.	$12\frac{1}{3}\times 2\frac{1}{3}$	$28\frac{7}{9}$
18.	$8\frac{1}{2}\times 5\frac{2}{3}$	$48\frac{1}{6}$	19.	$10\frac{4}{5}\times 4\frac{4}{9}$	48	20.	$6\frac{3}{5}\times 10\frac{2}{3}$	$70\frac{2}{5}$
21.	$16\frac{1}{3}\times 7\frac{1}{2}$	$122\frac{1}{2}$	22.	$41\frac{1}{2}\times 56\frac{3}{4}$	$2355\frac{1}{8}$	23.	$38\frac{1}{4}\times 16\frac{2}{3}$	$637\frac{1}{2}$

ORAL EXERCISES

1. How do you divide a fraction by an integer?

2. Give the quotients: $\frac{2}{3}\div 2$, $\frac{2}{3}\div 3$, $\frac{3}{5}\div 2$, $\frac{6}{7}\div 12$.

3. How do you divide any number by a fraction?

Find the quotients of the following:

4. $\frac{3}{5}\div 6$	6. $\frac{4}{5}\div 4$	8. $\frac{8}{9}\div 4$	10. $\frac{5}{8}\div 15$
5. $\frac{4}{7}\div 2$	7. $\frac{7}{9}\div 3$	9. $\frac{5}{12}\div 10$	11. $\frac{9}{16}\div 6$

WRITTEN EXERCISES

1. $\frac{2}{3}\div\frac{1}{2}$ $1\frac{1}{3}$	5. $\frac{3}{5}\div\frac{2}{3}$ $\frac{9}{10}$	9. $\frac{5}{6}\div\frac{3}{5}$ $1\frac{7}{18}$	13. $6\div\frac{3}{5}$ 10
2. $\frac{3}{4}\div\frac{2}{3}$ $1\frac{1}{8}$	6. $\frac{5}{6}\div\frac{3}{4}$ $1\frac{1}{9}$	10. $\frac{7}{8}\div\frac{5}{16}$ $2\frac{4}{5}$	14. $3\div\frac{5}{7}$ $4\frac{1}{5}$
3. $\frac{3}{8}\div\frac{3}{4}$ $\frac{1}{2}$	7. $\frac{5}{6}\div\frac{3}{8}$ $2\frac{2}{9}$	11. $\frac{9}{16}\div\frac{3}{4}$ $\frac{3}{4}$	15. $4\div\frac{7}{8}$ $4\frac{4}{7}$
4. $\frac{5}{8}\div\frac{3}{4}$ $\frac{5}{6}$	8. $\frac{4}{5}\div\frac{7}{12}$ $1\frac{13}{35}$	12. $\frac{7}{9}\div\frac{7}{8}$ $\frac{8}{9}$	16. $9\div\frac{4}{5}$ $11\frac{1}{4}$

Example 1. Divide $8\frac{3}{4}$ by 3.

$$8\frac{3}{4}\div 3=\frac{35}{4}\div 3=\frac{35}{12}=2\frac{11}{12}.$$

Divide the following:

17. $4\frac{1}{3}\div 2$ $2\frac{1}{6}$	20. $9\frac{4}{5}\div 3$ $3\frac{4}{15}$	23. $8\frac{1}{2}\div 3$ $2\frac{5}{6}$	26. $8\frac{3}{4}\div 5$ $1\frac{3}{4}$
18. $7\frac{2}{3}\div 3$ $2\frac{5}{9}$	21. $12\frac{3}{4}\div 3$ $4\frac{1}{4}$	24. $4\frac{5}{6}\div 2$ $2\frac{5}{12}$	27. $17\frac{2}{3}\div 4$ $4\frac{5}{12}$
19. $4\frac{3}{8}\div 6$ $\frac{35}{48}$	22. $18\frac{4}{5}\div 5$ $3\frac{19}{25}$	25. $19\frac{3}{7}\div 6$ $3\frac{5}{21}$	28. $24\frac{1}{2}\div 3$ $8\frac{1}{6}$

Example 2. Divide $1\frac{2}{3}$ by $2\frac{4}{5}$.

$$1\frac{2}{3}\div 2\frac{4}{5}=\frac{5}{3}\div\frac{14}{5}=\frac{5}{3}\times\frac{5}{14}=\frac{25}{42}$$

Divide the following:

29. $3\frac{1}{4}\div 2\frac{2}{3}$ $1\frac{7}{32}$	33. $7\frac{1}{8}\div 5\frac{3}{4}$ $1\frac{11}{46}$	37. $6\frac{2}{3}\div 8\frac{1}{2}$ $\frac{40}{51}$
30. $7\frac{1}{5}\div 9\frac{2}{3}$ $\frac{108}{145}$	34. $12\frac{1}{4}\div 2\frac{3}{5}$ $4\frac{37}{52}$	38. $16\frac{7}{8}\div 2\frac{3}{4}$ $6\frac{3}{22}$
31. $8\frac{1}{3}\div 3\frac{1}{4}$ $2\frac{22}{39}$	35. $6\frac{3}{4}\div 2\frac{1}{4}$ 3	39. $4\frac{5}{16}\div 2\frac{1}{2}$ $1\frac{29}{40}$
32. $3\frac{4}{5}\div 1\frac{1}{3}$ $2\frac{17}{20}$	36. $24\frac{3}{4}\div 6\frac{1}{3}$ $3\frac{69}{76}$	40. $12\frac{5}{8}\div 4\frac{7}{8}$ $2\frac{23}{39}$

91. Reducing Fractions to Decimals. Since a common fraction may be regarded as an indicated division (see page 152), it may be reduced to a decimal by carrying out the division.

Example 1. Reduce $\frac{3}{8}$ to a decimal.

$$8\overline{)3.000}\quad .375$$

Example 2. Reduce $\frac{3}{7}$ to a decimal.

$$7\overline{)3.0000}\quad .4285, \text{ remainder } 5.$$

On dividing we find .4285 with a remainder 5. Hence, .4286 is the decimal value of $\frac{3}{7}$ carried to 4 places of decimals. (At this point read again page 82.)

WRITTEN EXERCISES

Reduce the following to three-place decimals:

$\frac{7}{9}$, $\frac{4}{7}$, $\frac{8}{13}$, $\frac{5}{11}$, $\frac{8}{19}$, $\frac{5}{31}$, $\frac{7}{23}$, $\frac{9}{29}$, $\frac{14}{23}$.

.778; .571; .615; .455; .421; .161; .304; .310; .609.

ORAL EXERCISES

Find the products in the following:

1. $2\times\frac{2}{5}$	$\frac{4}{5}$	7. $2\times\frac{3}{4}$	$1\frac{1}{2}$	13. $3\times\frac{5}{8}$	$1\frac{7}{8}$	19. $2\times\frac{3}{8}$	$\frac{3}{4}$
2. $2\times\frac{2}{7}$	$\frac{4}{7}$	8. $4\times\frac{3}{4}$	3	14. $4\times\frac{3}{8}$	$1\frac{1}{2}$	20. $4\times\frac{5}{8}$	$2\frac{1}{2}$
3. $2\times1\frac{1}{2}$	3	9. $2\times1\frac{1}{3}$	$2\frac{2}{3}$	15. $2\times1\frac{2}{3}$	$3\frac{1}{3}$	21. $2\times2\frac{1}{3}$	$4\frac{2}{3}$
4. $6\times2\frac{1}{4}$	$13\frac{1}{2}$	10. $6\times2\frac{3}{4}$	$16\frac{1}{2}$	16. $6\times3\frac{1}{8}$	$18\frac{3}{4}$	22. $6\times3\frac{5}{8}$	$21\frac{3}{4}$
5. $3\times2\frac{1}{4}$	$6\frac{3}{4}$	11. $3\times2\frac{3}{4}$	$8\frac{1}{4}$	17. $3\times5\frac{3}{16}$	$15\frac{9}{16}$	23. $3\times5\frac{9}{16}$	$16\frac{11}{16}$
6. $\frac{1}{2}\times\frac{1}{2}$	$\frac{1}{4}$	12. $\frac{1}{2}\times\frac{1}{3}$	$\frac{1}{6}$	18. $\frac{1}{2}\times\frac{2}{3}$	$\frac{1}{3}$	24. $\frac{1}{2}\times\frac{1}{4}$	$\frac{1}{8}$

WRITTEN EXERCISES

Perform the indicated operations in the following:

1. $\frac{1}{2}+\frac{1}{2}+\frac{3}{4}$	$1\frac{3}{4}$	4. $\frac{7}{8}-\frac{6}{7}+\frac{5}{12}$	$\frac{73}{168}$	7. $10\frac{1}{2}+4\frac{3}{4}$	$15\frac{1}{4}$
2. $\frac{4}{5}+\frac{6}{7}+\frac{2}{5}$	$2\frac{2}{35}$	5. $2\frac{1}{2}+4\frac{1}{3}+1\frac{5}{8}$	$8\frac{11}{24}$	8. $\frac{5}{8}+\frac{3}{4}+\frac{5}{6}$	$2\frac{5}{24}$
3. $\frac{3}{5}-\frac{2}{7}+\frac{3}{4}$	$1\frac{9}{140}$	6. $12\frac{1}{3}+2\frac{1}{2}$	$14\frac{5}{6}$	9. $\frac{5}{6}+\frac{7}{10}+\frac{8}{15}$	$2\frac{1}{15}$

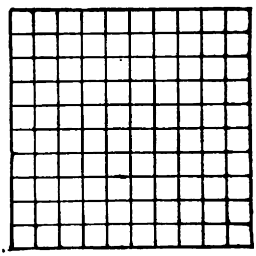
92. Percentage. In very many of the ordinary affairs of life fractions are expressed as *hundredths* or *per cent.* "Per cent" means *in the hundred,* or *hundredths.* The sign of per cent is %. Thus, 1% = .01, or 1/100.

That part of arithmetic which deals with per cents is called *percentage.*

Per cents may always be expressed as decimals. Thus, 2% = .02, 7% = .07, 25% = .25.

ORAL EXERCISES

1. Read the following per cents as decimals: 8%, 4%, 12%, 35%, 40%, 75%.



2. How many small squares are there in this figure?

3. Point out 1% of this figure, 2%, 5%, 10%.

4. Draw a figure like this on the board and point out 20% of it, 30%, 40%, 60%, 80%, 100%.

To find a certain per cent of a number is simply to find so many hundredths of it.

Thus, 5% of 400 = $.05 \times 400 = 20$.

And 9% of 360 = $.09 \times 360 = 32.40$.

5. Find 2% of 200, of 300, of 400.

6. Find 3% of 200, of 500, of 600, of 400.

7. Find 4% of 600, of 800, of 300, of 400.

8. Find 5% of 200, of 600, of 300, of 400.

It is easily seen that percentage involves no new principles of arithmetic. It is simply another name given to what is already known from the study of common fractions and decimals.

93. **Importance of Percentage.** It is not easy to appreciate fully the importance of the use of percentage. Thus, when a farmer takes the milk of two cows to the creamery, he is told that one sample contains 3.8% of butter fat, and the other 4.5%. Suppose that he had been told that one contained $\frac{19}{500}$ and the other $\frac{9}{200}$ of butter fat!

The miner has ore tested, and is told it contains so many % of metal. We say that 33% of the population of Massachusetts are foreign born, that 50.8% of the total population in that state are women, and so on. Through constant use we have become so accustomed to per cent (hundredths) that 8%, for example, has a more direct meaning to us than the equal fraction, $\frac{2}{25}$.

94. **Work in Percentage.** A large share of the work in arithmetic from now on will consist of the solution of problems involving percentage. Your success will depend mainly on two things:

95. **Conditions of the Problem.** The first condition on which your success will depend is a clear understanding of the problems to be solved and of the business practices that give rise to them.

Every effort must be made to learn just what the problem is. You should be on the constant lookout to learn the methods used in business in your own community. Try to visit places of business to see how things are done. The text book will give you descriptions of each topic as it is taken up. You will obtain information from your teacher, your father, and from others who have had practical experience.

96. **No New Principles of Arithmetic.** The second condition on which your success will depend, is a full appreciation of the fact that no new processes of arithmetic are involved.

If percentage and its applications puzzle you, remember that you are only dealing with decimal fractions. Never forget that all you need in solving business problems is the arithmetic you have already studied, some common sense, and a clear understanding of the conditions which give rise to your problems.

97. Base, Rate, Percentage. In the statement "5% of 600 is 30," 600 is called the *base*, 5% or .05 the *rate per cent*, or merely the *rate*, and 30 the *percentage*.

The number of which a certain per cent is taken is called the *base;* the number of per cent is called the *rate per cent*, or simply the *rate;* the result is called the *percentage.*

ORAL EXERCISES

1. Name the base, the rate, and the percentage in each of the following: 7% of 100 is 7, 7% of 300 is 21, 15 is 5% of 300.
2. What is 10% of 100? of 200? of 300? of 400? of 500? of 600? of 700? of 800? of 900?
3. What is 20% of each of the numbers in exercise 2?
4. What is 30% of each of the numbers in exercise 2?
5. What is 50% of each of the numbers in exercise 2?
6. What is 100% of each of the numbers in exercise 2?

Give the following:

7. 8% of 4000, 6% of 6000, 4% of 5000.
8. 3% of 4000, 7% of 2000, 12% of 3000.
9. 4% of 7000, 10% of 1800, 6% of 8000.
10. 6% of 250, 4% of 450, 8% of 650.
11. 3% of 50, of 150, of 450, of 5000.
12. 2% of 100, of 400, of 600, of 1000.
13. 1% of 200, of 300, of 400, of 500.
14. 7% of 300, of 500, of 600, of 800.
15. 40% of 200, of 400, of 500, of 700.
16. 5% of 100, of 500, of 700, of 800.

Example. If the base is 340, and the rate is 15%, what is the percentage?

340 (base) .15 (rate) 1700 340 51.00 (percentage)	This requires us to find 15% or .15 of 340.

WRITTEN EXERCISES

Find the percentage in the following:

	Base	Rate	Percentage		Base	Rate	Percentage
1.	280	12%	33.60	18.	86425	40%	34570
2.	6700	16%	1072.	19.	29640	25%	7410
3.	1245	25%	311.25	20.	12400	36%	4464
4.	3775	20%	755.	21.	38000	4%	1520
5.	194	5%	9.7	22.	5400	18%	972
6.	486	6%	29.16	23.	1140	60%	684
7.	395	9%	35.55	24.	690	80%	552
8.	1516	8%	121.28	25.	428	52%	222.56
9.	2840	7%	198.8	26.	3460	67%	2318.2
10.	3650	12%	438.	27.	4900	14%	686.
11.	2490	16%	398.4	28.	5400	9%	486
12.	5480	25%	1370.	29.	6420	12%	770.4
13.	6870	45%	3091.5	30.	1940	15%	291
14.	3918	13%	509.34	31.	6780	35%	2373
15.	2400	42%	1008	32.	9500	48%	4560
16.	6600	64%	4224	33.	8450	55%	4647.5
17.	27400	35%	9590	34.	1250	62%	775.

98. Suggestions for Solving Problems. Read the problem with care. If the statement consists of more than one sentence, read each sentence by itself, and insist that you understand it. You can not answer a question unless you know what the question is, and you can not solve a problem unless you know just what the problem is.

In a problem involving percentage, decide which number is the rate, which the base, and which the percentage. In such a problem two of these will be given, and you will be required to find the third.

WRITTEN WORK

1. During one year a certain cow yields 8640 pounds of milk, containing 4% butter fat. How many pounds of butter fat are there in this milk? 345.6
2. If 44% of the population are voters, how many voters are there in a state having a population of 2,640,000? 1,161,600
3. The weight of the flour obtained from a certain kind of wheat is 72% of the weight of the wheat. How many pounds of flour will be obtained from one bushel (60 pounds) of wheat? 43.2 lb.
4. A fruit dealer is obliged to sell a certain lot of fruit for 80% of what it cost him. For how much does he sell it if the cost was $20? $16.00
5. A bankrupt firm find they can pay 65% of their debts. How much can they pay if their total debt is $48,600? $31,590
6. A piece of land worth $5600 increased 15% in value. How much did the land increase in value? $840
7. If bread made from a certain brand of flour weighs 35% more than the flour, how much bread can be made from a sack containing 98 pounds of flour? 132.3 lb.
8. If 76% of a certain kind of potato is water, how many pounds of water are there in a bushel of potatoes weighing 60 pounds? 45.6

WRITTEN WORK

1. If 32% of a certain ore is copper, how many pounds of copper are there in one ton (2000 pounds)? What is the value of this copper at 19 cents a pound? **640 lb.; $121.60**

2. Corn left in the crib 4 months shrinks 7% in weight. How much will 1680 bushels shrink in this time? **117.6 bu.**

3. In a certain school there were 260 children in the 4th grade, 245 in the 5th grade, 230 in the 6th grade. In the 4th grade of this school 85% passed in arithmetic. How many passed in arithmetic in this grade? **221**

4. In the 5th grade 80% passed in arithmetic. What per cent did not pass? How many did not pass in this grade? **20%; 49**

5. In the 6th grade 90% passed in arithmetic. How many per cent did not pass? How many children passed? **10%; 207**

6. A family living in a city calculate to spend about 15% of their income for rent. How much can they pay for rent if their income is $1800? About how much rent per month can they pay? **$270; $22.50**

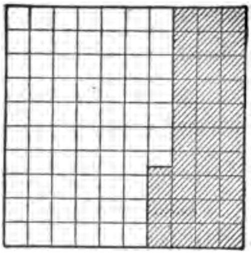
7. This family spend 24% of their income for food. How much a month is this? **$36**

8. A certain grade of automobile is supposed to depreciate 40% in value the first year. How much does such a car depreciate in one year if it was worth $2150 when new? How much is it worth at the end of one year? **$860; $1290**

9. In one year a certain cow yields 10480 pounds of milk, 4% of which is butter fat. At 38 cents a pound, how much is this butter fat worth? **$159.30**

10. In a school with 1200 pupils 19% of them are in the first grade, 17% in the second, 15% in the third, 14% in the fourth, 12% in the fifth, 10% in the sixth, 7% in the seventh, and 6% in the eighth grade. How many are there in each grade? **228, 204, 180, 168, 144, 120, 84. 72**

ORAL EXERCISES



1. Point out one per cent of the large square. Also point out 10% of it.

2. How many per cent. of the large square are shaded? ($\frac{1}{3}$ of the small square is shaded.)

3. What common fraction of the large square is shaded?

4. How many per cent of the large square are not shaded?

5. What common fraction of the large square is not shaded?

In reducing $\frac{1}{3}$ to a decimal we get $3\overline{)1.00}^{\,.33\frac{1}{3}}$, which also shows that $\frac{1}{3}=33\frac{1}{3}\%$.

Similarly, $\frac{2}{3}=66\frac{2}{3}\%$.

To reduce a common fraction to per cents, reduce it to a decimal to the nearest hundredth and write as per cent.

$\frac{1}{2}=.50=50\%$	$\frac{1}{12}=.08\frac{1}{3}=8\frac{1}{3}\%$
$\frac{1}{3}=.33\frac{1}{3}=33\frac{1}{3}\%$	$\frac{1}{15}=.06\frac{2}{3}=6\frac{2}{3}\%$
$\frac{2}{3}=.66\frac{2}{3}=66\frac{2}{3}\%$	$\frac{1}{16}=.06\frac{1}{4}=6\frac{1}{4}\%$
$\frac{1}{4}=.25=25\%$	$\frac{1}{20}=.05=5\%$
$\frac{1}{5}=.20=20\%$	$\frac{1}{25}=.04=4\%$
$\frac{1}{6}=.16\frac{2}{3}=16\frac{2}{3}\%$	$\frac{1}{40}=.02=2\frac{1}{2}\%$
$\frac{1}{8}=.12\frac{1}{2}=12\frac{1}{2}\%$	$\frac{1}{50}=.02=2\%$

Also notice that:

$$\frac{3}{8}=3\times 12\tfrac{1}{2}\%=37\tfrac{1}{2}\%$$
$$\frac{2}{15}=2\times 6\tfrac{2}{3}\%=13\tfrac{1}{3}\%$$
$$\frac{3}{16}=3\times 6\tfrac{1}{4}\%=18\tfrac{3}{4}\%$$
$$\frac{5}{16}=5\times 6\tfrac{1}{4}\%=31\tfrac{1}{4}\%$$

These instances suggest a method for finding the per cent equivalents of many fractions.

Example 1. $\frac{7}{12}$ equals how many per cents?

Since $\frac{1}{12}=8\frac{1}{3}\%$, $\frac{7}{12}=7\times8\frac{1}{3}\%=58\frac{1}{3}\%$.

ORAL EXERCISES

Find the missing numbers in the following:

1. $\frac{3}{4}=?\%$	6. $\frac{3}{8}=?\%$	11. $\frac{2}{15}=?\%$	16. $\frac{7}{16}=?\%$
2. $\frac{2}{5}=?\%$	7. $\frac{5}{8}=?\%$	12. $\frac{4}{15}=?\%$	17. $\frac{9}{16}=?\%$
3. $\frac{3}{5}=?\%$	8. $\frac{7}{8}=?\%$	13. $\frac{7}{15}=?\%$	18. $\frac{11}{16}=?\%$
4. $\frac{4}{5}=?\%$	9. $\frac{5}{12}=?\%$	14. $\frac{3}{16}=?\%$	19. $\frac{3}{20}=?\%$
5. $\frac{5}{6}=?\%$	10. $\frac{7}{12}=?\%$	15. $\frac{5}{16}=?\%$	20. $\frac{6}{25}=?\%$

Example 2. Find $6\frac{1}{4}\%$ of 6400.

Since $6\frac{1}{4}\%=\frac{1}{16}$, we take $\frac{1}{16}$ of 6400, which equals 400.

Example 3. Find $37\frac{1}{2}\%$ of 56.

Since $37\frac{1}{2}\%=\frac{3}{8}$, we find $\frac{3}{8}$ of 56, which equals 21.

Find the percentage in each of the following:

	Base	Rate		Base	Rate
21.	240	50%	31.	48	$6\frac{1}{4}\%$
22.	96	$33\frac{1}{3}\%$	32.	72	$37\frac{1}{2}\%$
23.	24	$66\frac{2}{3}\%$	33.	48	$62\frac{1}{2}\%$
24.	400	25%	34.	60	$13\frac{1}{3}\%$
25.	800	75%	35.	128	$12\frac{1}{2}\%$
26.	480	20%	36.	32	$18\frac{3}{4}\%$
27.	360	$16\frac{2}{3}\%$	37.	160	$2\frac{1}{2}\%$
28.	640	$12\frac{1}{2}\%$	38.	80	$12\frac{1}{2}\%$
29.	2400	$8\frac{1}{3}\%$	39.	48	$8\frac{1}{3}\%$
30.	3000	$6\frac{2}{3}\%$	40.	450	$6\frac{2}{3}\%$

99. Uses of Fractional Per Cents. In many practical applications fractional per cents other than those given on the preceding pages are used. Such per cents are usually expressed as decimal fractions. (For some practical examples see page 169.)

Example. Find 14.7% of 840.

$$\begin{array}{r} 840 \\ .147 \\ \hline 5880 \\ 3360 \\ 840 \\ \hline 123.480 \end{array}$$

Since 14.7% = .147, the required percentage may be found by multiplying the base by .147.

WRITTEN EXERCISES

1. Find 7.3% of 390. 28.47
2. Find 8.9% of 7140. 635.46
3. Find 13.1% of 3140. 411.34
4. Find 21.7% of 9360. 2031.12
5. Find 84.4% of 2240. 1890.56
6. Find 41.7% of 6120. 2552.04
7. Find 52.3% of 735.8 384.8234
8. Find 62.8% of 9272. 5822.816
9. Find 49.4% of 417.40. 206.1956
10. Find 76.3% of 26.31. 20.07453

ORAL DRILL

Find the following by using common fractions:

1. Find $33\frac{1}{3}$% of 60, of 120, of 300, of 600.
2. Find 25% of 16, of 40, of 200, of 400.
3. Find 20% of 50, of 80, of 200, of 500.
4. Find $16\frac{2}{3}$% of 60, of 300, of 600, of 720.
5. Find $12\frac{1}{2}$% of 48, of 72, of 120, of 600.
6. Find $8\frac{1}{3}$% of 48, of 72, of 120, of 600.
7. Find $6\frac{1}{3}$% of 60, of 90, of 300, of 600.
8. Find $6\frac{1}{4}$% of 64, of 128, of 800, of 1600.

WRITTEN WORK

1. During one year a certain cow yields 9420 pounds of milk averaging 4.1% of butter fat. How many pounds of butter fat is this? At 42 cents a pound, how much is it worth?
386.22; $162.21

On dairy farms the milk from each cow is tested, and the amount of milk weighed. The table below shows the results for one year for a herd of 12 cows.

No. of cow	Milk, lbs.	% fat	Fat, lbs.	No. of cow	Milk, lbs.	% fat	Fat, lbs.
1	7930	4.3%	340.99	7	10700	3.9%	417.3
2	8370	4.2%	351.54	8	9874	4.3%	424.582
3	9470	4.1%	388.27	9	8960	4.1%	367.36
4	6390	4.4%	281.16	10	7530	3.6%	271.08
5	7690	3.9%	299.91	11	6590	4.6%	303.14
6	8760	4.0%	350.4	12	11800	3.8%	448.4

2. Find the number of pounds of butter fat yielded by each cow.

Rule a sheet of paper properly, copy the table given above, and fill in the columns giving the number of pounds of butter fat.

Such a table enables the farmer to decide which ones of his cows are the more valuable.

3. Ordinary butter contains about $85\frac{1}{2}\%$ of butter fat. How many pounds of butter fat are there in 685 pounds of butter?
Suggestion: $85\frac{1}{2}\% = 85.5\% = .855$. 585.675

4. The United States government pays $4\frac{3}{4}\%$ each year in interest on the last war loan. How much interest does a man receive on $8550 of this loan? $406.13

5. An article valued at $60.00 is sold at a loss of $16\frac{2}{3}\%$. How much was the loss? $10

6. A certain ore contains 4.7% gold. How many pounds of gold are there in 10 tons of such ore? 940

Example 1. .42 is equal to how many per cent?

Since *per cent* means hundredths, .42 = 42%.

Example 2. .378 is equal to how many per cent?

.378 contains 37 hundredths and .8 of one hundredth. Hence, .378 = 37.8%.

Rule. To reduce a decimal fraction to per cent, move the decimal point two places to the right.

That is, 1.96 = 196%, and .0745 = 7.45%.

ORAL EXERCISES

Read the following as per cents:

1. .29	5. .52	9. .024	13. .82
2. .06	6. .124	10. .075	14. 1.081
3. .191	7. .242	11. .0094	15. .078
4. .314	8. .006	12. 1.04	16. .942

Example. Express $\frac{3}{7}$ as per cent, correct to the nearest tenth of one per cent.

$$7\overline{)3.000}\quad .429 = 42.9\%$$

Since $\frac{1}{10}$ of 1% is equal to one-thousandth, we reduce $\frac{3}{7}$ to a decimal to the nearest thousandth.

WRITTEN EXERCISES

Reduce the fractions in the following to per cents, giving each to the nearest tenth of 1%.

1. $\frac{3}{8}$ 37.5%	5. $\frac{3}{13}$ 23.1%	9. $\frac{9}{16}$ 56.3%	13. $\frac{8}{19}$ 42.1%	17. $\frac{8}{75}$ 10.7%
2. $\frac{4}{7}$ 57.1%	6. $\frac{7}{13}$ 53.8%	10. $\frac{9}{11}$ 81.8%	14. $\frac{11}{21}$ 52.4%	18. $\frac{9}{65}$ 13.8%
3. $\frac{7}{8}$ 87.5%	7. $\frac{9}{13}$ 69.2%	11. $\frac{8}{15}$ 53.3%	15. $\frac{5}{34}$ 14.7%	19. $\frac{12}{89}$ 13.5%
4. $\frac{5}{11}$ 45.5%	8. $\frac{5}{16}$ 31.3%	12. $\frac{9}{17}$ 52.9%	16. $\frac{6}{43}$ 14.0%	20. $\frac{15}{98}$ 15.3%

Example 1. 1 is how many per cent of 4?

First Solution: Since 1 is $\frac{1}{4}$ of 4, reduce $\frac{1}{4}$ to .25, which equals 25%.

Second Solution: 1% of 4 = .04, and .04 is contained 25 times in 1.

These solutions are not essentially different, but the first seems to be more direct. Choose one, and use that only.

Example 2. 3 is how many per cent of 8?

Solution: $\frac{3}{8}$ = .375 = 37.5%, or .08 is 1% of 8, and .08 is contained 37.5 times in 3.

ORAL EXERCISES

1. 5 is how many per cent of 20?
2. 6 is how many per cent of 30?
3. 12 is how many per cent of 24?
4. 8 is how many per cent of 40?
5. 7 is how many per cent of 28?
6. $\frac{1}{2}$ is how many per cent of 5?

Example. 17 is how many per cent of 24? *Suggestion*: Express $\frac{17}{24}$ as per cent, or divide 17 by 1% of 24.

WRITTEN EXERCISES

In the following find the rates to the nearest per cent:

1. 35 is how many per cent of 47? 74%
2. 47 is how many per cent of 82? 57%
3. 64 is how many per cent of 689? 9%
4. 43 is how many per cent of 37? 116%
5. 140 is how many per cent of 468? 30%
6. 365 is how many per cent of 468? 78%
7. 160 is how many per cent of 280? 57%

WRITTEN WORK

In the problems on this page in which the rate is required find the rate to the nearest tenth of one per cent.

1. A roast of beef weighed 6 pounds before roasting, and $4\frac{3}{4}$ pounds after roasting. How many pounds in weight were lost in roasting? How many per cent. of the first weight was this? *Suggestion:* $1\frac{1}{4}$ or 1.25 is how many per cent of 6? **$1\frac{1}{4}$ lb.; 20.8%**

2. In testing a certain ore, it was found that 250 pounds of ore contained 23 pounds of metal. What per cent of the ore was metal? **9.2%**

3. In a school containing 760 pupils 114 were graduated. How many per cent of the whole school were graduated? **15%**

4. In a town having a population of 15,640, there were 1960 pupils in the public schools. How many per cent of the total population were in the school? **12.5%**

5. A bankrupt firm owes $18,630, and has $15,850 with which to pay its debts. What per cent of its debts can this firm pay? **85.1%**

6. An automobile worth $3450, when new, decreases in value 30 per cent the first year. How much is it worth at the end of the year? **$2415**

7. If you spend $4\frac{1}{2}$ hours (270 minutes) in school each day, and if you spend 25 minutes of this time on arithmetic, what per cent of the whole time do you spend on arithmetic? **9.3%**

8. During one week a boy solved 64 examples, of which 57 were right. What per cent of the examples did he get right? **89.1%**

9. In a school there are 124 pupils in the first grade, 112 in the second, 106 in the third, 98 in the fourth, 92 in the fifth, 86 in the sixth, 72 in the seventh, and 58 in the eighth. What per cent of the total number are in each grade? What is the sum of these per cents? *Suggestion:* First find the total number in the school. **16.6; 15.0; 14.2; 13.1; 12.3; 11.5; 9.6; 7.8; Sum, 100.1%**

WRITTEN EXERCISES

Reduce the following fractions to per cents, correct to the nearest tenth of one per cent:

1. $\frac{3}{7}$ 42.9%	8. $\frac{14}{17}$ 82.4%	15. $\frac{15}{19}$ 78.9%	22. $\frac{5}{32}$ 15.6%
2. $\frac{8}{9}$ 88.9%	9. $\frac{7}{9}$ 77.8%	16. $\frac{4}{13}$ 30.8%	23. $\frac{7}{16}$ 43.8%
3. $\frac{3}{5}$ 60%	10. $\frac{8}{11}$ 72.7%	17. $\frac{13}{21}$ 61.9%	24. $\frac{9}{64}$ 14.1%
4. $\frac{6}{7}$ 85.7%	11. $\frac{6}{13}$ 46.2%	18. $\frac{41}{64}$ 64.1%	25. $\frac{19}{40}$ 47.5%
5. $\frac{3}{8}$ 37.5%	12. $\frac{5}{16}$ 31.3%	19. $\frac{38}{93}$ 40.9%	26. $\frac{47}{56}$ 83.9%
6. $\frac{5}{9}$ 55.6%	13. $\frac{8}{17}$ 47.1%	20. $\frac{64}{87}$ 73.6%	27. $\frac{29}{47}$ 61.7%
7. $\frac{5}{11}$ 45.5%	14. $\frac{5}{18}$ 27.8%	21. $\frac{12}{17}$ 70.6%	28. $\frac{38}{57}$ 66.7%

Find the missing numbers in the following, giving the required rates correct to the nearest tenth of one per cent:

	Base	Rate	Percentage		Base	Rate	Percentage
29.	47	6%	28.2	41.	$4\frac{1}{2}$	44.4%	2
30.	680	$4\frac{1}{2}$%	30.6	42.	12	27.1%	$3\frac{1}{4}$
31.	560	50%	280	43.	43	39.5%	17
32.	4960	16.9%	840	44.	670	$16\frac{2}{3}$%	111.7
33.	350	$12\frac{1}{2}$%	43.75	45.	590	$12\frac{1}{2}$%	73.75
34.	6240	2.2%	140	46.	12900	$6\frac{1}{4}$%	806.25
35.	7900	15%	1185	47.	4960	17%	843.2
36.	14800	$33\frac{1}{3}$%	4933.3	48.	8400	25%	2100
37.	9400	$12\frac{1}{2}$%	1175	49.	17640	$5\frac{1}{4}$%	926.1
38.	6400	$12\frac{1}{2}$%	800	50.	9400	9%	846
39.	4500	$33\frac{1}{3}$%	1500	51.	18400	9.0%	1650
40.	560	$12\frac{1}{2}$%	70	52.	5960	11.4%	680

WRITTEN WORK

1. A merchant estimates that of the accounts owing to him 15% cannot be collected. How much does he expect to lose of $8500? **$1275**

2. If 15% of a merchant's accounts cannot be collected, how many per cent are collectible? How much of $8500 does the merchant expect to collect? **85%; $7225**

3. A church subscription amounts to $1245.60. If it is estimated that 95% of this can be collected, how much is it supposed can be collected? What per cent of the total subscription is supposed not to be collectible? **$1183.32; 5%**

4. A merchant pays his lawyer 10% for collecting a note of $1500. How much does the lawyer receive? What per cent of the value of the note does the merchant receive? **$150; 90%**

5. A farmer puts 1450 bushels of corn in his crib in the fall. How many bushels does he expect to sell if he expects the corn to shrink 9% before selling? **1319.5**

6. The corn which a farmer places in his crib shrinks 12% before it is sold. What per cent of the amount put into the crib does he sell? How many bushels will he have to sell from 1200 bushels put into the crib in the fall? (Corn shrinks more the longer it is left in the crib.) **88%; 1056 bu.**

7. Only 35% of the electric current supplied to an electric toaster is used in toasting the bread; the remainder is lost. What per cent of the electric current supplied the toaster is lost? How much of $1.50 worth of electric current is lost? **65%; 97.5c**

8. Water expands 9% when it freezes into ice. How much will 10 gallons of water expand in freezing into ice? **.9 gal.**

9. A factory valued at $14,000 is insured for 65% of its value, what would the owner lose if the factory were completely destroyed by fire? **$4,900**

ORAL WORK

1. What is the meaning of the words, "per cent"?

2. Give an example in percentage, stating what number is the base and what number is the rate.

3. How may a given rate per cent be expressed as a decimal?

4. If the base and the rate are given how may the percentage be found?

5. If the per cent spoiled of a consignment of fruit is given, how may the per cent not spoiled be found?

6. If you know a man's income and what per cent of it he spends, how do you find the amount which he spends? How do you find the per cent of his income which he saves? How do you find how much he saves?

7. If you know the cost of a new automobile and the per cent decrease in value in one year, how do you find its value at the end of one year?

8. How may a decimal fraction be reduced to per cents?

9. How may a common fraction be reduced to per cents?

10. If you know the number of pupils in a grade, and also the number that passed, how do you find how many per cent passed? How do you find how many per cent failed?

11. If you know how many problems a boy had to solve in one month, and also how many he really did solve, how do you find how many per cent of his problems he solved?

12. How can you find your spelling grade in per cent for a month? A perfect grade is how many per cent?

13. If you know the base and the percentage, how may the rate be found?

100. The Elements of Percentage. There are three numbers involved in every problem in percentage, viz: *Base*, *Rate*, and *Percentage.* In any such problem, two of these numbers are given, directly or indirectly, and we are to find the third.

We have learned that when the base and rate are given, the percentage is found by multiplying the base by the rate expressed as a decimal. That is:

$$\textit{rate} \times \textit{base} = \textit{percentage} \qquad \text{(P)}$$

We shall refer to this equation as equation (P), meaning the equation of percentage.

The first step in solving a problem in percentage is to find which two of the three elements, base, rate, and percentage, are given. The remaining one can then always be found by referring to equation (P).

If the rate and the base are given, we need to find the missing member in

$$\text{rate} \times \text{base} = ?$$

That is, we have a problem in multiplication.

If the base and percentage are given, we need to find the missing members in

$$? \times \text{base} = \text{percentage}$$

That is, we have a problem in division.

If the rate and percentage are given, we need to find the missing number in

$$\text{rate} \times ? = \text{percentage}$$

That is, we again have a problem in division.

ORAL EXERCISES

1. State in words the problem represented by the equation: rate $\times$ base = ? How is it solved?

2. State in words the problems represented by the equations: ? $\times$ base = percentage, rate $\times$? = percentage. How are they solved?

Compare the three problems in percentage with finding the missing numbers in:

$$3\times4=?\,,\ 3\times?=12,\ ?\times4=12$$

Also compare these problems with the problems on price, cost, and number of articles sold, or with those on length, width, and area. (See pages 24, 26, 28.)

You should now be able to see that the same idea runs through all these problems. If you understand this you will have less trouble with any of them than you otherwise would.

ORAL EXERCISES

In each of the following state which is the base, which is the rate, and which is the percentage.

1. 5% of 800 is 40
2. 5% of 900 is 45
3. 75 is 5% of 1500
4. 12½% of 800 is 100
5. 1650 is 20% of 8250
6. 15% of 6500 is 975.
7. 25% of 64 is 16.
8. 50 is 5% of 1000
9. 48 is 5% of 960
10. 360 is 6% of 6000
11. 18% of 780 is 140.40
12. 33⅓% of 1560 is 520
13. 24% of 45 is 10.80
14. 224 is 40% of 560
15. 768 is 80% of 960
16. 35% of 1250 is 437.5
17. Give a rule for finding the percentage when the rate and the base are given.
18. Give a rule for finding the rate when the base and the percentage are given.

The problem of finding the base when the rate and percentage are given occurs less frequently than the other two. While finding the base is not more difficult than finding the rate, the problems which naturally involve this case come up in the seventh grade, and exercises of this sort will be deferred until then.

101. List Price and Marking Price. Goods sold at wholesale (and some goods sold at retail) are described in catalogues in which prices are given. Such prices are called *list prices.* Goods for sale in retail stores usually have price tags attached. The price shown on these is called *marking price.*

102. Discount. Any reduction from list or marking price is *commercial discount,* or simply *discount.* Discounts are allowed for many reasons. The list price is always high enough to cover any rise in the market price which is likely to occur for some time. Hence goods are regularly sold at a discount. Special discounts are also given on large sales, for prompt payment, and for other reasons. Discount is figured as *so many per cent of the list price or marking price.*

ORAL EXERCISES

1. Try to find out the reasons why stores in your city or neighborhood offer goods at a discount at certain times of the year. Also try to find out whether goods are sold at a discount for other reasons.

2. Give equivalents in common fractions of the following discounts: 50%, 25%, 20%, 10%, $33\frac{1}{3}$%. (See page 166.)

3. A chair marked $18, is reduced 25%. By how much is the price of the chair reduced?

4. If the price of a chair is reduced by $\frac{1}{4}$, for how many per cent of its original price does it sell?

5. What per cent of the list price is paid for goods sold at a discount of 10%? of 20%? of 30%? of 40%? of 15%? of 25%?

6. What per cent of the marking price is paid for goods sold at a discount of 51%? of 35%? of 45%? of 55%?

7. Find advertisements in the papers offering discounts, and make problems from them.

Problem. What is the selling price of goods listed at $125 if sold at a discount of 15%?

$125 .85 625 1000 $106.25 selling price.	Since the price is discounted 15%, the selling price is 85%, or .85 of the list price.

This problem may also be solved by finding 15% of $125 (the list price) and subtracting this from the list price. The method given here is shorter, however.

ORAL EXERCISES

1. If goods are reduced 12%, how do you find the selling price?

2. If goods are reduced 35%, how do you find the selling price?

WRITTEN EXERCISES

Find the selling price of each of the following:

1. Silk listed at $2.25 a yard; discount 25%. **$1.69**

2. Woolen goods listed at $1.80 a yard; discount 15%. **$1.53**

3. Men's suitings listed at $2.80 a yard; discount 45%. **$1.54**

4. Men's shirts listed at $18.00 a dozen; discount 40%. **$10.80**

5. Collars listed at $1.50 a dozen; discount 35%. **$.98**

6. Knives and forks listed at $16.00 a dozen; discount 45%. **$8.80**

7. Spoons listed at $12.00 a dozen; discount 18%. **$9.84**

8.

1 sideboard	$70.00
A china closet	85.00
Discount 12%	**$136.40**

9.

1 rug	$145.00
6 pairs of curtains	48.00
2 pieces of tapestry	34.00
4 pictures	120.00
Discount 8%	**$319.24**

103. Short Methods. The shortest method for finding the selling price is not the same for all cases. The following examples illustrate this:

Example 1. Discount 25%. Deduct $\frac{1}{4}$ of the list price.

Example 2. Discount 20%. Deduct $\frac{1}{5}$ of the list price.

Example 3. Discount 10%. Deduct $\frac{1}{10}$ of the list price.

Example 4. Discount $12\frac{1}{2}$%. Deduct $\frac{1}{8}$ of the list price.

Example 5. Discount $16\frac{2}{3}$%. Deduct $\frac{1}{6}$ of the list price.

Example 6. Discount $33\frac{1}{3}$%. Deduct $\frac{1}{3}$ of the list price.

Example 7. Discount 15%. Find 85% of the list price.

Example 8. Discount 45%. Find 55% of the list price.

ORAL EXERCISES

Give the shortest method for finding the selling price for each of the following discounts: 30%, 40%, 35%, $66\frac{2}{3}$%, 18%.

WRITTEN EXERCISES

Find the selling price in each of the following:

1.	5,000 envelopes at $1.55 per thousand.............	$
	10,000 envelopes at $1.35 per thousand...........	
	5,000 sheets of stationery at $1.75 per thousand.....	
	Discount 15%	$25.50
2.	6 office chairs at $12	$
	3 filing cases at $45...........................	
	4 office desks at $35	
	Discount $16\frac{2}{3}$%	$289.17
3.	1 set of account books...........................	$30.00
	10,000 reams of copy paper.........................	55.00
	1 doz. cases carbon paper.........................	18.00
	Discount 20%	$82.40

WRITTEN EXERCISES

1. Goods marked $70 were sold at a discount of 5%. What was the selling price? **$66.50**

2. Goods listed at $45 were sold at a discount of 60%. What was the selling price? **$18.00**

3. Goods listed at $250 were sold at a discount of 45%. What was the selling price? **$137.50**

4. Goods listed at $100 were sold for $62. What was the discount per cent? **38%**

5. Goods marked $40 were sold for $30. What was the discount per cent? **25%**

6. Goods listed at $60 were sold for $40. What was the discount per cent? **33⅓**

7. Goods listed at $2.50 were sold at a discount of 60%. What was the selling price? **$1.00**

Find the selling prices in the following, using short methods:

	List price	Rate discount			List price	Rate discount	
8.	$260.00	35%	**$169.**	18.	$90.00	35%	**$58.50**
9.	1.50	70%	**$.45**	19.	75.00	45%	**$41.25**
10.	4.00	65%	**$1.40**	20.	60.00	15%	**$51**
11.	9.50	55%	**$4.28**	21.	1250.00	10%	**$1125**
12.	45.00	35%	**$29.25**	22.	4.50	40%	**$2.70**
13.	7.50	75%	**$1.88**	23.	3.00	65%	**$1.05**
14.	90.00	15%	**$76.50**	24.	35.00	30%	**$24.50**
15.	45.00	12½%	**$39.38**	25.	40.00	33⅓%	**$26.67**
16.	30.00	16⅔%	**$25.**	26.	60.00	18¾%	**$48.75**
17.	120.00	6¼%	**$112.50**	27.	150.00	22½%	**$116.25**

104. **Profits.** People engage in business for the purpose of making money. Merchants make money by selling goods at a higher price than they pay for them. The difference between the buying and the selling price is called *gross profit*, or simply *profit*. In large transactions the profit is always computed as a certain per cent of the dealer's buying price. Thus, if a dealer buys a farm for \$10,000 and sells it for \$11,000, his profit is \$1000. The \$1000 profit is 10% of the buying price.

In retail business the rate of gain is frequently computed as a rate per cent on the selling price. Thus, if an article is bought for 60c and sold for \$1.00 the gain is 40c, which is 40% of the selling price. The usage is by no means uniform, even among retail dealers. Unless otherwise stated, the rate of profit will be regarded as a rate on the buying price.

ORAL EXERCISES

1. An article is bought for \$40 and sold for \$60. What is the gain?

2. An article is bought for \$50 and sold at a gain of \$25. What is the selling price?

3. An article is sold for \$25, which is a gain of \$8.00. Find the buying price?

4. If you know the buying and the selling price, how do you find the gross profit?

5. If you know the buying price and the gain, how do you find the selling price?

6. If you know the selling price and the gain, how do you find the buying price?

Find the buying price of each of the following:

7. Selling price \$60. Gain, 20%.

8. Selling price \$20. Gain, 50%.

If an article is sold at a gain of 25%, the selling price is 125% of the buying price, since the buying price is 100% of itself.

If an article is sold at a loss of 10%, the selling price is 90% of the buying price.

ORAL EXERCISES

1. If goods are sold at a gain of 30%, how many per cent of the buying price is the selling price?

2. If goods are sold at a gain of 45%, how many per cent of the buying price is the selling price?

3. If goods are sold at a loss of 15%, how many per cent of the buying price is the selling price?

4. If goods are sold at a loss of 5%, how many per cent of the buying price is the selling price?

5. If goods are sold at a gain, is the selling price more or less than 100% of the buying price?

6. If goods are sold for the price at which they were bought, is the selling price more or less than 100% of the buying price?

7. If goods are sold at a loss, is the selling price more or less than 100% of the buying price?

In the following, how many per cent of the selling price is the buying price, the rate gain or loss being based on the selling price?

Gain	Gain	Loss	Loss
8. 20%	15. 30%	22. 12%	29. 20%
9. 15%	16. 55%	23. 18%	30. $33\frac{1}{3}$%
10. 25%	17. 45%	24. $12\frac{1}{2}$%	31. 45%
11. 45%	18. 60%	25. $8\frac{1}{3}$%	32. 50%
12. $12\frac{1}{2}$%	19. 40%	26. $16\frac{1}{3}$%	33. 75%
13. $16\frac{1}{3}$%	20. 50%	27. 15%	34. 80%
14. 27%	21. 75%	28. 25%	35. 41%

Problem 1. A man buys a farm for $14,800, and sells it at a gain of 8%. What is the selling price?

$$\begin{array}{r} \$14{,}800 \\ 1.08 \\ \hline 118400 \\ 14800 \\ \hline \$15984.00 \end{array} = \text{selling price.}$$

The selling price is 108% of the buying price. Hence the selling price is found by multiplying the buying price by 1.08.

Problem 2. A man buys an automobile for $1600 and sells it at a loss of 35%. What is the selling price?

$$\begin{array}{r} \$1600 \\ .65 \\ \hline 8000 \\ 9600 \\ \hline \$1040.00 \end{array}$$

$1600 = buying price. $1040.00 = selling price.

The selling price is 65% of the buying price; hence the selling price is found by multiplying the buying price by .65.

WRITTEN EXERCISES

1. A house and lot are bought for $14,500, and are sold at a loss of 5%. Find the selling price. $13,775

2. A man buys an automobile for $2800, and sells it at a loss of 15%. Find the selling price. $2380

Find the selling price in each of the following:

	Buying price	Rate of gain			Buying price	Rate of gain	
3.	$1450.00	25%	$1812.50	10.	$1250.00	35%	$1687.50
4.	$1900.00	15%	$2185.	11.	$412.00	18%	$486.16
5.	$3700.00	28%	$4736.	12.	$380.00	12%	$425.60
6.	$4900.00	12%	$5488.	13.	$4200.00	27%	$5334.
7.	$800.00	55%	$1240.	14.	$8960.00	5%	$9408.
8.	$24,600.00	4%	$25584	15.	$12,000.00	14%	$13680.
9.	$1860.00	8%	$2008.80	16.	$7400.00	7%	$7918.

105. The Rate of Gain or Loss. The problem of finding the *rate of gain or loss,* when the buying price and the selling price are given, occurs frequently.

Problem 1. A man bought a house for $3400, and sold it for $3700. What was the gain per cent?

$$\begin{array}{r} .088 = 8.8\% \\ 3400\overline{)300.00} = 34\overline{)3.00} \\ 2\,72 \\ \hline 280 \end{array}$$

The gain was $300, and this we find to be 8.8% of $3400, the buying price. This result is correct to the nearest tenth of one per cent.

In practice the same number of zeros in the dividend and divisor are omitted before starting the dividing.

Problem 2. A man bought a steamboat for $180,000, and sold it two years later for $145,000. What was the loss per cent on this transaction?

$$\begin{array}{r} .194 = 19.4\% \text{ rate of loss.} \\ 18\overline{)3.5} \\ 18 \\ \hline 170 \\ 162 \\ \hline 80 \end{array}$$

The loss was $35,000, and this we find to be 19.4% of $180,000, the buying price. The result is correct to the nearest tenth of one per cent. Divide both dividend and divisor by 10000 before starting the dividing.

> *To find the rate per cent, gain or loss, find what per cent of the buying price the gain or loss is.*

WRITTEN EXERCISES

Find the rate, gain or loss, in each of the following, to the nearest tenth of one per cent, using buying price as the base:

	Buying price	Selling price			Buying price	Selling price	
1.	$1900.00	$1400.00	26.3%	5.	$1.50	$2.75	83⅓%
2.	$1200.00	$1500.00	25%	6.	$3.00	$5.00	66⅔%
3.	$35.00	$65.00	85.7%	7.	$0.15	$0.25	66⅔%
4.	$45.00	$75.00	66⅔%	8.	$0.60	$1.00	66⅔%

106. Commission. An agent or employee sometimes receives as his compensation a certain fraction or per cent of the amount involved in a transaction. This compensation is called a *commission.*

107. Business Transacted on Commission. There is a large variety of business transactions on which a commission is paid.

A real estate agent sells a piece of ground or a building for a customer and receives a certain per cent of the selling price as his commission. An architect draws the plans for a building, and superintends its erection, and receives for his work a certain per cent of the amount paid for the building. A commission merchant in a city receives eggs, butter, fresh veal, etc., from the farmers and sells these to local dealers. The pay for his work is a certain per cent of the amount involved in the sale. All these are said to receive a commission.

108. Commission, How Computed. Commission is always computed as so many per cent on the amount involved in the transaction; that is,

> *An agent's commission for selling is computed as a certain per cent of the selling price. An agent's commission for buying is computed as a certain per cent of the purchase price.*

It should be clear that the subject of commission involves no new principles of arithmetic. (Read again page 161.)

ORAL EXERCISES

1. Find the agent's commission for selling a piece of land for $4,000, if the rate is 5%.

2. Find an agent's commission for selling a city building for $50,000, if the rate is 2%.

3. Find commission for selling land for $8000, if the rate is 4%.

WRITTEN WORK

1. A real estate man sells a lot for $2400. At the rate of 5%, what is his commission? $120

2. A broker sells a motor boat for $2350 and deducts a commission of 5%. How much does the owner get from the sale? $2232.50

3. A house is to be built for $8500. The architect's fee for drawing the plans is 6% of this sum. What is his fee. $510

4. A commission merchant sells 50 dozen eggs for 38¢ a dozen, and deducts 10% for his commission. How much does he transmit to the producer? $17.10

5. At the rate of 12½%, what is the commission for selling 180 pounds of veal at 13½¢ a pound? How much does the farmer get for the veal? $3.04; $21.26

6. At 4% what is the architect's commission on a building for which the building contractor gets $125,000? How much does the building cost when the architect's commission is added? $5,000; $130,000

7. An agent sells a second-hand automobile for $1250. How much does the owner get if the agent deducts 8% as his commission? $1150

8. An agent buys a motor boat for $2750. How much does the boat cost the purchaser if the agent charges a commission of 3% for buying. $2832.50

9. A property is sold for $15,000. If the commission for selling is 2½%, how much does the seller get, after the commission for selling is deducted? $14,625

10. A man is offered $5750 for a house and lot, with a 5% commission to be deducted. He can sell the house for $5500 without commission. Which is the better offer? Latter

11. Make and solve other problems on commission. Find out what work is done on commission in your neighborhood.

109. Interest. If you loan a man \$100 for one year, then at the end of the year he owes you not only the \$100 which you loaned him, but a few dollars besides. That is, the man will pay you for the use of the money during the year.

Such payment for the use of money is called *interest.* It is rent paid for the use of money, just as rent is paid for the use of a house or a farm. If the man pays \$5 for the use of \$100 for one year, it is said that the *rate of interest* is 5%, and if he pays \$6 for the use of \$100 for one year, the rate of interest is 6%.

If a business man borrows money from a bank he is obliged to pay the bank a certain number of dollars per year for every hundred dollars he borrows; that is, he pays a certain % of the amount borrowed.

These are instances of a universal custom. Anyone who borrows money must pay interest for the use of it, and anyone who loans money receives interest on it. Interest is always reckoned as a certain number of dollars per year for each \$100 borrowed—that is, at a certain rate of per cent per year.

Problem. Find the interest for one year on \$600 at 5%.

Suggestion: Find 5% of \$600.

ORAL EXERCISES

1. Find 5% of 100, 200, 300. What is the interest on \$100 for one year at 5%? on \$200? on \$300?

2. Find the interest on \$300 for one year at 6%.

Find the interest for one year on each of the following:

3. \$800 at 5%	7. \$1500 at 5%	11. \$10,000 at 4%
4. \$600 at 4%	8. \$2000 at 4%	12. \$8000 at 6%
5. \$300 at 7%	9. \$5000 at 5%	13. \$12,000 at 7%
6. \$1000 at 6%	10. \$2500 at 6%	14. \$20,000 at 5%

Problem 1. What is the interest on \$6840 at 6% for one year?

$$\begin{array}{r} \$6840 \\ .06 \\ \hline \$410.40 \end{array}$$

Find 6% of \$6840, which is \$410.40, the required interest.

Frequently the interest charged involves a fraction of a per cent. Thus, $4\frac{1}{2}\%$ and $5\frac{1}{2}\%$ are common rates of interest.

Problem 2. What is the interest on \$4580 for one year at $5\frac{1}{2}\%$?

$$\begin{array}{r} \$4580 \\ .055 \\ \hline 22900 \\ 22900 \\ \hline \$251.900 \end{array}$$

You should notice that the solution of the problem consists in multiplying 4580 by $.05\frac{1}{2}$ or .055.

$$\begin{array}{rl} \$45.80 & 1\% \\ \hline 22.90 & \frac{1}{2}\% \\ 229.00 & 5\% \\ \hline \$251.90 & 5\frac{1}{2}\% \end{array}$$

Example 2 may also be solved by taking 1% of 4580, then $\frac{1}{2}\%$, and then 5%, and adding the $\frac{1}{2}\%$ and 5%.

WRITTEN EXERCISES

Find the interest for one year on each of the following:

1. \$645 at 5%	\$32.25	13. \$15000 at $5\frac{1}{2}\%$	\$825.
2. \$4260 at 6%	\$255.60	14. \$84200 at $4\frac{1}{2}\%$	\$3789.
3. \$7600 at 5%	\$380.	15. \$670 at $6\frac{1}{2}\%$	\$43.55
4. \$375 at 7%	\$26.25	16. \$7500 at $4\frac{1}{2}\%$	\$337.50
5. \$960 at 8%	\$76.80	17. \$42000 at $5\frac{1}{2}\%$	\$2310.
6. \$12250 at 6%	\$735.	18. \$260000 at $5\frac{1}{4}\%$	\$13650.
7. \$500 at 9%	\$45.	19. \$35600 at $5\frac{1}{2}\%$	\$1958.
8. \$9240 at 4%	\$369.60	20. \$13800 at $4\frac{3}{4}\%$	\$655.50
9. \$25400 at 5%	\$1270.	21. \$4980 at $5\frac{1}{2}\%$	\$273.90
10. \$35650 at 4%	\$1426.	22. \$16500 at $6\frac{1}{2}\%$	\$1072.50
11. \$3600 at $4\frac{1}{2}\%$	\$162.	23. \$12800 at $5\frac{1}{2}\%$	\$704.
12. \$5700 at $5\frac{1}{2}\%$	\$313.50	24. \$2500 at 7%	\$175.

WRITTEN EXERCISES

1. What is the interest for one year on $1600 at 6%? **$96**

2. What is the interest for one year on $2500 at $5\frac{1}{2}$%? **$137.50**

3. What is the interest for one year on $5000 at $4\frac{3}{4}$%? **$237.50**

4. A man borrows $14,000 to invest in a farm. What is the yearly interest if the rate is $5\frac{1}{2}$%? **$770**

5. A city borrows $145,000 at $5\frac{1}{2}$% to build a bridge. What is the yearly interest on this loan? **$797.5**

6. A capitalist finds that during one year his investments net him $5\frac{3}{4}$% interest. At this rate, what is his income from an investment of $480,000? **$27,600**

7. A schoolhouse in a large city costs $860,000. At $5\frac{1}{4}$%, what is the yearly interest on this sum? If this building holds 2400 pupils how much does the interest amount to for each pupil? **$45,150; $18.81**

In deciding whether it pays to buy a house or to rent one, we must figure interest on the investment.

8. A family paying $60 a month rent find that they can build an equally good house for $8000. If the rate of interest is 6%, and if taxes and repairs on the house amount to $260 yearly, which is cheaper, to rent or to build? **Rent $20 cheaper**

9. A man buys an automobile for $1650. At the end of one year it is worth $1200. If he allows interest on the investment at 7%, and $470 for gasoline, oil, tires, and repairs, how much does it cost him for the use of the machine for one year, including the depreciation? **$1035.50**

10. A man buys a farm of 240 acres at $110 an acre. At 6% what is the yearly interest on the buying price? If taxes are $180, and repairs on buildings $250, each year, what is the cost per year of running this farm? **$1584; $2014**

11. Make and solve other problems in reckoning interest.

ORAL EXERCISES

Find the selling price of the following:

	List price	Rate discount		List price	Rate discount
1.	\$100	25%	6.	\$25	10%
2.	\$5	20%	7.	\$40	8%
3.	\$12	$33\frac{1}{3}$%	8.	\$1.50	$33\frac{1}{3}$%
4.	\$30	20%	9.	\$8.00	30%
5.	\$2.50	50%	10.	\$75	10%

Find the selling price of the following:

	Cost	Rate gain		Cost	Rate gain
11.	\$10.00	40%	16.	\$0.25	100%
12.	\$35.00	50%	17.	\$80.00	25%
13.	\$0.50	100%	18.	\$300.00	10%
14.	\$3.00	25%	19.	\$5000	20%
15.	\$30.00	50%	20.	\$1.25	20%

Find the agent's commission on the following:

	Selling price	Rate com.		Selling price	Rate com
21.	\$200	10%	26.	\$25	10%
22.	\$4000	5%	27.	\$60	5%
23.	\$800	5%	28.	\$700	10%
24.	\$50,000	2%	29.	\$480	$12\frac{1}{2}$%
25.	\$1200	20%	30.	\$6000	5%

Find the yearly interest on the following:

	Principal	Rate		Principal	Rate
31.	\$200	6%	34.	\$3000	7%
32.	\$4000	5%	35.	\$5000	8%
33.	\$600	6%	36.	\$4500	6%

FINDING THE RATE OF INTEREST

110. **Principal, Rate, Interest.** The sum on which interest is computed is called the *principal.* The number of dollars paid for the use of $100 for one year is called the rate of interest, or simply the *rate,* and the sum paid for the use of the principal is called the *interest.* The principal is the amount loaned or borrowed.

111. **Finding the Rate.** Problems occur frequently in which the principal and the interest are given and the rate is required.

Problem 1. What is the rate if the interest for one year on $600 is $30?

Solution: As on pages 176, 177, we find that 30 is 5% of 600. Hence the required rate is 5%.

Problem 2. What is the rate if the interest for one year on $5000 is $275?

$$5000\overline{)275.00} = 5\overline{).275}\quad .055 = 5.5\%$$

We find that $275 is 5.5% of $5000. Hence the required rate is 5½%.

ORAL EXERCISES

Find the rate in each of the following:

	Principal	Interest		Principal	Interest
1.	$1000	$60	10.	$3000	$240
2.	$800	$40	11.	$400	$32
3.	$1500	$150	12.	$600	$30
4.	$2000	$100	13.	$3000	$210
5.	$4000	$240	14.	$5000	$250
6.	$6000	$300	15.	$2000	$120
7.	$600	$25	16.	$700	$56
8.	$1000	$50	17.	$400	$32
9.	$200	$12	18.	$900	$72

The problem of finding the rate of interest occurs very frequently. Anyone who saves money and invests it, thereby getting a certain income, is interested in knowing what rate of interest he is getting on his investment.

Problem. A man invested $1080 and received a yearly income of $60. What rate of interest did he receive on the investment? Find the result to the nearest tenth of one per cent.

```
       .0555 or 5.6%
1080)60.00
     5400
      6000
```

We find that 60 is 5.6% (nearly) of 1080. Hence, this is the required rate of interest.

WRITTEN EXERCISES

Find the rates of interest to the nearest tenth of one per cent in each of the following:

	Principal	Interest			Principal	Interest	
1.	$6500	$480	7.4%	8.	$5900	$460	7.8%
2.	$1800	$100	5.6%	9.	$14000	$1200	8.6%
3.	$3600	$225	6.3%	10.	$35000	$2800	8%
4.	$450	$35	7.8%	11.	$20000	$1800	9%
5.	$2500	$300	12%	12.	$7800	$600	7.7%
6.	$4000	$360	9%	13.	$4500	$400	8.9%
7.	$5000	$640	12.8%	14.	$1250	$150	12%

15. A man invested $6500 in bank stock and received yearly income of $550. What was the rate of income on the investment? 8.5%

16. A man invested $15000 in a business. His yearly share of the profits was $2000. What was the rate of income on this investment. 13.3%

17. A man paid $150,000 for an office building. His net yearly income from it was $10,000. What was the rate of income on the investment? 6.7%

WRITTEN WORK

1. What is the rate if the interest on $4200 for one year is $220? 5.2%
2. What is the rate if the interest on $6500 for one year is $260? 4%
3. A capitalist has investments amounting to $800,000 and the yearly income from them is $41,600. What is the average rate of interest? 5.2%
4. In a recent year the total income-bearing property of a great university was $31,748,000, and the income from this property was $1,526,400. Find the rate to the nearest tenth of one per cent. 4.8%
5. If an investment of $109.50 yields an income of $6 what is the rate of interest? 5.5%
6. A man buys a property for $9850, which yields a net yearly income of $500. What is the rate of interest on the investment? 5.1%
7. A farm is bought at $175 an acre. If the farm yields a net income of $12.50 per acre, what is the rate of income on the investment? 7.1%
8. A house costing $7600 is rented for $50 a month. If taxes, repairs, depreciation and so on, amount to $175 a year, what is the rate of income on the investment? 5.6%
9. An apartment building erected at a cost of $146,000 yielded a net income above all expenses of $11,500. What was the rate of income on the investment? 7.9%
10. A security bringing in a yearly income of $8.00 is bought for $139.00. What is the rate of income on the investment? 5.8%
11. A man improves a rented house at an expense of $2480, and the rent is thereby increased $10 a month. What is the rate of interest obtained on this investment? 4.8%
12. Make and solve other problems on finding the rate of interest.

When the time for which interest runs is two or more years, the interest is found by multiplying the interest for one year by the number of years.

ORAL EXERCISES

1. At the rate of 5% what is the interest on $600 for one year? for 2 years?

2. At the rate of 8% what is the interest on $300 for one year? for 3 years?

3. At the rate of 6% what is the interest on $800 for one year? for 3 years? for 10 years?

Find the interest on each of the following:

	Principal	Rate	Time
4.	$300	5%	One year, 3 years, 6 years.
5.	$200	6%	One year, 4 years,
6.	$400	6%	One year, 2 years, 5 years.
7.	$800	5%	One year, 6 years

WRITTEN EXERCISES

Find the interest on the following:

	Principal	Rate	Time	
1.	$7800	$5\frac{1}{2}$%	Two years, 5 years.	$858; $2145
2.	$3200	$6\frac{1}{2}$%	One year, 6 years.	$208; $1248
3.	$3800	$5\frac{3}{4}$%	Two years, 7 years.	$437; $1529.50
4.	$5800	6%	Two years, 7 years.	$696; $2436
5.	$3900	7%	Five years, 8 years.	$1365; $2184
6.	$25,000	$4\frac{1}{2}$%	Three years, 5 years.	$3375; $5625
7.	$2700	8%	Four years, 9 years.	$864; $1944
8.	$35,800	$4\frac{1}{2}$%	Two years, 5 years.	$3222; $8055

112. **Time Given in Months.** In computing interest it is customary to count 360 days to the year and 30 days to the month. One month is regarded as $\frac{1}{12}$ of a year, whether the month contains 28 days or 31 days. Two months equals $\frac{1}{6}$ of a year, and so on.

The time from any date in one month to the same date in the month following is counted as one month or thirty days.

Thus, from January 5 to February 5 is one month, and from February 5 to March 5 is also one month, although the latter is 28 days (except in a leap-year, when it is 29 days), while the former is always 31 days.

ORAL EXERCISES

1. What fraction of one year is there in 1 month? in 2 months? in 3 months? in 4 months? in 5 months? in 6 months? in 7 months? in 8 months? in 9 months? in 10 months? in 11 months?

To find the interest for one month take $\frac{1}{12}$ of the interest for one year; to find the interest for 2 months take $\frac{1}{6}$ of the interest for one year, and so on.

2. How may interest be found for 3 months? for 4 months? for 5 months? for 6 months?
3. How may interest be found for 7 months? for 8 months? for 9 months? for 10 months? for 11 months?

WRITTEN EXERCISES

Find the interest on the following:

	Principal	Rate	Time		Principal	Rate	Time
1.	\$500.	6%	6 months \$15	7.	\$1600.	8%	10 months \$106.67
2.	\$1200.	5%	6 months \$30	8.	\$3700.	7%	11 months \$237.42
3.	\$800.	6%	4 months \$16	9.	\$5800.	7%	9 months \$304.50
4.	\$2500.	6%	3 months \$37.50	10.	\$4300.	8%	7 months \$200.67
5.	\$4500.	7%	8 months \$210	11.	\$7200.	6%	5 months \$180
6.	\$6400.	5%	9 months \$240	12.	\$5400.	5%	4 months \$90

113. **Time Given in Days.** The simplest rule for finding interest for any time less than a year is to express the time as a fraction of a year and multiply the interest for one year by that fraction.

Since one year is regarded as 360 days, 20 days is $\frac{20}{360}$, or $\frac{1}{18}$ of a year.

Again, since every month is regarded as having 30 days, 1 month 20 days is 50 days or $\frac{50}{360}=\frac{5}{36}$ of one year.

Similarly, 18 days $=\frac{18}{360}=\frac{1}{20}$ of one year.

Example. At 7% find the interest on $800, for 3 mo. 15 da.

Solution:

$$\frac{7}{\cancel{24}_{3}}\times\cancel{56}^{7}=\frac{49}{3}=16.33\tfrac{1}{3}\text{ (dollars)}$$

3 mo. 15 da. is 105 da., or $\frac{105}{360}=\frac{7}{24}$ of a year. Hence, multiply the interest for one year ($56) by $\frac{7}{24}$.

WRITTEN EXERCISES

Express as fractions of a year and in the simplest form. If the time is given as months and days first reduce to days.

1. 70 days, 40 days, 50 days, 80 days, 100 days. $\frac{7}{36}$; $\frac{1}{9}$; $\frac{5}{36}$; $\frac{2}{9}$; $\frac{5}{18}$
2. 65 days, 25 days, 160 days, 130 days. $\frac{13}{72}$; $\frac{5}{72}$; $\frac{4}{9}$; $\frac{13}{36}$
3. 2 months and 15 days. $\frac{5}{24}$
4. 2 months and 21 days. $\frac{9}{40}$
5. 1 month and 5 days. $\frac{7}{72}$
6. 4 months, 24 days. $\frac{2}{5}$
7. 7 months, 18 days. $\frac{19}{30}$
8. 9 months, 15 days. $\frac{19}{24}$

Find the interest in each of the following:

	Principal	Rate	Time	
9.	$1700.	5%	50 days	$11.81
10.	$3500.	6%	2 mo. 15 da.	$43.75
11.	$4200.	5%	3 mo. 10 da.	$58.33
12.	$6800.	7%	4 mo. 20 da.	$185.11
13.	$9500.	6%	7 mo. 6 da.	$342

114. Fractional Per Cent. When the rates of interest contain fractions of one per cent, the fractions are usually very simple, such as $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{4}$.

Example 1. Find the interest on \$3600 at $5\frac{1}{2}\%$ for 9 mo. 18 da.

$36|00 = 1%
18|00 = $\frac{1}{2}\%$
180|00 = 5%
$198|00 = $5\frac{1}{2}\%$
4
5)792
158|40 = interest in dollars.

First find the interest for one year. To find $5\frac{1}{2}\%$ of \$3600 first find 1%, then $\frac{1}{2}\%$, and then 5%. Add the $\frac{1}{2}\%$ and 5%.

9 mo. 18 da. is $\frac{4}{5}$ of a year.

Multiply the interest for one year by $\frac{4}{5}$. In such examples the vertical line is used to separate dollars and cents.

Example 2. Find the interest on \$670 at $6\frac{3}{4}\%$ for 10 mo. 6 da.

$6|70 = 1%
1|675 = $\frac{1}{4}\%$
5|025 = $\frac{3}{4}\%$
40|20 = 6%
$45|225 = $6\frac{3}{4}\%$
17
316|575
452|25
20)768|825
38|44 = interest in dollars.

First find interest for one year. Find 1% of \$670, then $\frac{1}{4}\%$, then $\frac{3}{4}\%$, and finally 6%. Add the $\frac{3}{4}\%$ and 6%.

10 mo. 6 da. = $\frac{17}{20}$ of a year.

Multiply the interest for one year by $\frac{17}{20}$. Express the final result to the nearest cent.

WRITTEN EXERCISES

Find the interest on each of the following:

	Principal	Rate	Time	
1.	\$4500	$5\frac{1}{4}\%$	8 months, 15 days.	\$167.34
2.	\$780	$5\frac{3}{4}\%$	4 months, 12 days.	\$16.45
3.	\$34,000	$4\frac{3}{4}\%$	9 months, 24 days.	\$1318.92
4.	\$160	$6\frac{1}{2}\%$	3 months, 9 days.	\$2.86
5.	\$2400	$5\frac{3}{4}\%$	10 months, 7 days.	\$117.68

115. Drill in Interest. In the examples on this page the time is given as days, months or years, but not as combinations of these.

WRITTEN EXERCISES

1. What is the interest on \$750 at $5\frac{1}{2}\%$ for 3 years? \$123.75
2. What is the interest on \$350 at 6% for 7 years? \$147.00
3. What is the interest on \$9000 at 5% for 9 months? \$337.50
4. What is the interest on \$1600 at $5\frac{1}{2}\%$ for 4 years? \$382.00
5. What is the interest on \$1850 at $5\frac{1}{4}\%$ for 75 days? \$20.23

Find the interest for each of the following:

Principal	Rate	Time		Principal	Rate	Time	
6. \$1850	$5\frac{1}{2}\%$	3 yrs.	\$305.25	18. \$800	5%	126 da.	\$14.00
7. \$900	$5\frac{3}{4}\%$	5 yrs.	\$258.75	19. \$220	6%	90 da.	\$3.30
8. \$2400	6%	8 mos.	\$96.00	20. \$5300	$5\frac{1}{2}\%$	130 da.	\$105.26
9. \$7500	$5\frac{1}{4}\%$	6 yrs.	\$2362.50	21 \$5600	5%	85 da.	\$66.11
10. \$9400	5%	3 yrs.	\$1410.00	22. \$650	6%	260 da.	\$28.17
11. \$5000	6%	7 mos.	\$175.00	23. \$3400	4%	180 da.	\$68.00
12. \$245	6%	3 mos.	\$3.68	24. \$460	$3\frac{1}{2}\%$	230 da.	\$10.29
13. \$760	$6\frac{1}{4}\%$	5 mos.	\$19.79	25. \$1500	5%	300 da.	\$62.50
14. \$3400	$5\frac{1}{2}\%$	4 yrs.	\$748.00	26. \$940	$5\frac{3}{4}\%$	170 da.	\$25.52
15. \$850	5%	11 mos.	\$38.96	27. \$8500	$5\frac{1}{2}\%$	135 da.	\$175.31
16. \$460	$5\frac{1}{4}\%$	9 mos.	\$18.11	28. \$12,000	5%	280 da.	\$466.67
17. \$3500	6%	5 mos.	\$87.50	29. \$16,500	$4\frac{3}{4}\%$	75 da.	\$163.28

Remark: To find the interest when the time is given as years and fractions of a year, first find the interest for the whole years and then for the fraction of a year, and add. Such problems occur rarely in practice.

Problem. A man borrows $400 on October 10th at 6%, and pays it back on February 25th of the following year. How much must he pay?

To solve this problem we must find the number of months and days from October 10th to February 25th. From October 10th to February 10th is 4 months, and from February 10th to February 25th is 15 days. Hence, the time is 4 months 15 days or $\frac{3}{8}$ of a year.

The interest is $\frac{3}{8} \times \$24 = \$9$.

No one rule will give the shortest method for reducing months and days to a fraction of a year. Thus, 4 months 15 days is $\frac{1}{3} + \frac{1}{24}$ of a year $= \frac{9}{24} = \frac{3}{8}$ of a year. You should be on the alert to see short cuts of this sort.

WRITTEN EXERCISES

How many months and days are there between each of the following pairs of dates? Express each as a fraction of a year.

1. Jan. 15 to Sept. 24.
   8 mo. 9 dys.; $\frac{83}{120}$ yr.
2. April 10 to Nov. 18.
   7 mo. 8 dys.; $\frac{109}{180}$ yr.
3. Sept. 7 to May 1.
   7 mo. 24 dys.; $\frac{13}{20}$ yr.
4. July 24 to Dec. 15.
   4 mo. 21 dys.; $\frac{47}{120}$ yr.
5. May 12 to Nov. 21.
   6 mo. 9 dys.; $\frac{21}{40}$ yr.
6. June 15 to Dec. 27.
   6 mo. 12 dys; $\frac{8}{15}$ yr.
7. April 24 to Aug. 27.
   4 mo. 3 dys.; $\frac{41}{120}$ yr.
8. March 12 to Sept. 3.
   5 mo. 21 dys.; $\frac{19}{40}$ yr.
9. A man borrowed $550 Nov. 1, and paid it back May 1 the next year. How much interest did he pay if the rate was 6%? $16.50
10. A man borrowed $750 June 3, and paid it back December 1 of the same year. How much interest did he pay if the rate was $6\frac{1}{2}$%? $24.10
11. A man borrowed $8500 to buy a lot and build a house. How much interest did he pay each year if the rate was $5\frac{1}{2}$%? $467.50
12. Find the interest on $2600 at 6% from March 1 to July 10. $55.90

Make and solve other problems like these. See who can make the most interesting problems.

116. The Savings Bank. Money is frequently deposited in savings banks, which offer a high degree of security, but pay a low rate of interest, usually from 3% to 4%.

117. Interest Days. Interest Period. The savings bank computes the interest twice or four times a year. The days on which interest is computed are called *interest days*, and the periods between interest days are called *interest periods*.

Interest is usually computed on the smallest amount on deposit during the interest period; interest is not allowed on a fraction of one dollar.

Problem. In a certain savings account the smallest amount on deposit for the interest period from January 1st to April 1st was $284.60. Find the interest for this period, the rate being $3\frac{1}{2}\%$.

Solution: $3\frac{1}{2}\%$ of $284 = $9.94. The time is $\frac{1}{4}$ of a year.
$\frac{1}{4}$ of $9.94 = $2.49. The 60 cents in the principal is neglected.

WRITTEN EXERCISES

Find the interest:

	Minimum deposit	Interest period	Rate			Minimum deposit	Interest period	Rate	
1.	$227.50	6 mo.	3%	$3.41	5.	$124.20	6 mo.	4%	$2.48
2.	$47.80	3 mo.	$3\frac{1}{2}\%$	$.42	6.	$680.50	3 mo.	$3\frac{1}{4}\%$	$5.53
3.	$45.40	6 mo.	4%	$.91	7.	$362.70	3 mo.	3%	$2.72
4.	$134	3 mo.	$3\frac{1}{4}\%$	$1.09	8.	$145.60	6 mo.	4%	$2.91

Solve as many examples as you can on this page orally.

Reduce each of the following fractions to per cent:

1. $\frac{1}{2}$ 50%	4. $\frac{1}{5}$ 20%	7. $\frac{4}{5}$ 80%	10. $\frac{1}{8}$ $12\frac{1}{2}\%$	13. $\frac{7}{8}$ $87\frac{1}{2}\%$
2. $\frac{1}{4}$ 25%	5. $\frac{2}{5}$ 40%	8. $\frac{1}{3}$ $33\frac{1}{3}\%$	11. $\frac{3}{8}$ $37\frac{1}{2}\%$	14. $\frac{1}{6}$ $16\frac{2}{3}\%$
3. $\frac{3}{4}$ 75%	6. $\frac{3}{5}$ 60%	9. $\frac{2}{3}$ $66\frac{2}{3}\%$	12. $\frac{5}{8}$ $62\frac{1}{2}\%$	15. $\frac{5}{6}$ $83\frac{1}{3}\%$

Reduce the following to decimals, giving each to the nearest ten-thousandth:

16. $1\frac{1}{2}\%$ .015	19. $1\frac{1}{8}\%$ .0113	22. $4\frac{7}{8}\%$ .0488	25. $4\frac{1}{5}\%$ .042
17. $2\frac{1}{4}\%$ .0225	20. $6\frac{3}{8}\%$ .0638	23. $4\frac{1}{3}\%$ .0433	26. $5\frac{3}{5}\%$ .056
18. $3\frac{3}{4}\%$ .0375	21. $5\frac{5}{8}\%$ .0563	24. $7\frac{2}{3}\%$ .0767	27. $6\frac{4}{5}\%$ .068

Fill in the blank in each of the following:

	Base	Rate	Percentage		Base	Rate	Percentage
28.	460	6%	27.60	30.	8000	4%	320
29.	540	$4\frac{1}{2}\%$	24.3	31.	2450	$5\frac{1}{2}\%$	134.75

Express each of the following decimals in per cent:

32. .064 $6\frac{2}{5}\%$	34. .438 $43\frac{4}{5}\%$	36. .093 $9\frac{3}{10}\%$	38. 1.04 104%
33. .147 $14\frac{7}{10}\%$	35. .001 $\frac{1}{10}\%$	37. .1942 $19\frac{21}{50}\%$	39. 1.351 $135\frac{1}{10}\%$

Below is a season's record of a school baseball team.

40. Reduce the standing of each team to a decimal correct to the nearest thousandth. Also express each in per cent.

	Games played	Games won	Standing
Lincoln School...........	24	13	.542—$54\frac{1}{6}\%$
Grant School.............	25	11	.440— 44%
Marshall School..........	23	10	.435—$43\frac{1}{2}\%$
Ward School..............	24	12	.500— 50%
Emerson School..........	26	16	.615—$61\frac{1}{2}\%$
Calhoun School...........	22	10	.455—$45\frac{1}{2}\%$

ORAL AND WRITTEN WORK

1. Tell how the rate of gain in making a sale is computed. Give an example.
2. Tell how the rate of loss in making a sale is computed. Give an example.
3. On what price is the rate of reduction from the marking price computed?
4. A man buys real estate for $10,000 and sells it for $11,500. Disregarding incidental expense, what is his gain per cent? 15%
5. A merchant buys an article for $6.00 and marks it to sell 80% above the purchase price. At what price does he mark the article? $10.80
6. An article costing $10 is marked to sell at $14. What per cent of the purchase price is the gain? 40%
7. A real estate man buys property for $7600 and sells it at a gain of 7%. What is the selling price? $8132
8. A dealer buys goods for $730 and sells them at a gain of 12½%. What is the selling price? $821.25
9. A merchant buys goods for $100 and marks them to sell at a gain of 75%. What is the marking price? $175
10. At a special sale a merchant reduces the price of an article marked $175.00 by 50%. What is the selling price? $87.50
11. A housekeeper buys a box of soap containing 100 cakes of soap for $5.00. This soap sells at retail at 4 cakes for 30 cents. How much does the housekeeper save by buying the whole box instead of buying 100 cakes, 4 cakes at a time? How many per cent of the purchase price ($7.50) does she save? $2.50; 33⅓%
12. Find the interest on $1740 for 7 mo. 18 da., the rate being 6%. $66.12
13. Find the interest on $4300 for 3 mo. 12 da. The rate of interest is 6½%. $79.19

WRITTEN WORK

1. A man invests \$7500 and makes a net gain of \$860. What is the rate per cent of gain on this investment? 11.5%
2. A certain make of automobile depreciates 35% the first year. If it costs \$1350 when new, how much will it be worth at the end of one year? \$877.50
3. An agent sells a second-hand automobile for \$850. How much does he remit to the owner if he deducts a commission of 10%? \$765
4. A piece of real estate is sold for \$9600. How much does the owner get for it if the agent deducts a commission of 10%? \$8640
5. An agent buys a used automobile for \$1600. How much does it cost the buyer if the agent charges 3% as his commission? \$1648
6. A man buys land for \$165 an acre. This land yields a net income of \$11.55 an acre. What rate per cent does the man make on this investment? 7%
7. Books are sold at $\frac{1}{2}$ off. What per cent reduction is this? If books are sold at $\frac{1}{3}$ off, what is the rate per cent discount? 50%; $33\frac{1}{3}$%
8. $\frac{3}{4}$ is what per cent of $\frac{2}{5}$? $187\frac{1}{2}$%
9. Find 45% of $1\frac{3}{4}$. .7875
10. What is the interest on \$286 at $3\frac{1}{2}$% for 3 months? \$2.50
11. Find the interest on \$14,500 at $6\frac{1}{2}$% for 2 months, 21 days. \$212.06
12. Find the interest on \$3500 at $5\frac{3}{4}$% for 87 days. \$48.64
13. Find the interest on \$780 at $6\frac{1}{2}$% for one year and 7 months. \$80.28
14. A ship built at a cost of \$278,000 yielded a net return of \$97,400 the first year. What per cent of the cost was this? 35%

WRITTEN WORK

1. A property costing $19,600 yields a net yearly income of $1650. What is the rate of interest on this investment? 8.4%
2. In a school with 465 boys and 535 girls what per cent of the pupils are boys and what per cent are girls? 46½% boys; 53½% girls
3. An automobile is bought for $675. If the cost of running it one year is $286, and depreciation 25% of the original price, what is the total expense of using this machine one year? $454.75
4. A county was bonded for $280,000 to build roads, the rate of interest being 4¾%. What was the yearly interest? $13,300
5. If roads costing $280,000 depreciate 6¼% a year, what is the yearly depreciation? $17,500
6. The population of a certain city in 1900 was 26,840, and in 1910 it was 41,352. What was the increase per cent during this decade? 54.1%
7. If an investment of $147.50 yields an income of $9 a year, what per cent on the investment is this? 6.1%
8. At an average rate of 4.87%, what is the income from an investment of $2,860,000? $139,282
9. If an investment of $84,280 yields a net income of $7360, what is the rate of income on the investment? 8.7%
10. If an investment of $84 yields a net income of $5 a year, what is the rate on the investment? 5.95%
11. Goods listed at $12.50 are sold for $8. What is the per cent of discount? 36%
12. A firm has debts owing to it amounting to $14,850.60. Of this $12,500 is regarded as collectible. What per cent is regarded as collectible? 84.2%
13. Goods listed at $65 are sold at discount of 25%. What is the selling price? If the buyer sells them again at a gain of 10% what is his selling price? $48.75; $53.63

CHAPTER IV.

118. **The Solution of Problems.** In our study of Arithmetic we have learned to perform the operations of addition, subtraction, multiplication, and division on whole numbers and on fractions; and we have made use of these operations in the solution of problems. We will now make a more special study of the solution of problems.

Example. A farmer gets 560 bushels of wheat from a field containing 24 acres. At this rate, how much will he get from a field containing 64 acres?

Solution: $560 \div 24$ or $\frac{560}{24}$ = number of bushels from one acre.

$$\frac{560}{24} \times 64 = \text{number of bushels from 64 acres.}$$

The expression $\frac{560}{24} \times 64$ contains complete directions for solving the problem. All that is left is to carry out the operations indicated by this expression. This we do as follows:

$$\frac{560}{\underset{3}{\cancel{24}}} \times \overset{8}{\cancel{64}} \qquad \begin{array}{r} 560 \\ 8 \\ \hline 3\overline{)4480} \end{array} = 1493\tfrac{1}{3} = \text{number of bushels.}$$

First cancel 8 in 24 and 64. 3 is not a factor of 8 or of 560, so we multiply 560 by 8 and then divide by 3. Notice that we save trouble by multiplying by the 8 *before* dividing by 3.

WRITTEN EXERCISE

1. Solve the above problem by carrying out the operations as they arise. That is, divide 560 by 24 to get the yield per acre, and then multiply by 64. This gives you an idea of the importance of indicating all the operations before performing any of them.

ORAL AND WRITTEN WORK

1. If you know the number of acres in a field and the average yield per acre, how do you find the total yield?

The answer may be written as follows:

$$\textit{number of acres} \times \textit{yield per acre} = \textit{total yield}. \qquad (1)$$

2. If the number of acres and the total yield are given, how do you find the average yield per acre? (Compare with finding the missing number in $3 \times ? = 12$.)

3. The census of 1910 gives the total production of wheat in the United States for 1909 as 683,379,259 bushels, and the number of acres in wheat for that year as 44,262,592. The average yield in bushels per acre is given as 15.4. How was this average obtained by the Census Office? Find the average yourself, and see if it is correctly given.

4. If the total yield and the yield per acre are given, how do you find the number of acres? (Compare with $? \times 4 = 12$.) (Use equation 1.)

5. If you know the number of things bought or sold, and the price for each, how do you find the total cost?

The answer may be written:

$$\textit{number of articles} \times \textit{price} = \textit{cost}. \qquad (2)$$

6. If the total cost and the price per unit are given, how do you find the number bought? (Compare with $? \times 4 = 12$.)

7. If the total cost and the number of things bought are given, how do you find the price? (Compare with $3 \times ? = 12$.)

8. 435 horses were bought at a total cost of \$55,000. What was the average cost per head? \$126.44

9. If horses for the army could be bought at an average price of \$160 per head, how many could be bought for \$500,000? 3125

10. State in words the three problems which may be solved by using equation (2).

1. If you know the length and width of a rectangle, how do you find the area? The answer may be written:

$$\textit{length} \times \textit{width} = \textit{area}. \qquad \text{(A)}$$

2. If you know the length of a rectangle and its area, how do you find the width? (Compare $3 \times ? = 12$.)

3. If you know the width of a rectangle and its area, how do you find its length?

4. A farmer is plowing a field 120 rods long; how wide a strip must he plow to make 18 acres? Remember that one acre is 160 square rods. (Divide the area in square rods by the length.) **24 rd.**

5. A builder wants 6400 square feet for a building. How wide a strip must he buy if the piece from which he buys is 98 feet deep? **65.3 ft.**

6. A lot is 25 feet wide and 112 feet deep. Another lot is 28 feet wide. How deep must it be to have the same area as the first? **100 ft.**

Suggestion: $25 \times 112 =$ area in square feet of the first, and also of the second lot. Hence, $\frac{\text{area}}{\text{width}} = \frac{25 \times 112}{28} =$ depth in feet of the second lot.

7. A room is 16 feet wide and 18 feet long. How wide must a room be which is to have twice the area of this one, if it is 24 feet long? **24 ft.**

Suggestion: $2 \times 16 \times 18 =$ area in square feet of the larger room.

8. A farmer wishes to plant 6 acres in apple trees. How long will the orchard be if it is planted on a strip of land 30 rods wide? **32 rd.**

Suggestion: The area of the orchard is 6×160 square rods.

9. A certain crew can put in 1250 square feet of pavement in one day. How long a street can they pave in one week if the street is 42 feet wide? **178.6 ft.**

10. State in words the three problems that may be solved by using equation (A).

WRITTEN WORK

(Remember it is best to indicate the solution first.)

1. A lot 25 feet wide and 150 feet deep is sold for \$7500. What is the price per square foot? **\$2**

2. At \$1.75 a square foot what is the value of a lot 26.8 feet wide and 94.3 feet deep? Find the value to the nearest cent. **\$4422.67**

3. At \$70 an acre what is the value of a farm 145 rods wide and 216 rods long? **\$13702.50**

Suggestion: $\frac{145\times216}{160}$ is the number of acres in the farm.

4. A farmer is offered \$10,000 for a piece of land 182 rods wide and 290 rods long. What price per acre is this? **\$30.31**

5. A field 68 rods wide and 120 rods long yields 860 bushels of wheat. What is the average yield per acre from this field? Find the result to the nearest tenth of a bushel. **16.9 bu.**

Suggestion: The yield per acre is $860\div\frac{68\times120}{160}=860\times\frac{160}{68\times120}$.

6. At 25 cents a square foot what is the cost of laying a hardwood floor in a room 26 feet by 36 feet? **\$234**

7. A carpenter offers to put floors in a hall 80 feet by 105 feet for \$1800. What is the cost per square foot?

Suggestion: Indicate the solution by writing $\frac{1800}{80\times105}$. This will give result in dollars.
\$$\frac{3}{14}$ or 21.4c

8. A sidewalk 5 feet wide and 350 feet long was laid at a cost of \$260. What was the cost per square foot? per square yard? **14.9c; \$1.34**

9. At \$1.80 a square yard what is the cost of paving a street 35 feet wide and 680 feet long? **\$4760**

10. A field 120 rods long and 80 rods wide was cultivated one season for a total cost of \$325.00. What was the cost per acre? **\$5.42**

ORAL EXERCISES

1. A box is 8 inches long, 6 inches wide, and 4 inches deep. How many cubic inches does it hold?

2. If you know the length, the width, and the depth of a rectangular box, how do you find its volume? The answer may be written:

$$\textit{length} \times \textit{width} \times \textit{depth} = \textit{volume}. \qquad \text{(V)}$$

3. If you know the volume of a box and also its length and depth, how do you find its width? (Compare with finding the missing number in $2 \times 3 \times ? = 24$.)

4. If you know the volume of a box and also its length and depth how do you find its width?

5. If you know the volume of a box and also its width and depth, how do you find its length?

Problem. At 75c a cubic yard how much will it cost to excavate a basement 26 feet wide, 42 feet long, and 5 feet deep?

Solution: $\frac{26 \times 42 \times 5}{27}$ = number of cubic yards, and $\frac{26 \times 42 \times 5 \times 75}{27}$ = cost in cents.

That is, $$\frac{26 \times \overset{14}{\cancel{42}} \times 5 \times \overset{25}{\cancel{75}}}{\underset{\cancel{9}\ \ 3}{\cancel{27}}} = \frac{26 \times 14 \times 5 \times 25}{3} = 15167 = \text{cost in cents.}$$

Hence the cost to the nearest cent is $151.67.

Notice that the fraction $\frac{26 \times 42 \times 5}{27}$ is multiplied by 75 by multiplying its numerator by that number. It is also useful in indicating the solution of problems to note that a fraction is divided by a number by multiplying its denominator by that number.

WRITTEN EXERCISES

1. At 45 cents a cubic yard, find the cost to the nearest cent of making an excavation 49 feet wide, 50 feet long, and 4 feet deep. **$163.33**

2. A wheat bin in a grain elevator is 12 feet wide, 16 feet long, and 54 feet deep. How many bushels of wheat does it hold if one cubic foot is $\frac{4}{5}$ of a bushel? **8294.4**

Suggestion: The required number of bushels is $12\times16\times54\times\frac{4}{5}$.

3. A farmer has a wheat bin 9 feet by 14 feet. How many bushels of wheat are there in it if the grain is $7\frac{1}{2}$ feet deep? **756 bu.**

4. The excavation for a sewer is 7 feet wide and 10 feet deep. The contractor is removing 280 cubic yards each day. In how many days will he excavate one mile of sewer? Get result to the nearest day.

Suggestion: $7\times10\times5280$ = number of cubic feet to be removed, and 280×27 = number of cubic feet removed in one day.

Hence, $\dfrac{7\times10\times5280}{280\times27}$ is the required number of days. **48.9 days.**

5. A schoolroom 30 feet wide, 36 feet long, and 11 feet high seats 48 pupils. The large assembly hall, which is 84 feet wide and 96 feet long and 22 feet high, seats 678 pupils. Which room contains the larger number of cubic feet per pupil? How many? **Assembly; 14.2 cu. ft.**

Suggestion: The number of cubic feet of air per pupil in the two rooms are indicated by: $\dfrac{30\times36\times11}{48}$ and $\dfrac{84\times96\times22}{678}$.

6. A hall is 64 feet long, and 76 feet wide and 20 feet high. How many pupils may be seated in it if it is to contain 225 cubic feet of space per pupil? Get the result to the nearest whole number of pupils. **432**

Suggestion: The number is $\dfrac{64\times76\times20}{225}$.

ORAL EXERCISES

Now consider the three problems:

(*a*) $\frac{3}{8}$ of $\frac{4}{5}$ = what? This is equivalent to finding the missing number in $\frac{3}{8}\times\frac{4}{5}=?$

(*b*) $\frac{3}{8}$ of what number $=\frac{3}{10}$? Or $\frac{3}{10}$ is $\frac{3}{8}$ of what number? This is equivalent to finding the missing number in $\frac{3}{8}\times ? =\frac{3}{10}$.

(*c*) $\frac{3}{10}$ is what part of $\frac{4}{5}$? This is equivalent to finding the missing number in $?\times\frac{4}{5}=\frac{3}{10}$.

In (*a*) the product of two fractions is required. In (*b*) and (*c*) the product and one factor are given and the other factor is required.

WRITTEN EXERCISES

Indicate the solution of each of the following:

1. Find $\frac{4}{7}$ of $\frac{3}{14}$.
   Indicated solution: $\frac{4}{7}$ of $\frac{3}{14}=\frac{4}{7}\times\frac{3}{14}$.

2. $\frac{2}{3}$ is what part of $\frac{3}{4}$?
   Indicated solution: From $?\times\frac{3}{4}=\frac{2}{3}$ we know that the missing number is $\frac{2}{3}\div\frac{3}{4}$ or $\frac{2}{3}\times\frac{4}{3}$.

3. $\frac{1}{4}$ is what part of $\frac{7}{8}$? $\frac{1}{4}\times\frac{8}{7}$
4. $\frac{2}{3}$ is $\frac{1}{2}$ of what number? $\frac{2}{3}\times 2$
5. $\frac{1}{2}$ is $\frac{5}{4}$ of what number? $\frac{1}{2}\times\frac{4}{5}$
6. $\frac{3}{4}$ is $\frac{1}{2}$ of what number? $\frac{3}{4}\times 2$
7. $\frac{4}{5}$ is what part of $2\frac{1}{3}$? $\frac{4}{5}\times\frac{3}{7}$
8. $2\frac{1}{3}$ is what part of $6\frac{4}{5}$? $\frac{7}{3}\times\frac{5}{34}$
9. $1\frac{1}{2}$ is $\frac{2}{3}$ of what number? $\frac{3}{2}\times\frac{3}{2}$
10. $4\frac{1}{3}$ is $\frac{2}{3}$ of what number? $\frac{13}{3}\times\frac{3}{2}$
11. $12\frac{1}{2}$ is what part of 25? $\frac{25}{2}\times\frac{1}{25}$
12. $\frac{3}{4}$ is what part of $\frac{7}{8}$? $\frac{3}{4}\times\frac{8}{7}$
13. $1\frac{1}{2}$ is what part of $12\frac{2}{3}$? $\frac{3}{2}\times\frac{3}{38}$
14. $4\frac{1}{2}$ is $\frac{2}{3}$ of what number? $\frac{9}{2}\times\frac{3}{2}$
15. Find $\frac{3}{8}$ of $\frac{8}{15}$. $\frac{3}{8}\times\frac{8}{15}$
16. Find $\frac{5}{16}$ of $8\frac{2}{3}$. $\frac{5}{16}\times\frac{26}{3}$
17. $7\frac{2}{3}$ is $\frac{5}{6}$ of what number? $\frac{23}{3}\times\frac{6}{5}$
18. $4\frac{5}{6}$ is $\frac{4}{5}$ of what number? $\frac{29}{6}\times\frac{5}{4}$
19. $5\frac{2}{3}$ is what part of $17\frac{1}{2}$? $\frac{17}{3}\times\frac{2}{35}$
20. $2\frac{7}{8}$ is what part of $41\frac{1}{2}$? $\frac{23}{8}\times\frac{2}{83}$

Example. $\frac{5}{6}$ is what part of $2\frac{1}{3}$? Reduce the result to a decimal correct to the nearest thousandth. Also give it in per cent correct to the nearest tenth of one per cent.

Solution: From $? \times 2\frac{1}{3} = \frac{5}{6}$, we know that the required number is:

$$\frac{5}{6} \div 2\frac{1}{3} = \frac{5}{6} \div \frac{7}{3} = \frac{5}{\cancel{6}_{2}} \times \frac{\cancel{3}}{7} = \frac{5}{14} = .357 = 35.7\%.$$

We now see that the above problem is the same as "$\frac{5}{6}$ is how many per cent of $2\frac{1}{3}$?" The only difference is that in one case the result is given as a common fraction, and in the other as a decimal read as per cent.

WRITTEN EXERCISES

In each of the following give the result as a common fraction or mixed number and also as a decimal correct to the nearest thousandth, and in per cent correct to the nearest tenth of one per cent.

1. $1\frac{3}{4}$ is what part of $\frac{2}{3}$? $2\frac{5}{8}$; 2.625; 262.5%
2. $7\frac{1}{2}$ is what part of 47? $\frac{15}{94}$; .160; 16.0%
3. $2\frac{1}{3}$ is what part of $24\frac{3}{8}$? $\frac{56}{585}$; .096; 9.6%
4. $1\frac{5}{8}$ is what part of $3\frac{5}{16}$? $\frac{26}{53}$; .491; 49.1%
5. $\frac{3}{4}$ is what part of $\frac{5}{16}$? $2\frac{2}{5}$; 2.4; 240%
6. $3\frac{4}{5}$ is what part of $\frac{5}{8}$? $6\frac{2}{25}$; 6.08; 608%
7. A school year consists of 10 months. $3\frac{2}{5}$ months have passed. What fraction of the school year has passed? How many per cent of the year have passed? $\frac{17}{50}$; 34%
8. The one-cent piece weighs 48 grains, $2\frac{2}{5}$ grains of which is tin and zinc and the rest is copper. What fraction of the one-cent piece is pure copper and what fraction is tin and zinc? Also reduce these fractions to per cents. $\frac{19}{20} = 95\%$ copper; $\frac{1}{20} = 5\%$ tin and zinc
9. Brass is made of copper and zinc. 50 pounds of a certain variety of brass contains $32\frac{1}{2}$ pounds of copper and $17\frac{1}{2}$ pounds of zinc. What fraction of this brass is copper and what fraction is zinc? Also reduce these fractions to per cents. $\frac{13}{20} = 65\%$ copper; $\frac{7}{20} = 35\%$ zinc
10. Try to make other problems like these.

ORAL EXERCISES

1. What is meant by *per cent, base, rate, percentage?*

2. If the base and rate are given, how do you find the percentage? The answer may be written:

$$\textit{rate} \times \textit{base} = \textit{percentage}. \qquad \text{(P)}$$

3. State how you find the rate when the base and percentage are given. This problem is written:

$$? \times \textit{base} = \textit{percentage}.$$

4. State how you find the base when the rate and percentage are given. This problem is written:

$$\textit{rate} \times \; ? \; = \textit{percentage}.$$

5. What is meant by principal, rate, interest?

6. If the principal and rate are given how do you find the interest if the time is one year? The answer may be written:

$$\textit{rate} \times \textit{principal} = \textit{interest}. \qquad \text{(I)}$$

The rate and principal may then be found the same as the rate and base were found above.

7. State in words the problems that may be solved by using equation (P).

8. State in words the problems that may be solved by using equation (I).

WRITTEN EXERCISES

Find the missing numbers in the following:

	Base	Rate	Percentage		Base	Rate	Percentage
1.	450	5%	? 22.5	5.	240	? 6%	14.4
2.	3900	$6\frac{1}{4}$%	? 243.75	6.	52,600	? $6\frac{1}{2}$%	3419
3.	1250	? 4%	50	7.	7900	? $4\frac{1}{4}$%	335.75
4.	6400	? $5\frac{1}{2}$%	352	8.	94,200	$4\frac{3}{4}$%	4474.5 ?

ORAL WORK

1. If the principal, rate and time are given, how do you find the interest? The answer may be written:

$$principal \times rate \times time = interest.$$

2. If the interest, principal and rate are given, how do you find the time? (Compare with finding the missing number in $2 \times 3 \times ? = 24$. Also compare with problems on volume on pages 88, 89.)

3. If the interest, principal and time are given, how do you find the rate? (Again compare with $2 \times ? \times 4 = 24$ and with problems on volume, pages 88, 89.)

4. If the interest, rate and time are given, how do you find the principal?

5. If you know the list price and the rate of discount, how do you find the selling price?

6. If you know the list price and the selling price, how do you find the rate of discount?

7. If you know the commission and also the rate, how do you find the selling price?

8. If you know the buying price and the selling price, how do you find the rate of gain?

9. If you know the selling price and the rate of gain, how do you find the buying price?

10. If in a problem on commission you know the amount involved in the transaction and the rate, how do you find the commission?

11. State the four problems which can be solved by means of the equation:

$$length \times width \times depth = volume.$$

119. Equations in Arithmetic. It is important to see how few are the different ideas which really enter into the solution of problems. To assist you in this, we collect here the equations which we have used.

(S) *speed* × *time* = *distance*.

(C) *number of articles* × *price* = *cost*.

(A) *length* × *width* = *area*.

(V) *length* × *width* × *depth* = *volume*.

(P) *rate* × *base* = *percentage*.

(I) *principal* × *rate* × *time* = *interest*.

The letters S, C, A, V, P, and I indicate speed, cost, area, volume, percentage, and interest.

The ideas represented by these equations are absolutely fundamental in arithmetic and in practical life, and you must make sure that you master them perfectly. It is not enough that you remember them. You must see clearly why they are what they are. By means of these equations and others like them, which you can make as you need them, you can solve a large variety of problems. In general, a problem will give all but one of the numbers in such an equation, and your task will be to find that one. If you have trouble in finding the missing number just study the similar problem of supplying missing numbers in $3 \times ? = 12$, and $2 \times ? \times 4 = 24$.

ORAL WORK

1. Ask and answer the questions which can be answered from equation (C). (See page 207.)

2. Ask similar questions for equations (A) and (V).

3. Ask and answer similar questions for equations (P) and (I).

4. Ask and answer questions which can be answered from equation (S). (See page 28.)

WRITTEN WORK

1. A piece of land 100 rods wide and 150 rods long is sold for \$8400. What is the price per acre? **\$89.60**

2. If a carpet costs \$1.80 per square yard, allowing nothing for waste, what will be the cost of carpeting a room 20 feet wide and 24 feet long? **\$96**

3. If ice weighs 57 pounds to the cubic foot, how many tons of ice are there in a car 38 feet long, $8\frac{1}{2}$ feet wide, if it is piled 5 feet deep? **46.03 tons**

4. A farm in Illinois is bought for \$215 an acre. If taxes amount to \$1.40 an acre, and if interest on money is 6%, how much per acre do the interest on the purchase price and the tax amount to? **\$14.30**

5. In New York city a lot 40 feet wide and 86 feet deep is sold for \$1.15 per square foot. If the purchase price is borrowed at $5\frac{1}{2}$% interest what is the yearly interest on the purchase price? **\$217.58**

6. Goods sold at a reduction of 15% from the marking price are sold for how many per cent of the marking price? 85%

7. A machine listed at \$90 was sold for \$67.50. What was the rate per cent of discount? 25%

8. A house costing \$6500 to build is rented at \$50 a month. If repairs and taxes amount to \$175 yearly what is the net income from the house? What per cent of the cost of the house is this net income? **\$425.00; 6.5%**

9. At $6\frac{1}{2}$% find the interest on \$3400.00 from May 1st to November 15. **\$119.09**

10. On April 19, 1919, Captain E. F. White made the first non-stop trip between Chicago and New York in an aeroplane, flying 727 miles in 6 hours and 50 minutes. Find his average speed in miles per hour. **106.4 mi.**

WRITTEN WORK

In many of the following problems the complete solution should first be indicated. Then cancel as far as possible, and finally obtain the required answer. Discuss the advantages of first indicating the solution. Are these advantages equally great for all problems?

1. If it costs \$2.40 to pave one square yard, how much does it cost to pave a street 40 feet wide and 1280 feet long? **\$13653.33**

2. If one bushel of grain is counted as $1\frac{1}{4}$ cubic feet, how many bushels are there in a bin 8 feet long, 6 feet wide and 5 feet deep? **192 bu.**

3. A man buys 140 sheep at \$9.50 apiece and sells them at a gain of 15%. How much does he gain? **\$199.50**

4. If 36 tons of hay are sufficient to winter 19 cows, how many tons will be required to winter 53 cows? **100.4 tons**

5. At 32 pounds of oats to the bushel, how many bushels are there in a load weighing 2380 pounds? **74.4 bu.**

6. If oats are selling for 65 cents a bushel, what is the value of a load weighing 2640 pounds. **\$53.63**

7. At \$8.25 a ton, what is the value of a load of coal weighing 6700 pounds? **\$27.64**

8. 272 is how many per cent of 531? Find the result to the nearest tenth of one per cent. **51.2%**

9. What is the rate of per cent of income on an investment of \$648,000 if the income is \$54,000 yearly? **8.3%**

10. An article listed at \$40 is sold for \$25. What is the rate of discount? **$37\frac{1}{2}$%**

11. In the first non-stop aeroplane flight across the Atlantic Alcock and Brown flew 1900 miles in 16 hours, 20 minutes. What was their speed in miles per hour? **116.3**

WRITTEN WORK

1. A furniture dealer bought a table for $20, and marked it to sell at a gain of 40%. What was the marking price? **$28**

2. If the dealer of the preceding problem was obliged to sell the table at a reduction of 50% from the marking price, what was his selling price? Did he gain or lose? **$14; lost**

3. If one loaf of bread requires 12 ounces of flour, how many loaves can be made from a barrel containing 196 pounds? How much does the flour in each loaf cost if the barrel costs $12.15? **261⅓ loaves; 4.65c**

4. At 60¢ a square yard, how much will it cost to sod a lawn 40 feet by 55 feet? **$146.67**

5. $\frac{1}{2}$ is how many per cent of $\frac{3}{4}$? **$66\frac{2}{3}$%**

6. Find 50% of $1\frac{1}{3}$. **$\frac{2}{3}$**

7. Find $12\frac{1}{2}$% of $34\frac{2}{3}$. **$4\frac{1}{3}$**

8. What is the interest on $260 at 6% for 1 year, 5 months and 12 days? **$22.62**

9. What is the interest on $3650 at $5\frac{1}{2}$% for 2 years, 7 months and 21 days. **$530.32**

10. A potato field 18 rods by 40 rods yields 670 bushels. What is the yield per acre? **148.9 bu.**

11. If the tax rate on personal property is $\frac{7}{8}$ of 1 per cent of its value, how much tax does a man pay whose personal property is valued at $7400? **$64.75**

12. If the tax rate on farm land is .92% of the value, what must a man pay on a farm containing 170 acres, valued at $95 an acre? **$148.58**

13. A very high wind blowing 45 miles an hour exerts a pressure of 8.1 pounds per square foot. How many tons pressure will it exert on a wall 54 feet by 22 feet? **4.81 tons**

120. Units of Measure. The unit of measure used depends upon the kind of thing to be measured.

Thus, in answer to the questions in exercise 1 below, you say you weigh so many *pounds*, that you are so many *inches* tall, that you are so many *years* old, and that you live so many *blocks* or *rods* or *miles* from the school.

That is, in measuring any quantity whatever, use is made of a certain quantity as a *unit*, and we then find out how many times that unit is contained in the quantity to be measured.

ORAL EXERCISES

1. What is your weight? your height? How old are you? How far from school do you live?

2. Name some units used in measuring time, weight, length, area, volume.

121. Importance of Measurement. The art of measuring is one of the most important that man has learned, and you will find it very interesting to observe how much use is made of it.

Have you ever seen a stone building in the process of construction? How are the stones made to fit into their places? Are they cut near the building? If not, how does it happen that they are cut just the right size?

In digging tunnels, such as those under the Hudson River in New York, it is customary to begin from both sides and dig toward the middle. The engineers, by making careful measurements and calculations, are able to direct the work so that the tunnels meet exactly. In such cases as the building of these tunnels the measurements made are very exact, and very fine instruments are used.

Without careful measuring it would be impossible to construct large buildings, railways, steamships, or any sort of complicated machinery. Indeed, if the art of measuring were lost we should be reduced practically to the level of the savage, so far as our material comforts are concerned.

122. Reasons for Studying Measurements. It is necessary that we should study measurements both because of their practical usefulness to each of us and also because they are of such fundamental importance in our civilization that every intelligent and well-informed person should have some understanding of them.

123. Denominate Numbers. A number expressed in terms of a standard unit of measure, such as 5 gallons, 3 feet, etc., is called a *denominate number*.

A number expressed in terms of more than one unit of measure, such as 3 gallons and 2 quarts, or 2 feet 6 inches, is called a *compound denominate number*.

A number expressed in terms of one unit is a *simple denominate number*.

It is frequently necessary to reduce a compound denominate number to a simple denominate number. Thus, we may need to reduce yards and feet to feet; pounds and ounces to ounces. We may also need to reduce inches to a compound number, such as feet and inches.

ORAL EXERCISES

1. Give a number of examples of the use of compound denominate numbers. Can you state your height as a compound denominate number? your age?

2. Express 2 minutes, 30 seconds in terms of seconds. Is the result a simple or compound denominate number?

3. Express 3 feet 8 inches as a simple denominate number.

4. Express 36 hours as a compound denominate number.

5. Express 13 feet as a compound denominate number.

Drill in Fundamentals. Play game No. 3, page 30. Multiply integers for the first event, divide integers for the second event, solve examples in interest for the third event, solve examples in finding the rate in percentage for the fourth event.

ORAL AND WRITTEN WORK

1. What things do you know that are sold by liquid measure? by dry measure?

2. Give the table of liquid measure; of dry measure.

3. How many cubic inches are there in one gallon? in one quart? (See page 288.) **231 cu. in.; $57\frac{3}{4}$ cu. in.**

4. How many cubic inches are there in one bushel? in one peck? in one dry quart? **2150.42; 537.60; 67.20**

5. One cubic foot of water weighs 62.5 pounds. From this find the weight of one gallon of water. Give result to the nearest hundredth of a pound. **8.36 lb.**

Suggestion: One cubic inch of water weighs $\frac{62.5}{1728}$ pounds.

$$\text{Hence, one gallon weighs } \frac{62.5}{\underset{576}{\cancel{1728}}} \times \overset{77}{\cancel{231}} \text{ pounds} = ?$$

Milk delivered at a creamery is weighed instead of measured with a liquid measure. Why is this? One gallon of milk weighs 8.62 pounds.

6. During one week a farmer delivered milk as follows to a creamery:

Sunday,	384.3 pounds	Thursday	412.4 pounds
Monday,	406.6 pounds	Friday	405.8 pounds
Tuesday,	414.2 pounds	Saturday	416.3 pounds
Wednesday	397.5 pounds		

How many gallons did he deliver during the entire week? At $18\frac{2}{3}$¢ a gallon, how much was this milk worth?

329.1 gal.; $61.43

7. A milk can filled with milk weighs 90.4 pounds. When empty it weighs 20.3 pounds. How many gallons of milk did it contain? Give the result to the nearest tenth of a gallon. **8.1 gal.**

ORAL AND WRITTEN WORK

1. A bushel of potatoes should weigh 60 pounds. What should be the weight of a peck of potatoes? **15 lb.**

2. If a driving horse is fed 12 quarts of oats a day, how many bushels of oats will he consume in one year? At 45 cents a bushel how much will these oats cost? **136.9 bu.; $61.61**

3. If a farmer sows $3\frac{1}{2}$ quarts of timothy seed to the acre, how many bushels will he require to seed a meadow containing 52 acres? **5.7 bu.**

4. A grocer buys green peas by the bushel at $1.30 a bushel, and sells them at 6¢ a quart. How much does he gain by selling 4 bushels of green peas? **$2.48**

5. By how many cubic inches does .804 of a bushel differ from one cubic foot? (For the number of cubic inches in one bushel see page 288.) **.93768 cu. in.**

In estimating the contents of boxes and bins, it is customary to regard one cubic foot as containing .8 of a bushel.

6. How many bushels are there in a bin 14 feet long and 10 feet wide, if the grain in it is $5\frac{3}{4}$ feet deep? **644 bu.**

7. A bin is 12 feet long and 8 feet wide. How deep must the grain be in it to make 400 bushels? Get result to the nearest hundredth of a foot. Also, find the result in feet and inches to the nearest tenth of an inch. **5.21 ft.; 5 ft. 2.5 in.**

8. Sometimes coal is sold by the bushel. The bushel used, however, is different from the ordinary bushel. One bushel of bituminous (soft coal) weighs 76 pounds. How many bushels of coal are there in a ton? At 25¢ a bushel, how much does one ton cost? **26.3 bu.; $6.58**

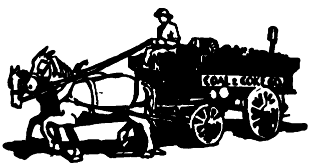
9. Measure the inside dimensions of a chalk box. Will this box hold more or less than one quart? How much? (See problem 3 on the opposite page.)

WRITTEN WORK

Hay is conveniently weighed by weighing the wagon and load together and then weighing the wagon separately.

1. A farmer hauled six loads of hay which, with the wagon weighed 3480 lbs., 3520 lbs., 3360 lbs., 3630 lbs., 3580 lbs. and 3430 lbs. respectively. The wagon, including the hayrack and the man, weighed 1480 lbs. How many tons of hay did the farmer haul? Give result to nearest hundredth. **6.06 tons**

2. At $12.50 a ton what was the value of the hay in problem 1? **$75.75**



Coal at the mine is usually weighed by the long ton. A long ton is 2240 pounds. It is sold by the ordinary short ton, which contains 2000 pounds.

3. A car at the mine is loaded with 46 long tons of coal. How many tons (short) can a dealer sell from this car if he deducts 3% from the weight at the mine for loss in transportation? **49.97 tons**

4. At $7.80 per ton (short tons) what is the value of ten loads of coal which weigh 3880 lbs., 4060 lbs., 3850 lbs., 3820 lbs., 3960 lbs., 3930 lbs., 4020 lbs., 4040 lbs., 3970 lbs., and 3890 lbs.? **$153.74**

5. If one ton of coal occupies 35 cubic feet of space, how many tons are there in a car $8\frac{1}{2}$ feet wide and 42 feet long, if the coal is $4\frac{1}{2}$ feet deep? **45.9 tons**

Flour, live cattle, hogs, etc., are often bought and sold by the hundredweight. If you look up the market reports, in the daily paper, you will see that hogs and cattle are quoted at so much per hundred pounds. This means live weight.

Cattle

Beef steers, good to prime	$8.25a	$9.50
Beef steers, medium to good	7.40a	8.25
Beef steers, common to medium	6.65a	7.40
Yearlings	7.50a	9.35
Beef cows	5.00a	7.25
Fat heifers, good to selected	6.00a	8.00
Stock steers	4.70a	6.85
Feeders	5.75a	7.40
Canners and cutters	3.75a	4.70
Good to prime veals	10.25a	11.50

Hogs

Bulk of sales	$8.40a	$8.55
Common to good mixed	8.35a	8.50
Fair to choice medium weights	8.50a	8.55
Light weights	8.40a	8.55
Fair to choice butchers	8.45a	8.60
Select 200a 300 packers	8.45a	8.50

The above is a clipping from the market report of a Chicago paper. Look up market reports in the papers to see how present prices compare with these.

WRITTEN WORK

If a steer weighs 1000 pounds before killing, and if the dressed meat weighs 580 pounds, the steer is said to dress off 420 pounds or 42% in butchering.

1. What will be the average cost per pound of a dressed steer, which dresses off 38% in butchering, if the live animal is sold at $8.50 per hundredweight? 13.7c

2. Poorer grades of steers dress off 45% in butchering. What will be the average cost per pound of the meat from such a steer if the live animal is sold at $7.25 per hundredweight? 13.2c

3. Fine hogs dress off about 25% in butchering. What will be the average cost per pound of a dressed hog, if the live animal sells for $8.45 per hundredweight? 11.3c

4. Using a newspaper report of prices of live cattle and hogs, make other problems and solve them. Remember that hogs dress off about 25%, while cattle dress off from 35% to 50%.

ORAL WORK

1. In measuring the width of this page, what unit of measure do you use? What unit do you use in measuring the length of your schoolroom?

2. What unit of measure is used in giving the distance between two cities like Chicago and New York?

3. Name lengths which are measured by the inch, by the foot, by the rod, by the mile.

4. Give the table of linear measure. (See page 288.)

5. How many rods are there in one mile? How many feet are there in one rod?

WRITTEN WORK

1. How many yards are there in one rod? in one mile? If you step $2\frac{1}{2}$ feet, how many steps must you take to walk one mile? **$5\frac{1}{2}$; 1760; 2112**

2. If a military step is $2\frac{1}{2}$ feet, how many steps must a soldier take on a day's march of 22 miles? **46464.**

3. If fence posts are set 12 feet apart, how many posts are used in one mile of fence, including one post at each end of the fence? **441.**

4. If the cost of the posts, including the setting, is 23 cents apiece, what is the cost for the posts for this fence? **$101.43**

5. The distance between railway ties from center to center is 2 feet. How many ties are used to the mile? At 80 cents apiece what is the cost of ties for one mile? **2640; $2112**

6. A heavy railway rail weighs 105 pounds to the yard. How many tons of such rails are required for a mile of track? At $28.50 a ton what is the cost of these rails? **184.8 tons; $5266.80**

7. Find the average length of your step by walking a distance of known length and counting the steps.

ORAL AND WRITTEN WORK

Answer as many as you can of the following questions without using pencil and paper:

1. How many square feet are there in one square yard? 9

2. How many square inches are there in one square foot? 144

3. How many square yards are there in one square rod? If you do not know this, how can you find out? 30¼

4. How many square rods are there in one square mile? 102,400

5. How many square rods are there in one acre? 160

6. How many acres are there in one square mile? 640

A township is a square piece of land six miles on each side.

7. How many square miles are there in a township? 36

A section of land is one mile square. A quarter-section is one-fourth of a section.

8. How many rods of fence are required to fence in a section of land? a quarter-section? 1280; 640

If this question causes trouble, draw a figure to represent a section and a quarter-section of land.

9. Give the table of square measure. (See page 288.)

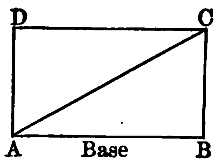
10. A field is 80 rods long and 60 rods wide; how many acres are there in it? 30

11. The total number of miles of railway in the United States in 1912 was 244,180 miles. The average width of the *right-of-way* was $4\frac{1}{4}$ rods. How many square miles of land was occupied by these railways? $3243\frac{1}{64}$ sq. mi.

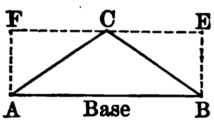
12. A contract is let to cover a lawn with sod for 65 cents a square yard. What is the cost if the lawn is $34\frac{1}{2}$ feet wide and 94 feet long? $234.22

124. Area of Triangle. We know that the area of a rectangle is equal to the product of the length and width. We sometimes call one side of a rectangle the *base* and the other side the *altitude*. That is,

area of rectangle = base × altitude.



In the first figure, opposite corners of a rectangle are connected, forming two equal triangles, ABC, and ACD. Each of these triangles is equal to one-half of the rectangle.



In the second figure, a triangle ABC is given, and the lines BE, EF, and FA are drawn so as to form a rectangle. It is clear that the triangle ABC is one-half of the rectangle ABEF.

One side of the triangle is called its base, and the line drawn from the opposite corner to the base and making a right angle with it is called the altitude. Then we have:

area of triangle = one-half base × altitude.

Notice that in the upper figure BC is the altitude of the triangle and also of the rectangle. In the lower figure a line drawn from C to the base AB and making right angles with it is the same length as BE, which is the altitude of the rectangle.

The rule given here for finding the area of any triangle is important, and you should make sure you understand it.

ORAL EXERCISES

Find the areas of triangles whose bases and altitudes are:

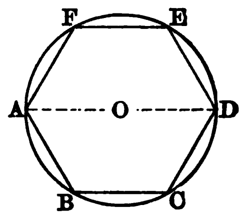
	Base	Altitude		Base	Altitude		Base	Altitude
1.	12	6	4.	16	10	7.	15	8
2.	8	10	5.	20	10	8.	18	7
3.	10	6	6.	40	12	9.	13	6

125. Circle, Diameter, Radius. A curve all of whose points are the same distance from one point, called the *center*, is a *circle*.

The word circle is also used to mean the area inside the curve. There is nothing strange in this, however. There are plenty of words with more than one meaning.

The distance from the center to the curve is the *radius* of the circle. The distance straight across the circle through the center is the *diameter*.

In the figure, OD is a radius and AD is a diameter. The figure ABCDEF placed inside the circle has all its sides equal to the radius. Hence the distance around this figure is 6 times the radius or 3 times the diameter.



The distance around the circle is called the *circumference*.

ORAL WORK

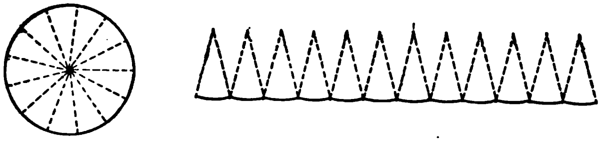
1. Is the circumference of a circle more or less than 3 times its diameter?

2. Measure the diameter of a circular object such as a tin can and also its circumference. Divide the circumference by the diameter to find the number by which the diameter must be multiplied to get the circumference. Make a number of such measurements, using different circular objects. Then, take the average of the quotients.

> *If the diameter of the circle is multiplied by* $3\frac{1}{7}$ *the product is very nearly its circumference.*

3. If the diameter of an automobile wheel is 34 inches, what is the length of the outside of the tire? (Multiply $3\frac{1}{7}$ by 34.) How far will this wheel run in making one revolution? (When a wheel turns around once it is said to make a revolution.)

4. If you have an automobile in your family find how many revolutions its wheels make in going one mile.



126. Area of Circle. From these figures we see that the interior of a circle may be cut into small pieces, each shaped nearly like a triangle. If they are cut narrow enough, we may regard them as triangles. The altitude of each triangle will then be the same as the radius of the circle, and the bases will be parts of the curve itself. The sum of the bases is the same as the circumference of the circle. So the area of all these triangles will be the sum of these bases, or the circumference of the circle, multiplied by half its radius. That is,

area of circle = circumference × half the radius.

ORAL AND WRITTEN WORK

1. If the diameter of a circle is given, how do you find the circumference? Then how do you find the area?

2. Find the circumference and the area of a circular flower bed whose diameter is 6 feet. **$18\frac{6}{7}$ ft.; $28\frac{2}{7}$ sq. ft.**

3. The distance across a circular pond is 36 feet. What is the distance around it? What is its area? **$113\frac{1}{7}$ ft.; $1018\frac{2}{7}$ sq. ft.**

4. The diameter of the earth is about 8000 miles. Find the distance around it to the nearest thousand miles. **25,000 mi.**

5. The diameter of the moon is about 2160 miles. How many miles are there around the moon? **$6788\frac{4}{7}$ mi.**

6. The diameter of the sun is about 866,300 miles. How many miles are there around the sun? **$2,722,657\frac{1}{7}$ mi.**

7. Find the area of a circular disk one foot in diameter. How much less than a square foot is this? **$113\frac{1}{7}$ sq. in.; $30\frac{6}{7}$ sq. in.**

8. Draw a circle on the board with radius 8 inches. Find its area. **$201\frac{1}{7}$ sq. in.**

WRITTEN WORK

1. A large gas tank is 180 feet in diameter. How many feet are there around it? **$565\frac{5}{7}$ ft.**

2. Find the number of square feet of ground covered by the gas tank of the preceding problem. **$25,457\frac{1}{7}$ sq. ft.**

3. The diameter of the wheels of a certain automobile is 35 inches. How many revolutions do the wheels of this machine make in going from New York to Boston, a distance of 250 miles?

   Suggestion: The distance from New York to Boston in inches is $250 \times 5280 \times 12$, and the circumference of the wheel is $35 \times 3\frac{1}{7}$ or $35 \times \frac{22}{7}$. Hence, the required number of revolutions is:

$$\frac{250 \times 5280 \times 12}{35 \times \frac{22}{7}} = \frac{250 \times 5280 \times 12 \times 7}{35 \times 22}.$$

144,000 revs.

NOTE: While for practical purposes the distance from New York to Boston is never given in inches, this problem is very much simplified by so expressing it, since the circumference of wheel with which the distance is to be compared is most naturally expressed in inches.

4. The diameter of a standard wheel on a railway passenger coach is 36 inches. How many times will this wheel turn around in going from Chicago to Boston a distance of 1120 miles? **627,200 revs.**

5. During one season an automobile was driven 8480 miles. How many revolutions did its wheels make if the tires are 34 inches in diameter? **$5,028,141\frac{3}{17}$ revs.**

ORAL AND WRITTEN WORK

1. What is meant by a cubic foot? a cubic yard?

2. How many cubic inches are there in one cubic foot? How many cubic feet are there in one cubic yard? **1728 cu. in.; 27 cu. ft.**

3. The interior measurements of a box are: Length, 4 feet 5 inches; width, 3 feet 2 inches; depth, 2 feet 3 inches. Find its volume in terms of cubic feet.

Solution: 4 feet 5 inches $=4\frac{5}{12}$ feet $=\frac{53}{12}$ feet $=$ length.
3 feet 2 inches $=3\frac{1}{6}$ feet $=\frac{19}{6}$ feet $=$ width.
2 feet 3 inches $=2\frac{1}{4}$ feet $=\frac{9}{4}$ feet $=$ depth.

Hence the volume in cubic feet $=$

$$\frac{53}{12}\times\frac{19}{6}\times\frac{9}{4}=\frac{1007}{32}=31.47,$$

which is the result within one-hundredth of a cubic foot.

4. What measurements must be made to find the volume of a rectangular solid? When these measurements have been made, how do you find the volume?

The answer to this question may be stated:

length $\times$ *width* $\times$ *depth* $=$ *volume.*

5. If the dimensions of the solid are measured in inches in what terms will the volume be given?

6. If the dimensions of a solid are measured in feet in what terms will the volume be given? In what terms will the volume be given if the dimensions are measured in yards?

7. If the dimensions of the solid are given in feet, how do you find the volume in terms of cubic yards?

8. How many cubic yards of dirt must be taken out in making an excavation 40 feet long, 25 feet wide and 5 feet deep? Get the result to nearest integer. **185 cu. yd.**

WRITTEN WORK

The exact volume of a bushel is 2150.42 cubic inches.

1. If you know the number of cubic inches in a box, how can you find the *exact* number of bushels it will hold?

2. If you know the number of cubic feet in a box, how do you find the number of cubic inches in it?

Hence, we see that to find the exact number of bushels a box will hold, we must multiply the number of cubic feet by 1728, and divide the result by 2150.42. This is the same as multiplying by $\frac{1728}{2150.42} = .8036$.

For practical purposes it is sufficiently accurate to multiply the number of cubic feet by .8 to obtain the number of bushels. (See page 223.)

3. Find the number of bushels in a bin containing 1000 cubic feet by multiplying by .8 and then by .8036. By how many bushels do the results differ? **800 bu.; 803.6 bu.; 3.6 bu.**

A grain elevator is a great building used for the storage of grain. Grain elevators are very common in the central and western parts of the United States.

4. A bin in a grain elevator is 16 feet long and 14 feet wide. How many bushels of wheat are there in it if the wheat is 48 feet deep? (Use .8036 as multiplier.) **8640.3 bu.**

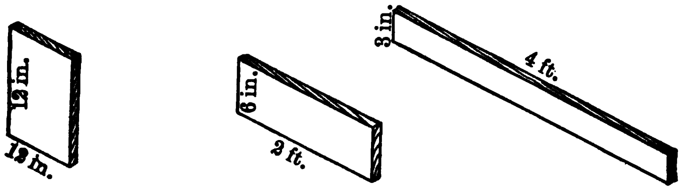
5. One cubic foot of clear water weighs 1000 ounces. How many pounds is this? **$62\frac{1}{2}$ lb.**

6. A tank used for hauling water to a threshing machine is 12 feet long, $3\frac{1}{2}$ feet wide, and 18 inches deep. How many pounds of water will it hold? how many tons? **$3937\frac{1}{2}$ lb.; 1.97 tons.**

7. The concrete foundation of a tall chimney is 10 feet 8 inches square and 7 feet deep. What is its cost at 35¢ a cubic foot? **$278.76**

127. The Board Foot. The unit for measuring lumber is the *board foot*. A board foot consists of a piece of board, one foot long, one foot wide, and one inch thick, or of any other piece of lumber having the same volume. A board foot is usually called a *foot* of lumber.

A board 1 inch thick, 6 inches wide, and 2 feet long contains one board foot, as does a board 1 inch thick, 3 inches wide, and 4 feet long.



Rule. To find the number of board feet in a piece of lumber, express two dimensions in feet and one in inches, and multiply.

Lumber dealers say: "*To get board feet, multiply feet by feet by inches.*"

Thus, to get the number of board feet in a piece of studding 14 feet by 4 inches by 2 inches change the 4 inches to $\frac{1}{3}$ feet and then find the product, $14\times\frac{1}{3}\times2=9\frac{1}{3}$.

A board less than one inch thick is regarded as being one inch thick, while in pieces more than one inch thick, the actual thickness in inches and fractions of an inch is used.

In giving the dimensions of lumber, a foot is indicated by $'$ and an inch by $''$.

Thus the dimensions of a piece of lumber 2 inches thick, 8 inches wide and 16 feet long are given by $2''\times8''\times16'$.

Problem. The dimensions of a piece of lumber are: $3''\times7''\times14'$. Give two of these dimensions in feet and one in inches.

Solution: 3 inches is more readily expressed in feet than is 7 inches. But $3''=\frac{1}{4}'$. Hence, $\frac{1}{4}'\times7''\times14'$ are the required dimensions.

ORAL EXERCISES

Express two dimensions of each of the following pieces of lumber in feet and one dimension in inches:

1. $1\frac{1}{2}'' \times 12'' \times 10'$	4. $4'' \times 4'' \times 10'$	7. $1'' \times 8'' \times 16'$
2. $1'' \times 8'' \times 14'$	5. $1'' \times 4'' \times 14'$	8. $2'' \times 8'' \times 14'$
3. $2'' \times 4'' \times 8'$	6. $1'' \times 5'' \times 12'$	9. $3\frac{1}{2}'' \times 8'' \times 12'$

Problem 1. Find the number of board feet in a piece of lumber $4'' \times 10'' \times 16'$.

Solution: First: express $4'' \times 10'' \times 16'$ as $\frac{1}{3}' \times 10'' \times 16'$.
Second: multiply $\frac{1}{3} \times 10 \times 16 = 53\frac{1}{3}$ (board feet).

Problem 2. Find the number of board feet in 150 pieces, $1\frac{3}{4}'' \times 10'' \times 14'$.

Solution: The number of board feet is $\frac{7}{\underset{2}{\cancel{4}}} \times \frac{5}{\cancel{6}} \times \overset{7}{\cancel{14}} \times \overset{25}{\cancel{150}} = 3062\frac{1}{2}$.

WRITTEN EXERCISES

Find the number of board feet in the following pieces of lumber:

1. $6' \times 12'' \times 1''$, $8' \times 6'' \times 2''$, $10' \times 8'' \times 1\frac{1}{2}''$. **6; 8; 10**

2. $12' \times 14'' \times 2''$, $14'' \times 8'' \times 2''$, $15' \times 12'' \times 4''$. **28; $1\frac{5}{9}$; 60**

3. Fill out the blank spaces in the following:

Description	No. pieces	Dimension	Board feet
Rough boards............	360	$12' \times 12'' \times 1''$........	**4320**
Rough boards............	120	$16' \times 10'' \times 1''$........	**1600**
Oak flooring.............	580	$8' \times 3'' \times 1''$..........	**1160**
Oak flooring.............	340	$6' \times 3'' \times 1''$..........	**510**
Pine planks..............	84	$14' \times 10'' \times 2\frac{1}{8}''$......	**$2082\frac{1}{2}$**
Studding................	142	$10' \times 4'' \times 2''$.........	**$946\frac{2}{3}$**
Siding..................	275	$8' \times 6'' \times 1''$..........	**1100**
Sills	14	$12' \times 6'' \times 6''$.........	**504**
Rafters.................	40	$10' \times 6'' \times 2''$.........	**400**

128. Plastering and Painting. Plastering and painting are usually undertaken at a certain price per square yard. Sometimes allowance is made for doors and windows, and sometimes not. Only seldom is full allowance made for these, because working around them is slower than working over a clear surface.

1. Find the area in square yards of a wall 16 feet long and 9 feet high. **16 sq. yd.**

2. Find the area in square yards of a ceiling 18 feet long and 15 feet wide. **30 sq. yd.**

3. A room is 15 feet long, 12 feet wide, and 9 feet high. How many square yards are there in its four walls, making no allowance for openings? How many square yards are there in the ceiling? What is the total area of the walls and ceiling? **54 sq. yd.; 20 sq. yd.; 74 sq. yd.**

4. Find the area in square yards of a ceiling 16 feet long and 14 feet wide. **$24\frac{8}{9}$ sq. yd.**

5. Find the total area in square yards of the walls and ceiling of a room 20 feet long, 16 feet wide, and 9 feet high, allowing 12 square yards for doors and windows. **$95\frac{5}{9}$ sq. yd.**

6. What is the area in square yards of the walls and ceiling of a room 22 feet by 18 feet and 9 feet high? At 15 cents a square yard what does it cost to plaster this room? **124 sq. yd.; $18.60**

7. The first story of a house is 9 feet high (above the base-board). At 12 cents a square yard, how much does it cost to plaster rooms (walls and ceiling) in this house having the following dimensions: 14′×12′; 18′×15′; 14′×10′; 16′×12′; 12′×8′? No allowance is made for openings. **$42.99**

8. Make other problems like these and solve them.

129. **Estimating Wall Paper.** Wall paper is sold in single and double rolls, which are 8 yards and 16 yards long, respectively. The standard width is 18 inches.

Following are the steps in estimating the number of rolls required for papering a room:

First: Find the distance around the room, omitting the width of the doors and windows.

Second: Find how many strips ($1\frac{1}{2}$ feet wide) will cover this distance.

Third: Figure the number of rolls required to furnish this number of strips, keeping in mind that pieces will be needed to fill in above and below windows and above the doors, and that some allowance must be made for matching the patterns.

Problem. How many rolls of paper are needed for the walls of a room $18' \times 14'$, the length of the strips being 8 feet (distance from base-board to moulding), allowing $13'$ for doors and windows.

Solution: The length around the room, making allowance for doors and windows, is $18+18+14+14-13=51$ (ft.).

$51 \div 1\frac{1}{2} = 34$, which is the number of strips.

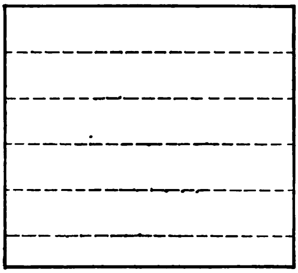
Only 5 strips can be cut from a 16-yard roll because of allowance for matching the pattern. Hence, 7 rolls will be needed.

WRITTEN EXERCISES

Find the number of rolls of paper required for each of the following rooms:

	Size of rooms	Length of strips	Allowance for openings	
1.	$18' \times 13'$	8 feet	14 feet	7 rolls
2.	$19' \times 14'$	$8\frac{1}{2}$ feet	16 feet	7 rolls
3.	$23' \times 17'$	9 feet	18 feet	9 rolls
4.	$17' \times 16'$	$8\frac{1}{2}$ feet	11 feet	8 roll

130. **Estimating Carpets.** In estimating the number of yards of carpet needed for a room the first thing to be kept in mind is that one cannot buy a fractional width of carpet. Thus, using carpet 1 yard wide, 6 strips will be needed for a room 16 feet wide, if the carpet is laid lengthwise.



Problem. How many yards of carpet one yard wide are needed for a room 20 feet long and 17 feet wide, if the carpet is laid lengthwise of the room? Allow 6 inches on the length of each strip for matching the pattern.

Solution: First, find the number of strips. Since 17 feet is more than 5 yards (15 feet) and less than 6 yards, 6 strips are needed.

Second, the length of each strip must be 20 ft. + 6 inches, or $20\frac{1}{2}$ feet. Hence, the total length of carpet is $6 \times 20\frac{1}{2} = 123$(feet) or 41 yards.

WRITTEN WORK

1. How many yards of carpet one yard wide are needed for a room 17 feet long and 14 feet wide, if the strips are laid lengthwise of the room? Allow 6 inches on each strip for matching the pattern. **$29\frac{1}{6}$ yd.**

2. How many yards of carpet, one yard wide, are needed to carpet a lodge hall 64 feet long and 48 feet wide, no allowance being made for matching pattern, and the carpet being laid lengthwise of the hall? **$341\frac{1}{3}$ yd.**

3. How many yards of carpet will be needed to carpet the hall in Example 2 if the carpet is laid crosswise of the hall? **352 yd.**

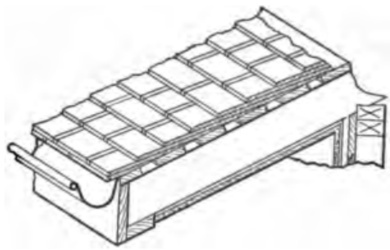
4. How many yards of carpet 27 inches ($\frac{3}{4}$ yards) wide are required to carpet a room 21 feet long and 18 feet wide, if 8 inches ($\frac{2}{3}$ feet) are allowed on each strip for matching the pattern? The carpet is to be laid lengthwise of the room. **$57\frac{7}{9}$ yd.**

131. **Estimating Lumber for a Floor.** In estimating the number of board feet of lumber needed for a floor it is customary to add $\frac{1}{6}$ of the area of the floor to allow for waste and matching the boards.

Problem. How many board feet of oak flooring are required to cover one floor of a house 54 feet by 36 feet?

Solution: The total floor area is $54 \times 36 = 1944$ square feet. Adding $\frac{1}{6}$ of the area, we have 2268, the required number of board feet.

$$\begin{array}{r} 54 \\ \underline{36} \\ 324 \\ \underline{162} \\ 6)\overline{1944} \\ \underline{324} \\ 2268 \end{array}$$



132. **Shingles for Roof.** In estimating shingles for roof, 100 square feet of roof is called *one square*. The number of shingles required to cover a square of roof varies from 800 to 950. A *bunch* of shingles contains 250 shingles.

WRITTEN WORK

1. How many board feet are needed to lay one floor in a building 45 feet long and 32 feet wide? At $85 a thousand, what is the value of this lumber? 1680: $142.80

2. The first and second floors of a building 42 feet wide and 48 feet long are to be laid with oak flooring costing $78.00 a thousand feet. What is the cost of this flooring? $366.91

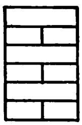
3. The two sides of the roof of a barn are $24' \times 46'$ each. How many squares of roofing are there in this roof? At 935 shingles to the square, how many shingles will be needed for this roof? 22.08 squares; 20,645 shingles

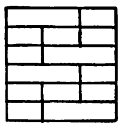
4. At $3.25 a thousand, find the cost of shingles for a roof containing $24\frac{1}{2}$ squares, if 950 shingles are used for a square. $75.64

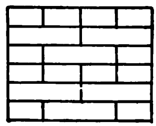
133. Number of Bricks in a Wall. Ordinary building bricks are $8\frac{1}{4}$ inches long, 4 inches wide, and 2 inches thick. The following are a builder's estimates of the number of bricks needed per square foot of wall for various thicknesses:

$8\frac{1}{4}$-inch wall, 14 bricks per square foot of wall.
$12\frac{3}{4}$-inch wall, 21 bricks per square foot of wall.
17-inch wall, 28 bricks per square foot of wall.
$21\frac{1}{2}$-inch wall, 35 bricks per square foot of wall.

The thicknesses of the walls are determined by the size of the brick. Thus, in the $8\frac{1}{4}$-inch wall one brick is laid crosswise or two


End of 1 brick wall


End of 1½ brick wall


End of 2 brick wall

side by side, with $\frac{1}{4}$ inch of mortar between. How are the bricks laid in the $12\frac{3}{4}$-inch wall? In the 17-inch? In the $21\frac{1}{2}$-inch?

WRITTEN WORK

The total length of wall around the house shown in the figure is 134 feet. Find the number of bricks needed to build this house if the walls are $12\frac{3}{4}$ inches thick and the house is 19 feet high. (No allowance is made for doors and windows, since some extra brick will be needed for chimneys and gables.) **53,466**

ORAL AND WRITTEN WORK

See how many of the following you can solve orally:

1. How many hours are there from noon to noon of the next day? 24

2. What is the number of days in a common year? in a leap-year? 365; 366

The real length of a year (solar year) is the time it takes the earth to complete its journey around the sun. This is nearly $365\frac{1}{4}$ days. The error caused by calling 365 days a year is rectified by making every fourth year a leap-year.

The years whose numbers are divisible by 4, such as 1916, 1920, 1924, are leap-years. However, years whose numbers are divisible by 400, such as 1600, 2000, are not leap-years.

Ten years is called a decade, and 100 years a century.

3. Make a table of time measure, including seconds, minutes, hours, days, and weeks, months, years, leap-years, decade, and century.

4. How many Sundays are there in one year? Add to these the number of the holidays you can think of, and then find the number of working days in a year.

5. How many days do you go to school each year? What fraction of the year is this? what per cent?

6. Give the number of days in each month of the year. Recall the device suggested on page 166 of the primary arithmetic for remembering this.

7. How many days are there from January first to July first in a common year? in a leap-year? 181; 182

Find the number of days in each of the following:

8. April 3d to August 17 136

9. May 9th to December 3 208

10. July 27th to January 4 161

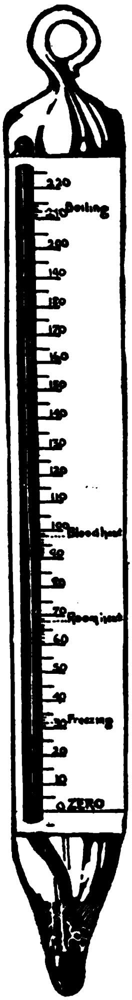
11. March 20th to October 14 208

12. February 3d to August 23 201

13. June 15th to March 7 265

ORAL EXERCISES

1. What is the name of the thermometer in common use in the United States?



2. At what temperature does water freeze? At what temperature does it boil?

3. How many degrees above freezing is the boiling-point?

4. The boiling-point of fat is 315 degrees. How much above the boiling-point of water is that?

5. What is the highest temperature you have ever seen? what is the lowest? What is the difference between these temperatures?

6. What is the highest temperature you have ever read about? what is the lowest? What is the difference between these temperatures?

On account of its higher boiling temperature, fat is used in cooking certain things which cannot be cooked in water.

The amount of heat required to raise the temperature of one pound of water one degree Fahrenheit is called a *heat unit*. Thus, 180 heat units are required to change one pound of water from the freezing-point to the boiling-point.

You are acquainted with the fact that even on a hot day ice will thaw out but slowly. To change one pound of ice into water, the water being just the same temperature as the ice, requires 144 heat units.

After a pound of water is just brought to a boil it requires 966 heat units to boil it away entirely.

Thus, to thaw one pound of ice, bring it to a boil, and boil it entirely away, requires $144+180+966=1290$ heat units.

WRITTEN WORK

1. How many heat units are required to bring a kettle containing 12 pounds of water to a boil, if it starts at a temperature of 54 degrees? 1896

2. How many heat units are required to boil away these 12 pounds of water after the water is brought to a boil? 11592

3. How many heat units are required to melt 16 pounds of ice? 2304

It is because the ice uses up so much heat in melting that it keeps the ice-box cool.

4. Which requires more heat, to melt 20 pounds of ice or to bring 15 pounds of ice-cold water to a boil? Former (2880; 2700)

5. Which requires more heat, to bring 20 pounds of ice-cold water to a boil or to boil away 4 pounds of water after it just starts boiling? Latter (3600; 3864)

6. How many units of heat are required to melt a block of ice weighing 65 pounds? How many pounds of water could be changed from 70° to the boiling-point with this amount of heat? 9360; 65.9 lb.

Because of decreased atmospheric pressure, water boils at a lower temperature as the altitude increases.

Altitude	Boiling-point	Altitude	Boiling-point	Altitude	Boiling-point
Sea level	212°	4169	204°	10685	192°
512 ft.	211°	5225	202°	11799	190°
1025	210°	6304	200°	12934	188°
1539	209°	7381	198°	14075	186°
2065	208°	8481	196°	15221	184°
3115	206°	9575	194°		

ORAL EXERCISES

1. At about what temperature does water boil at your altitude?
2. Can you think of a place where water boils at 206 degrees? at 200 degrees? at 190 degrees? at 184 degrees?

134. Examples of Practical Reductions. Compound denominate numbers may be reduced to simple denominate numbers as in the following:

Example 1. Reduce 5 yards, 2 feet, 8 inches to inches.

Solution: 5 yards = 15 feet. 15 feet + 2 feet = 17 feet = 204 inches. 204 + 8 = 212 = number of inches in 5 yards 2 feet 8 inches.

A simple denominate number may be reduced to another simple denominate number, as in the following:

Example 2. How many square feet are there in one acre?

$$\begin{array}{r} 16.5 \\ 16.5 \\ \hline 825 \\ 990 \\ 165 \\ \hline 272.25 \end{array} \qquad \begin{array}{r} 272.25 \\ 160 \\ \hline 1633500 \\ 27225 \\ \hline 43560.00 \end{array}$$

$16.5 \times 16.5 = 272.25$ = number of sq. ft. in one square rod.

$272.25 \times 160 = 43{,}560$ = number of sq. ft. in one acre.

Example 3. Reduce 788 feet to feet and yards.

$$3\overline{)\,788}\quad 262 - 2$$

Dividing 788 by 3 gives a quotient of 262 and a remainder of 2. Hence 788 feet = 262 yards 2 feet.

In some cases a simple denominate number is expressed in a higher denomination, even when that requires fractions, as in the following:

Example 4. Reduce 186470 square feet to acres.

1 acre = 43560 square feet and $186470 \div 43560 = 4.28$ = number of acres in 186470 square feet.

This problem occurs very frequently in foresters' measurement.

Example 5. Reduce 17840 pounds to tons.

$$2\overline{)17.840}\quad 8.92 = \text{number of tons.}$$

First divide by 1000 and then by 2. How do you divide 17840 by 1000?

This problem occurs frequently in selling coal, hay, and other things sold by the ton.

135. **Special Reductions.** Frequently it is necessary to reduce one kind of denominate number to another kind. Thus, we need to find the number of gallons in a given number of pounds of milk. We need to reduce cubic feet to gallons and to bushels, and so on.

8.6	pounds of milk	= 1 gallon (nearly)
8.4	pounds of water	= 1 gallon (nearly)
231	cubic inches	= 1 gallon
1	cubic foot	= .8 bushel
36	cubic feet of soft coal	= 1 ton (nearly)

WRITTEN EXERCISES

1. Reduce 43 feet 7 inches to inches. **523 in.**

2. Reduce 1340 inches to feet and inches. **111 ft. 8 in.**

3. Reduce 286,400 square feet to acres. **6.57 acres**

4. Reduce 42,650 square feet to square yards. **$4738\frac{8}{9}$ sq. yd.**

5. Reduce 6840 cubic feet to cubic yards. **$253\frac{1}{3}$ cu. yd.**

6. 3840 pounds of milk are how many gallons? **446.5 gal.**

7. How many gallons of water are there in a tank which holds 27.8 cubic feet? (First reduce to cubic inches.) **207.96 gal.**

8. How many bushels does a bin hold which contains 480 cubic feet? **384 bu.**

9. At $7.25 a ton, what is the value of 12,480 pounds of coal? **$45.24**

10. At $13.75 a ton, what is the value of 27,360 pounds of hay? **$188.10**

11. How many cubic yards are there in an excavation 42 feet long, 34 feet wide, and $5\frac{1}{2}$ feet deep? **$290\frac{8}{9}$ cu. yd.**

12. How many square yards are there in a surface $18\frac{1}{2}$ feet by $13\frac{1}{3}$ feet? **27.4 sq. yd.**

13. A tank $6\frac{1}{2}' \times 3\frac{1}{2}' \times 1\frac{1}{2}'$ will hold how many gallons? **$255\frac{3}{11}$ gal.**

Example 1. Add 8 ft. 6 in.
9 ft. 9 in.
18 ft. 3 in.

We first add the inches. The sum is 15 inches, which equals one foot and 3 inches.

Write 3 in. in inches column and carry 1 ft. The sum is 18 ft. 3 in.

Example 2. Multiply 4 gal. 2 qt. 1 pt. by 7.

4 gal. 2 qt. 1 pt.
7
32 gal. 1 qt. 1 pt.

7×1 pt. $= 7$ pts. or 3 qts. and 1 pt. We write 1 pt. in the pints column and carry 3 qt. 7×2 qt. $= 14$ qt. 14 qt. $+ 3$ qt. (carried) $= 17$ qt., which is 4 gal. and 1 qt. Write 1 qt. in the quarts column and carry the 4 gal. The product is 32 gal. 1 qt. 1 pt. Note that the carrying in multiplication is just the same as in addition.

EXERCISES

Add the following, doing as many as you can orally.

1. 2 gal. 2 qt.
1 gal. 1 qt.
3 gal. 3 qt.

2. 15 yd. 2 ft.
12 yd. 1 ft.
28 yds.

3. 12 bu. 3 pk.
7 bu. 2 pk.
20 bu. 1 pk.

4. 2 mi. 140 rd.
3 mi. 280 rd.
6 mi. 100 rd.

5. 18 gal. 3 qt.
7 gal. 1 qt.
26 gal.

6. 5 T. 980 lbs.
7 T. 270 lbs.
12 T 1250 lb.

7. 10 yd. 10 in.
24 yd. 8 in.
34 yds. 1 ft. 6 in.

8. 4 mi. 80 rd.
7 mi. 300 rd.
12 mi. 60 rd.

9. 12 cu. ft. 104 cu. in.
7 cu. ft. 93 cu. in.
19 cu. ft. 197 cu. in.

Multiply the following:

10. 3 bu. 1 pk.
6
19 bu. 2 pk.

11. 7 yd. 2 ft.
16
122 yd. 2 ft.

12 3 mi. 115 rds.
8
26 mi. 280 rd.

13. 9 T. 150 lbs.
80
726 T

14. 8 cu. ft. 120 cu. in.
72
581 cu. ft.

15. 3 qt. 1 pt.
10
35 qt.

16. 5 yd. 8 in.
16
83 yd. 1 ft. 8 in.

17. 6 gal. 3 qt.
27
182 gal. 1 qt.

18. 17 ft. 5 in.
24
418 ft.

Example 1. Find the difference between 8 gal. 2 qt. and 4 gal. 3 qt.

FIRST METHOD

8 gal. 2 qt.
4 gal. 3 qt.
3 gal. 3 qt.

Add 4 qt. (1 gal.) to both minuend and subtrahend. Then 6 qt. = 3 qt. + 3 qt. 8 gal. = 5 gal. + 3 gal.

SECOND METHOD

8 gal. 2 qt. = 7 gal. 6 qt.
4 gal. 3 qt. = 4 gal. 3 qt.
3 gal. 3 qt.

First reduce 8 gal. 2 qt. to 7 gal. 6 qt. and then subtract. Why must 8 gal. 2 qt. be reduced to 7 gal. 6 qt.?

Example 2. Divide 13 bus. 3 pk. 6 qt. by 3.

4 bu. 2 pk. $4\frac{2}{3}$ qt.
3)13 bu. 3 pk. 6 qt.

Dividing 13 by 3 gives a quotient 4 and remainder 1. Reduce the remainder (1 bu.) to the next lower denomination, which is pecks. We then have 7 pk. in all. Dividing gives a quotient 2 and remainder 1. Reduce the remainder (1 pk.) to qt. We then have 14 qt. in all. Dividing by 3 gives $4\frac{2}{3}$ qt. The result is 4 bu. 2 pk. $4\frac{2}{3}$ qt.

WRITTEN EXERCISES

Subtract the following:

1. 27 bu. 5 qt.
 8 bu. 7 qt.
 18 bu. 30 qt.

2. 160 a. 40 sq. rd.
 90 a. 64 sq. rd.
 69 a. 136 sq. rd.

3. 17 rd. 6 ft.
 8 rd. 12 ft.
 8 rds. 16 ft. 6 in.

4. 35 cu. ft. 264 cu. in.
 18 cu. ft. 396 cu. in.
 16 cu. ft. 1596 cu. in.

5. 84 cu. yd. 23 cu. ft.
 30 cu. yd. 26 cu. ft.
 53 cu. yd. 24 cu. ft.

6. 20 hr. 35 min. 23 sec.
 4 hr. 20 min. 45 sec.
 16 hr. 14 min. 38 sec.

Divide each of the following:

7. 6)8 t. 340 lb.
 1 ton $723\frac{1}{3}$ lbs.

8. 9)36 cu. yd. 24 cu. ft.
 4 cu. yd. 2 cu. ft. 1152 cu. in.

9. 7)24 bu. 2 pk.
 3 bu. 2 pks.

10. 12)65 a. 84 sq. rd.
 5 a. $73\frac{2}{3}$ sq. rd.

11. 8)14 hr. 38 min.
 1 hr. 49 min. 45 sec.

12. 6)21 mi. 145 rd.
 3 mi. 184 rd. 2 ft. 9 in.

136. Bills and Receipts. Before making out the bills on this and the next page, read again page 92.

A bill is receipted by writing "Paid" across the face of it and signing the name or initials of the person receiving the payment.

ORAL AND WRITTEN EXERCISES

1. What are the principal items that should be contained in a bill?
2. What is meant by "extending" and "footing" a bill? by "receipting" it?
3. Copy, extend and foot the following bill:

Mr. A. C. Blaine, Chicago, Ill.,
214 Lake Street, City. Feb. 8, 19—

Bought of LAKESIDE FRUIT Co.

Terms: 30 days.

140 boxes oranges	$4.00	$560			
120 doz. pineapples	.75	90			
20 bbls. greenings	3.25	65			
50 bbls. baldwins	3.80	190			
25 bbls. Kings	4.50	112	50		
75 bunches bananas	1.25	93	75		
20 crates strawberries	5.25	105			
45 bbls. sweet potatoes	3.63	163	35		
30 bbls. potatoes	3.50	105			
14 crates lemons	5.20	72	80		
15 crates grape fruit	3.00	45			
15 bbls. Northern spy	4.75	71	25		
17 boxes Floridas	3.12	53	04		
190 lbs. dates	.08	15	20		
125 bbls. wealthies	4.25	531	25		
560 baskets Concord grapes	.15	84			
350 baskets California grapes	.20	70			
80 boxes tangerines	1.50	120			
				2547	14

Paid March 3rd, '19

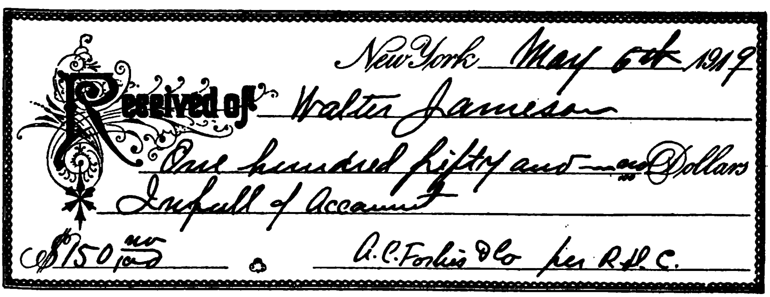
The first line in this bill indicates 140 boxes of oranges at $4 a box. The total cost of these oranges is entered in the next column. The total amount of the bill should be entered in the last column to the right. (See page 92.)

WRITTEN WORK

Arrange the following in the form of bills and find the amounts:

1. February 8, 1916, the following articles were bought of the Edwards Co. by Mrs. Frank Morse, both of Springfield, Mass.: 10 lbs. of sugar at $7\frac{1}{2}$¢ a lb., 1 sack of flour at \$2.20 a sack, $1\frac{1}{2}$ bushels of potatoes at \$1.25 a bu., 2 bags of salt at 6¢ a bag, 2 lbs. of lettuce at 16¢ a lb., 2 loaves of bread at 5c a loaf, $2\frac{1}{4}$ dozen eggs at 45¢ a dozen, 2 lbs. of Elgin butter at 40¢ a lb., $1\frac{3}{4}$ dozen oranges at 35¢ a dozen, and 2 cans of peas at 20¢ a can. **\$8.19**
2. Walter Simons, Syracuse, N. Y., bought of the Rudd Co. Buffalo, 1400 bushels of apples at 72¢ a bu., 580 boxes of oranges at \$3.14 a box, 15 bunches of bananas at \$3.98 a bunch. Date, August 9th, 1917. **\$2888.90**
3. Johnson Co., of Pittsburg, sold William Earle, Avon, Ohio. Jan. 17, 1917, 20 bushels of cloverseed at \$11.25 per bu., 18 bushels of alfalfa seed at \$11.75 per bu., 8 bushels of timothy seed at \$4.30 per bu. **\$470.90**

   Receipt the bill as having been paid, Feb. 8, 1917.
4. Allen, Frye & Co., St. Paul, sold to Johnson and Williams, of Fargo, North Dakota, 860 yards of gingham at $6\frac{1}{4}$¢ per yd., 1450 yds. of cashmere at 62¢ a yd., 580 yards of silk at \$1.15, a yd., 1840 yards toweling at $12\frac{1}{2}$¢ a yd. **\$1849.75**


New York May 6th 1917
Received of Walter Jameson
One hundred fifty and 00/100 Dollars
In full of Account
\$150 00/100   A. C. Forbes & Co per R. H. C.

In some cases a receipt of the form shown here is given for money paid. What does this receipt tell you?

137. The Cash Account. People who are in business are obliged to keep accounts, so they may know at any time the condition of their business. Different accounts are used for different purposes. In the *cash account* is put down every item of cash taken in or paid out. The cash received is placed on the left side of the account and the cash paid out on the right side of the account.

The account below shows the following receipts of cash: April 2d, \$12.40, April 3d, \$16.50; April 4th, \$25.65; April 5th, \$31.45; April 6th, \$25.25, and April 7th, \$46.45.

It also shows that at the beginning of this week there was \$246.78 on hand.

During this week cash was paid out as follows: April 2d, \$4.60; April 3d, \$10.70; April 4th, \$14.85; April 5, \$7.65; April 6th, \$10.72; April 7th, \$124.25. The total paid out was \$172.77, and at the end of the week there was \$231.71 on hand.

CASH

Dr.					Cr.
April 2	On hand	\$246.78	April 2	By cash	\$4.60
" 2	To cash	12.40	" 3	" "	10.70
" 3	" "	16.50	" 4	" "	14.85
" 4	" "	25.65	" 5	" "	7.65
" 5	" "	31.45	" 6	" "	10.72
" 6	" "	25.25	" 7	" "	124.25
" 7	" "	46.45			
		\$404.48			\$172.77
April 7	On hand	\$231.71		By balance	231.71
					\$404.48

The left side of an account is called the debit side, and is marked Dr. The right side is called the credit side, and is marked Cr. Whenever cash is received the amount is placed on the Dr. side of the cash account. We say we debit this account. Whenever cash is paid out we credit the cash account by entering the amount on the Cr. side.

138. **Account with Real Estate.** The account below is designed to show the expenditures and receipts in buying and selling a house. Copy the account and fill in the numbers as you solve the problems.

WRITTEN WORK

1. Bought a house for $9600. At 7% what is the interest on this sum for one year? **$672**

2. Taxes for one year $161.28.

3. At $1.75 a month water tax, what was this tax for one year? **$21**

4. Paid for repairs and improvements: $3.50, $46.50, $8.25, $12.65, $27.40. What is the total of these amounts? **$98.30**

5. The house was rented 11 months at $55.00 a month. What was the total rent? **$605**

6. At the end of one year, the house was sold for $11,000. What was the net gain?

House 9246 Langley Avenue.

Dr.			Cr.		
Purchase Price	$9600		Rent 11 mo.	605	
Interest	672		Selling price	11000	
Taxes	161	28			
Water Taxes	21				
Repairs	98	30			
	$10552	58		$11605	
Balance	1052	42			
			Gain	$1052	42

7. Make up problems like the above, using data which you can find in your neighborhood. Find out for how much a building or a farm was bought, for how much it was rented, and for how much it was sold.

See if you can make up this problem without help from the teacher.

139. Account with Selling Papers. The *Morning Sun*, the *Morning American*, the *News* and the *Star* are sold to the newsboys at the rate of 5 papers for 3 cents.

The *Evening Sun* is sold to the newsboys at the rate of 5 papers for 2 cents.

All these papers are sold at a cent apiece.

When the account on the next page is completed it will show just how much money James made in one week selling papers. The left side of the account shows what he paid for his papers, and the right side shows what he received for them.

During one week James sold afternoon papers as follows:

Monday		Tuesday		Wednesday	
Evening Sun	45	*Evening Sun*	40	*Evening Sun*	40
News	35	*News*	45	*News*	35
Star	30	*Star*	35	*Star*	35
Thursday		**Friday**		**Saturday**	
Evening Sun	40	*Evening Sun*	40	*Evening Sun*	50
News	35	*News*	30	*News*	45
Star	35	*Star*	35	*Star*	40

1. How much did he pay for his papers each day?

57c, 64c, 58c, 58c, 55c, 71c

Copy the account on the next page and fill in the numbers as you solve the problems.

2. How much did he get for his papers each day?

\$1.10, \$1.20, \$1.10, \$1.10, \$1.05, \$1.35

3. On Sunday morning James sold 36 Sunday *Americans* and 85 Sunday *Suns*. At 2 cents apiece what did he pay for these papers? How much did he get for them if he sold 24 papers at 5 cents apiece and the rest at 3 cents apiece? (The regular selling price for these Sunday papers was 3 cents, but early in the morning they were sold at 5 cents.) **\$2.42; \$4.11**

4. On Saturday morning James sold 55 copies of the *Morning Sun* and 40 copies of the *Morning American*. How much did these papers cost him? How much did he get for them? **\$.57; \$.95**

Extras are sold to the newsboys at the regular price, but the boys charge all they think they can get for them.

5. On Friday evening James sold 14 copies of extras at 5 cents apiece, 12 copies at 2 cents apiece, and 24 copies at 1 cent apiece. How much did the papers cost him? How much did he get for them? **\$.20; \$1.18**

The Saturday Evening Post is sold to the newsboys for 3 cents apiece. Unsold copies are returnable. The *Post* is sold everywhere at five cents a copy.

6. This week James sold 17 copies of the *Saturday Evening Post*. How much did they cost him? How much did he get for them? **\$.51; \$.85**

Dr. James' Account with his Papers. Cr.

Mon.	Paid for eve. pap.	$	57	Mon.	Rec. for eve. pap.	$1	10
Tues.	" " " "		64	Tues.	" " " "	1	20
Wed.	" " " "		58	Wed.	" " " "	1	10
Thurs.	" " " "		58	Thurs.	" " " "	1	10
Thurs.	*Sat Eve. Post*		51	Thurs.	*Sat. Eve. Post*		85
Fri.	Paid for eve. pap.		55	Fri.	Rec. for eve. pap.	1	05
Fri.	" " Extras		20	Fri.	" " Extras	1	18
Sat.	" " eve. pap.		71	Sat.	" " eve. pap.	1	35
Sat.	" " Morn. "		57	Sat	" " Morn. "		95
Sun.	" " " "	2	42	Sun.	" " " "	4	11
	Total	7	33		Total	13	99
	Balance	6	66		Gain	6	66

The prices used on these pages were taken from the city of Baltimore before the war. Make up the above account supposing that the daily papers cost the boys \$1.40 per hundred and sold for 2 cents apiece.

140. Account with a Vegetable Garden. Walter planted a vegetable garden, and sold vegetables from it. Copy the account on the next page, and fill in the items as you work the problems.

1. He bought a hoe for 80 cents, a rake for 50 cents, a hand-weeder for 15 cents, and a watering-can for 40 cents. How much did all these tools cost him? **$1.85**

2. Walter bought 5 packages of lettuce seed at 5 cents a package, 3 packages of radish seed at 5 cents a package, and 2 pounds of onion sets for 25 cents. How much did all these cost him? **$.65**

3. Walter worked 3 hours a day for 8 days, and one hour a day for 57 days. How many hours did he work altogether? At ten cents an hour, how much did this work amount to? We must enter the total pay for this work on the debit side of our account. **81 hr.; $8.10**

4. What was Walter's total expenditure on his garden, including his labor and $2.50 for a load of manure. **$13.10**

Walter sold vegetables from his garden as shown by the following table:

Crop	No. of days sale	Average sale per day	Price	
Radishes	15	2 bunches	2 bunches for 5 cents	.75
Lettuce	26	6 "	3 cents a bunch	4.68
Onions	16	4 "	2 bunches for 5 cents	1.60
Beet Greens	14	3 "	3 cents a bunch	1.26
Beets	17	6 "	2 bunches for 5 cents	2.55

How much did Walter get for each of his crops?

If his tools were worth $1.25 in the fall, how much did Walter make on his garden, besides getting paid for his work?

Dr.			Cr.		
Cost of tools	$1	85	Sold radishes for		75
" " seeds		65	" lettuce for	$4	68
" " labor	8	10	" onions	1	60
" " manure	$2	50	" beet greens	1	26
			" beets	2	55
			Value of tools	$1	25
Total	$13	10	Total	$12	09
			Loss	1	01
				$13	10

141. **Account with Belgian Hares.** Make up an account like the above, using the following data:

Tommy Weeks raised Belgian hares for sale.

From April to September he bought 12 bales of alfalfa at 90 cents a bale; 650 lbs. of bran at $1.10 per cwt. (hundredweight); 500 lbs. ground barley at $1.60 per cwt.

Tom believes that his work taking care of the hares is worth 10 cents a day. How much does this amount to for these 6 months? (Find how many days there are in these months.) Enter this amount on the debit side. $44.35

His sales were:

April,	37 lbs. of dressed meat at	38 cents	$14.06
May,	34 lbs. at	26 cents	8.84
June,	47 lbs. at	25 cents	11.75
July,	49 lbs. at	35 cents	17.15
August,	34 lbs. at	35 cents	11.90
September,	34 lbs. at	35 cents	11.90

Enter all expenses on the Dr. side, and all receipts on the Cr. side. From the account tell how much money he made these six months, besides receiving 10 cents a day for his work. $31.35

142. Classes of Mail Matter. Mail matter addressed to any post office in the United States or her possessions is divided into four classes: first, second, third, and fourth.

Ordinary letters, and any written matter whatsoever, is first class matter, and the rate is two cents an ounce. Second class matter includes newspapers and periodicals. The rate of postage is 1 cent a pound if mailed by the publishers or their agents; otherwise 4 cents a pound. Third class matter includes circulars, printed matter (not books or complete newspapers and magazines); the rate of postage is 1 cent for two ounces.

The rates for first, second, and third class matter are the same for all distances. Fourth class matter (parcel post) includes books, merchandise and other articles in very large variety. (For rates and weight of packages permitted, see page 259.)

A letter may be *registered* or a *special delivery* secured for an extra 10 cents in postage.

On October 3, 1917, a law levying special war taxes went into effect. By this law the rates of postage on nearly all classes of mail matter were increased.

It was understood, however, that these increases were purely temporary, and on July 1, 1919, the old rates were restored.

WRITTEN EXERCISES

1. Assuming that the average weight of a letter is $\frac{5}{8}$ of an ounce, how many letters weigh one ton? At 2 cents apiece, how much does the government get for transmitting one ton of such mail? **51,200; $1,024**

2. How much does the government get for carrying one ton of newspapers and periodicals in the mails? **$20**

3. How much does the government get for carrying one ton of third class matter in the mails? **$160**

4. During one month a large business house sent out 148,600 sealed letters (2c) and 56,400 open letters (1c). How much did this house pay for postage? **$3536**

143. **Money Orders.** Money may be sent to practically any post-office in the world by means of postal money orders.

The fees for money orders, payable in the United States are:

For orders from	$0.01	to	$2.50	3 cents
" " "	2.51	"	5.00	5 cents
" " "	5.01	"	10.00	8 cents
" " "	10.01	"	20.00	10 cents
" " "	20.01	"	30.00	12 cents
" " "	30.01	"	40.00	15 cents
" " "	40.01	"	50.00	18 cents
" " "	50.01	"	60.00	20 cents
" " "	60.01	"	70.00	25 cents
" " "	75.01	"	100.00	30 cents

The fees for foreign money orders payable in any country on which a money order can be drawn, other than those named above, may be ascertained upon inquiry at the post office.

The express companies sell money orders at the same rate as the post office.

ORAL EXERCISES

1. What is the cost of a money order for $20.01?

2. What is the cost of a money order for $3.50? for $6.00? for $1.75? for $9.65? for $17.45?

The total income of the Post Office Department for the years 1850, 1880, 1916 were:

	1850	1880	1916
Income	$5,499,985	$33,315,479	$312,057,688
Population	23,191,876	50,155,783	102,000,000

WRITTEN EXERCISES

1. Find the average amount expended for postage by each person in the United States for each of these years. Give each result to the nearest cent. 24c; 66c; $3.06

How do you account for the very large increase in the amount spent for postage?

144. **Fourth class matter** embraces that known as domestic parcel post mail, and includes merchandise, farm and factory products, seeds, cuttings, bulbs, roots, scions, and plants, books, (including catalogs), miscellaneous printed matter weighing more than 4 pounds, and all other mailable matter not embraced in the first, second, and third classes.

The rates are given in the table on the opposite page.

145. **Insurance.** Fourth class mail (*but no other*) may be insured against loss, on payment of the following rate: 3 cents for values up to $5; 5 cents for values up to $25; 10 cents for values up to $50, and 25 cents for values up to $100. Damages are paid only in cases of total loss or destruction of the insured articles. No damage is paid for partial destruction.

ORAL EXERCISES

1. What is the postage on a parcel weighing 2 lbs. sent a distance of 25 miles? 100 miles? 500 miles? 800 miles?

2. What is the postage on a parcel weighing 20 pounds sent a distance of 250 miles? 450 miles? 700 miles? 950 miles?

3. What is the cost of sending a package from New York to Chicago (912 miles) if it weighs 3 lbs.? 5 lbs.? 12 lbs.? 16 lbs.? 20 lbs.?

4. What is the rate from St. Paul to Boston (7th zone) of a package weighing 3 lbs.? 10 lbs.? 12 lbs.? 16 lbs.? 18 lbs.? 20 lbs.?

5. What is the rate from St. Louis to Cleveland (4th zone) on a package weighing 5 lbs.? 7 lbs.? 19 lbs.?

6. In what zone is New York from your home? St. Louis? Boston? Chicago? Denver? San Francisco? Estimate these as closely as you can. What is the postage on a 7-pound package from your home to each of these places?

7. Find out what you can about express rates, and compare them with parcel post rates.

Weight in pounds	Local	1st Up to 50 miles	2d 50 to 100 miles	3d 100 to 300 miles	4th 300 to 600 miles	5th 600 to 1000 miles	6th 1000 to 1400 miles	7th 1400 to 1800 miles	8th Over 1800 miles
1	$0.05	$0.05	$0.05	$0.06	$0.07	$0.08	$0.09	$0.11	$0.12
2	.06	.06	.06	.08	.11	.14	.17	.21	.24
3	.06	.07	.07	.10	.15	.20	.25	.31	.36
4	.07	.08	.08	.12	.19	.26	.33	.41	.48
5	.07	.09	.09	.14	.23	.32	.41	.51	.60
6	.08	.10	.10	.16	.27	.38	.49	.61	.72
7	.08	.11	.11	.18	.31	.44	.57	.71	.84
8	.09	.12	.12	.20	.35	.50	.65	.81	.96
9	.09	.13	.13	.22	.39	.56	.73	.91	1.08
10	.10	.14	.14	.24	.43	.62	.81	1.01	1.20
11	.10	.15	.15	.26	.47	.68	.89	1.11	1.32
12	.11	.16	.16	.28	.51	.74	.97	1.21	1.44
13	.11	.17	.17	.30	.55	.80	1.05	1.31	1.56
14	.12	.18	.18	.32	.59	.86	1.13	1.41	1.68
15	.12	.19	.19	.34	.63	.92	1.21	1.51	1.80
16	.13	.20	.20	.36	.67	.98	1.29	1.61	1.92
17	.13	.21	.21	.38	.71	1.04	1.37	1.71	2.04
18	.14	.22	.22	.40	.75	1.10	1.45	1.81	2.16
19	.14	.23	.23	.42	.79	1.16	1.53	1.91	2.28
20	.15	.24	.24	.44	.83	1.22	1.61	2.01	2.40
21	.15	.25	.25						
22	.16	.26	.26						
23	.16	.27	.27						
24	.17	.28	.28						
25	.17	.29	.29						
26	.18	.30	.30						
27	.18	.31	.31						
28	.19	.32	.32						
29	.19	.33	.33						
30	.20	.34	.34						
31	.20	.35	.35						
32	.21	.36	.36						
33	.21	.37	.37						
34	.22	.38	.38						
35	.22	.39	.39						
36	.23	.40	.40						
37	.23	.41	.41						
38	.24	.42	.42						
39	.24	.43	.43						
40	.25	.44	.44						
41	.25	.45	.45						
42	.26	.46	.46						
43	.26	.47	.47						
44	.27	.48	.48						
45	.27	.49	.49						
46	.28	.50	.50						
47	.28	.51	.51						
48	.29	.52	.52						
49	.29	.53	.53						
50	.30	.54	.54						

Special Rates:

(*a*) Parcels weighing 4 ounces or less, except books, seeds, plants, etc., 1 cent for each ounce any distance.

(*b*) Parcels weighing 8 ounces or less containing books, seeds, plants, etc., 1 cent for each 2 ounces any distance.

(*c*) Parcels weighing more than 8 ounces containing books, seeds, plants, etc., parcels of miscellaneous printed matter, weighing more than 4 pounds, and all other parcels weighing more than 4 ounces, are chargeable according to distance as shown on the table on this page.

146. Drawing to Scale. We are now fairly familiar with simple drawings representing simple objects to a definite scale. The scale used depends, among other things, upon the size of the object to be represented. In a drawing representing objects such as a door or a window 1 inch may represent one foot, while in a map of the United States one inch may represent 1000 miles.

ORAL EXERCISES

What scale would you use in making a drawing representing the floor of your schoolroom? the top of your desk? a farm? your state?

WRITTEN EXERCISES

1. Draw a figure representing the floor of your schoolroom. Let 1 inch of your drawing represent 4 feet of space. Put in marks to show the location of the seats and the teacher's desk. Take great care to get each object in its proper place. Thus if the teacher's desk stands 4 feet from the wall, it must be just 1 inch from the edge of your drawing. Underneath your drawing write very neatly:

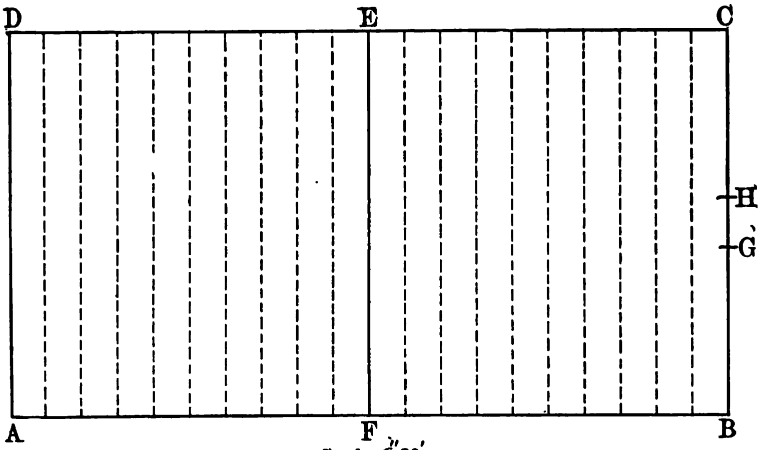
   Scale: 1 inch represents 4 feet.

2. Draw a figure representing a wall of your schoolroom. Put in all doors and windows. Make all the necessary measurements. Use the same scale as in Problem 1.

The drawings of buildings and rooms made by architects are always carefully drawn to scale.

3. Draw carefully to scale a map of your school grounds. By means of your map determine the distance between opposite corners of the yard. (See page 69.)

4. If you live in a city make a drawing representing several blocks and streets about your schoolhouse. If you live in the country make a drawing representing the farm on which you live. Be careful to select a convenient scale.



Scale 1":80'

The above is a diagram of a football field drawn to a scale in which 1 inch represents 80 feet.

WRITTEN EXERCISES

1. Measure the distances AB and BC carefully with a ruler, and then determine the dimensions of a football field. **300' by 160'**

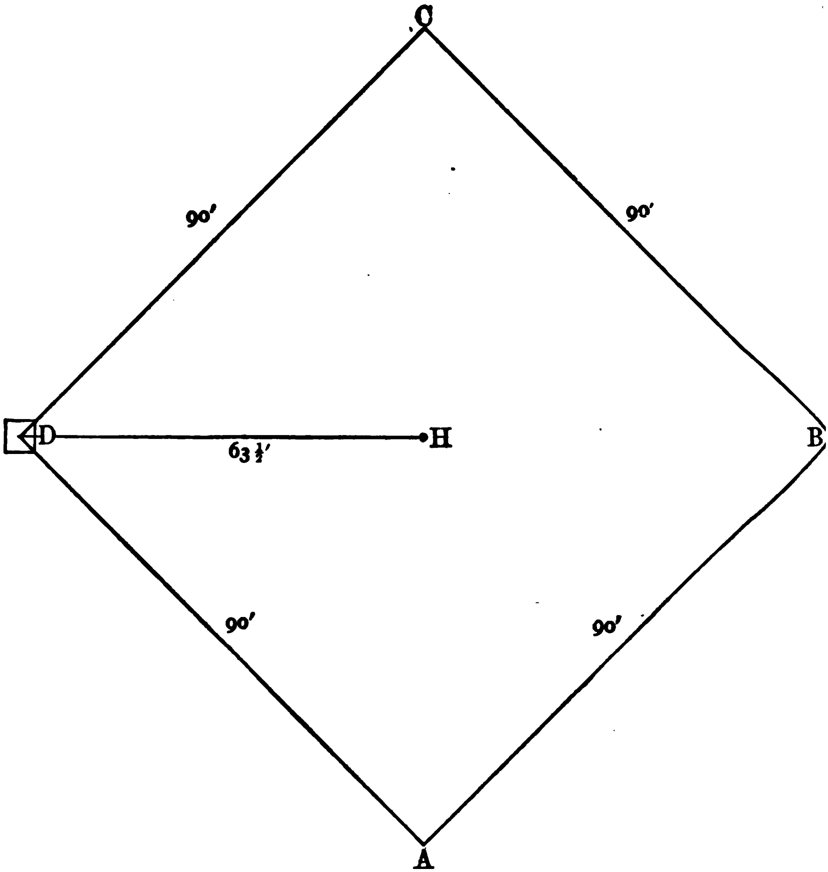
2. What is the distance from the center of the field to the goal-line? Give the distance in yards. **50 yds.**

3. The goal-posts are $18\frac{1}{2}$ feet apart. How far is it from the corner B to the nearest goal-post? **$70\frac{3}{4}$ ft.**

4. On the board draw a diagram of a football field, letting 1 inch represent 10 feet. Put in the cross-lines, the goal-posts, and the side-lines. Make it represent a real football field as completely and accurately as you can.

5. Make a drawing to a convenient scale representing a door in your schoolroom. Put in as many details as you can.

6. Make a drawing to a convenient scale representing a window in your schoolroom, showing the sizes of the window panes.



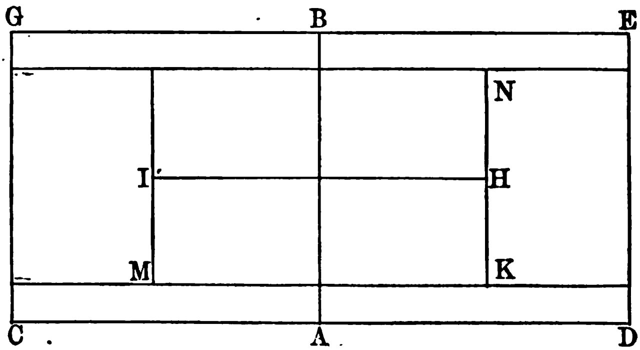
Scale 1″=30′.

The above drawing represents a baseball field. The point D marks the home plate. Name the other points that are lettered.

1. From the scale given on the drawing give the distances DA, AB, BC, CD, DB, and DH. **DB = 127′ nearly**

2. Draw this field to the scale 1″ to 15′.

This drawing represents a tennis court of standard size.



Scale 1″ = 24′.

ORAL EXERCISES

1. From the scale given on this drawing find the distances CD, DE, HN, IH.

2. If this drawing is "1″ to 6′," what will be the length of the line CD in the drawing? What will be the lengths of the lines DE, HN, IH?

WRITTEN EXERCISES

1. Draw a tennis court to the scale 1″ to 12′.

2. On the blackboard draw a tennis court to the scale 1″ to 6′. Make this drawing as nearly accurate as you can.

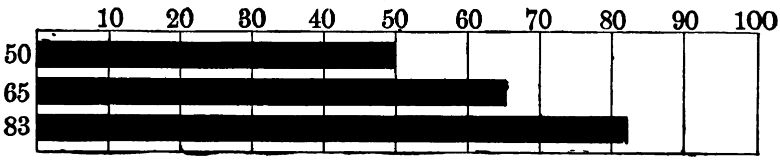
3. On the blackboard draw a baseball diamond to the scale "1 foot to 45 feet."

4. Draw a good map of your county, selecting a convenient scale, and find distances from your home to important points.

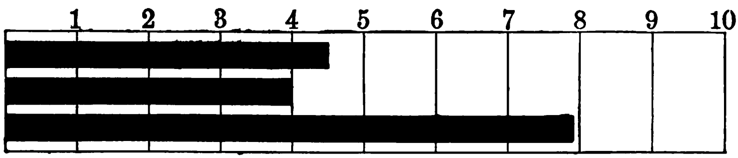
5. On a good map of your state find the straight line distances from your home to important points in the state. (First find the scale of the map.)

6. Draw to scale a good map of your state.

147. **Practical Uses of Graphic Representation.** Pictures of many kinds are used to represent numbers and sizes of things. All such pictures are called *graphs*. There are temperature charts which show the number of degrees of temperature for each hour of the day. The sizes of the armies and navies of different countries, prices of goods as they change from month to month and from year to year, populations of countries as they increase from decade to decade, and an endless number of other things are represented by graphs of one kind and another. Indeed, graphs are in such general use that popular magazines cannot be read intelligently without some understanding of them. On this and the next three pages one important kind of graph is explained.



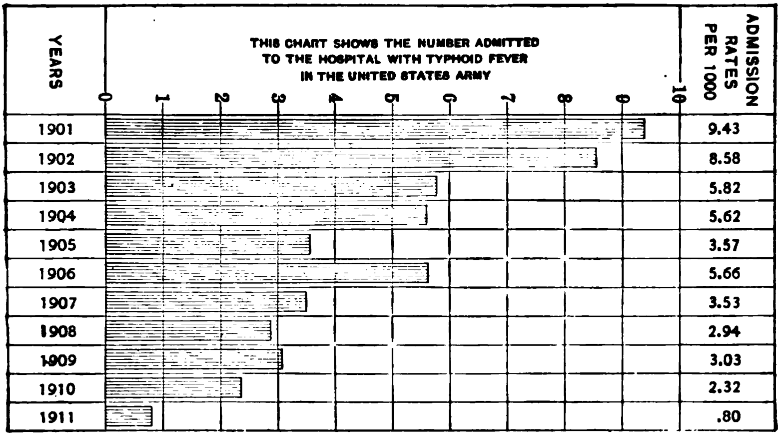
In the above figure the first black rectangle represents 50, the second 65, and the third 83.

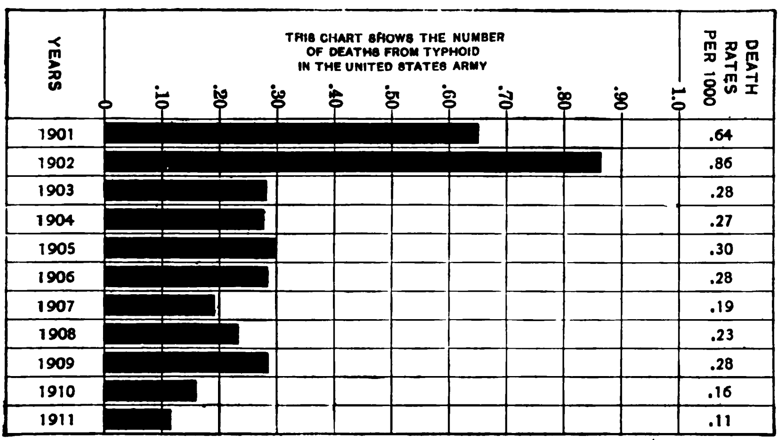


1. What number is represented by each of the black rectangles in the second figure?

2. Draw a rectangle like the first figure above, and by means of small rectangles within it represent each of the numbers 20, 80, 83, 57.

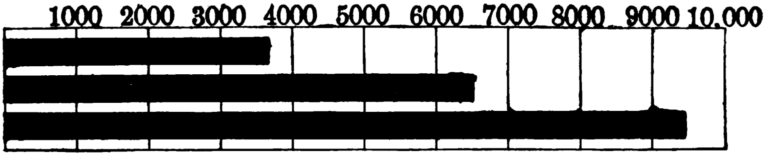
3. By means of a figure like the second one above represent the numbers 3, 5, 5, 7, 3, 9, 8.





During these years the sanitation in the army was greatly improved, and vaccination against typhoid came into general use.

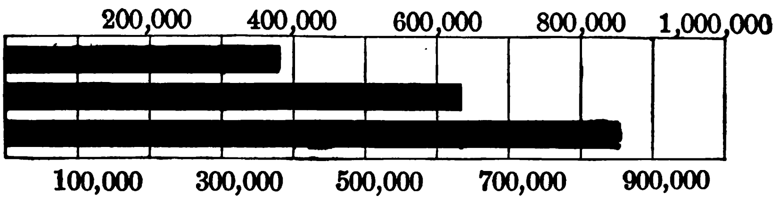
Which shows the facts more plainly, the column of figures to the right or the drawing?



1. State what numbers are represented by the black rectangles in the above graph.

A number like 3846 can be represented only approximately on such a graph. On the other hand, 4000 can be represented exactly, and 6300 may be represented very nearly exactly. That is, we can estimate or measure one-tenth of the spaces quite accurately. On a graph like the above we can therefore represent a number to the *nearest hundred.*

2. By means of such a figure represent 2500, 4600, 3900. To represent a number like 1680, in this figure, we simply take the number to the nearest hundred. That is, we take 1700



In this graph numbers may be represented to the nearest 10,000.

3. On such a graph represent 480,000, 820,000, 270,000.

4. On such a graph represent approximately 148,560, 257,942, 573,300, 911,841. Use the approximations 150,000, 260,000, 570,000, 910,000.

5. Find the number of people in your city now, 10 years ago, 20 years ago, and 30 years ago. Represent these four numbers on a graph like the above. Your teacher will help you find these numbers. Your graph will show the growth of your city during the last 30 years.

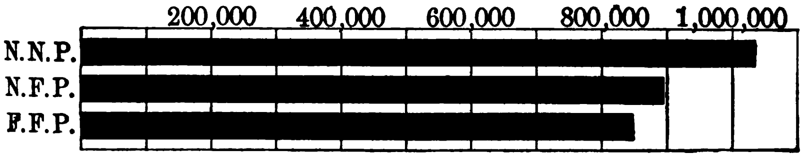
6. Find the number of pupils attending your school for each of the last few years, and make a graph to represent these numbers. What does this graph tell you about the growth of your school?

In the graph on this page the numbers are represented to the nearest 10,000.

In 1900 there were in Massachusetts, 1,032,264 persons who were native born of native parents, 897,386 were native born of foreign or mixed parents, and 840,114 were foreign born.

This is shown graphically as follows:

(N of N. P. means native born of native parents, N of F. P. means native born of foreign or mixed parents, and F. of F. P. means foreign born of foreign parents.)



Make graphs representing the information given below.

Compare these graphs to see what you can learn from them as to the changes in the character of the population in these states.

		1910	1900	1890
Mass.	N. of N. P.	1,103,429	1,032,264	995,430
	N. of F. P.	1,170,447	897,386	606,440
	F. of F. P.	1,051,050	840,114	653,503
Ill.	N. of N. P.	2,600,555	2,271,764	1,882,690
	N. of F. P.	1,723,847	1,498,473	1,044,804
	F. of F. P.	1,202,560	964,635	840,975
Ca.	N. of N. P.	2,568,382	2,179,395	1,804,778
	N. of F. P.	25,672	24,913	19,683
	F. of F. P.	15,072	12,021	11,892
N. D.	N. of N. P.	162,461	65,811	37,712
	N. of F. P.	251,236	133,311	63,347
	F. of F. P.	156,158	112,590	81,238

Make a graph to show the number of girls and the number of boys in your school.

See if you can find graphs like these in some magazine. If you do, bring it to school and explain what the graphs tell you.

1. A large wagon box is 10 feet 6 inches long, 3 feet 6 inches wide, and 26 inches deep, all inside measure. How many bushels will it hold? One cubic foot = .8 bu. **63.7 bu.**

2. A man bought lots for $1850, and sold them for $1926. What was his loss if his expenses, including taxes and interest on the money invested, was $245? What was his loss per cent? **$169; 9.1%**

3. At the rate of 6¼% commission, what is an agent's commission for selling a pleasure boat for $4600? **$287.50**

4. The drivers on a passenger locomotive are 6 feet in diameter. How many revolutions will these drivers make in going from Milwaukee to Chicago, a distance of 85 miles? **23,800**

5. A piece of furniture is bought by a dealer for $25, and marked 75% above the buying price. It is then sold at a discount of 25%. What is the marking price? What is the selling price? **$43.75; $32.81**

6. If 85% of butter is butter fat, how many pounds of butter fat are required to make 45 pounds of butter? **38¼ lbs.**

7. At $2.15 a square foot, what is the value of an oriental rug 9 feet 6 inches wide and 11 feet 8 inches long? **$238.29**

8. A field contains 48.5 acres. How wide is it if its length is 120 rods? **64⅝ rds.**

9. A family spends 18% of their total income for rent. What is the income if they pay $580 a year rent? **$3222.22**

10. In October a farmer put 2360 bushels of corn into his cribs. He sold it in February at 71 cents a bushel. How much did he receive for the corn if it shrank 8%? **$1541.55**

In a loaded freight car the weight of the car is called "dead weight," while the weight of the freight is called "live weight."

1. A wooden freight car weighs about 30,000 pounds, and carries 30 tons of freight. What per cent of the weight of a loaded car is "dead weight," and what per cent is "live weight"? $33\frac{1}{3}\%$; $66\frac{2}{3}\%$

2. A steel car weighs 36,000 pounds, and has a capacity of 50 tons. What per cent of the total weight is dead weight and what per cent is live weight? 26.5%; 73.5%

3. A steel coal car (gondola) weighs 46,000 pounds, and has a capacity of 70 tons. What per cent is live weight and what per cent is dead weight? 75.3%; 24.7%

4. A house which rents for $600 a year can be bought for $8000. Which is cheaper, to buy the house or to rent it if money can be borrowed at $5\frac{1}{2}\%$, and if taxes and repairs amount to $180 a year? **Rent (600; 620)**

5. An auctioneer sold $5630 worth of goods, charging a commission of $2\frac{1}{2}\%$ of the selling price. How much did the owner get for the goods? $5489.25

6. An agent made a sale for $360 and remitted $324 to his customer. What was the agent's rate of commission? 10%

7. A lot 25 feet wide and 95 feet deep sold for $1450. At this rate what is the value of a lot 45 feet wide and 85 feet deep? $2335.26

8. A dealer bought a furnace for $350 and sold it for $402.50. What was his rate of gain? 15%

9. A farmer bought 860 feet of steel fence at $.95 per foot. If he received discounts of 20% and 5%, what was the net cost? $620.92

10. A coal bin 14 feet long, $10\frac{1}{2}$ feet wide, is filled with coal to a depth of $5\frac{1}{2}$ feet. How many tons of coal are there in the bin? (One ton of coal occupies 36 cu. ft.) 22.46 tons

1. During one year an agent averages \$2450 monthly sales. What is his yearly income if his commission is $16\frac{2}{3}\%$ on all of his sales? **\$4900**
2. The inside measurements of an ice-box are 18 inches by 14 inches by 12 inches. How many cubic feet of ice does it hold, allowing one inch in each dimension because the ice does not fit the box? **1.4 cu. ft.**

   What is the weight of this ice if one cubic foot weighs 57 pounds? At 35 cents a hundred pounds how much does it cost? **79.8 lbs.; 28c**
3. A certain street in a large American city is 29 miles long and 66 feet wide. How many acres are there in this street? At 56 bushels to the acre, how much corn could be raised on this area? **232 a.; 12992 bu.**
4. A cattle dealer bought 78 head of cattle averaging 1173 pounds at $11\frac{1}{4}$ cents a pound. He sold them a few days later at $12\frac{3}{4}$ cents a pound. What was his buying price? What was his selling price if the cattle shrank 650 pounds in weight? His expenses were \$38.50. Did he gain or lose on the transaction? **\$10,293.08; \$11,582.61; Gained \$1251.03**
5. In a city containing 246,900 inhabitants there were 4024 deaths in one year. What was the death-rate per thousand? Give rate to the nearest tenth. **16.3**

   Suggestion: Divide 4024 by 246.9.
6. How many tons of ice can be cut from one acre of lake surface, if the ice is 16 inches thick. Allow 10% for waste in cutting and hauling. (One cubic foot of ice weighs 57 pounds.) **1489.75 tons**
7. Recently an American-made racing car was driven 1 mile in 24.02 seconds. How many miles per hour was this?

   Suggestion: At this speed the car would go $\frac{1}{24.02}\times 3600$ miles in 1 hour [1 hour equals $60\times 60 = 3600$ seconds]. **149.9 mi.**
8. In his report of the first aeroplane journey across the Atlantic Lieut.-Commander A. C. Reed says that the time from Trepassey to the Azores was 15 hours 18 minutes, and the average speed 94 miles per hour. Find the distance. **1438.2**

1. Find the interest on \$1400 at $6\frac{1}{2}\%$ for 3 years 4 months. \$303.33

2. A farmer fed 1180 bushels of corn to a drove of hogs, thereby making them gain 10,316 pounds. How many pounds of corn were required to make the hogs gain one pound? Give result to the nearest tenth of a pound. (1 bushel of corn weighs 56 pounds.) 6.4 lb.

3. If 1780 pounds of milk contain 75 pounds of butter fat, how many per cent of butter fat does this milk contain? 4.21%

4. Find the interest on \$850 at $5\frac{1}{2}\%$ for 4 months 15 days. \$17.53

5. A real estate agent sold a house and lot for \$5600. How much did the owner get if the agent deducted 5% as his commission? How much did the agent make if his expenses connected with the sale were \$84.75? \$5320; \$195.25

6. A lot in a city is valued at \$2300, and the house on it is valued at \$12,500. What is the tax on this property if it is 1.952% of the total value? \$288.90

7. Find the circumference of a circle 16 inches in diameter. What is the area of this circle? By how much does it differ from the area of a square whose sides are 14 inches?
$50\frac{2}{7}$ in.; $201\frac{1}{7}$ sq. in.; $5\frac{1}{7}$ sq. in.

8. A stationery bill sold by a wholesale house, amounting to \$1240, is discounted 26%. What is the net amount of the bill? \$917.60

9. Find the interest on \$180 at $3\frac{3}{4}\%$ for 4 months 27 days. \$2.76

10. What is the area of a triangle whose base is 16 inches and altitude 10 inches? 80 sq. in.

11. In making an excavation 45 feet wide, 72 feet long, and $4\frac{1}{2}$ feet deep, how many tons of earth must be removed if 18 cubic feet of solid earth weigh one ton? 810 tons

12. If there are 11 ounces of flour in a 14-ounce loaf of bread, how many pounds of flour will make 1000 loaves? $687\frac{1}{2}$ lb.

1. If bricks weigh on an average $4\frac{1}{2}$ pounds, how many bricks are there in a load of $4\frac{1}{4}$ tons? 1889

2. At $65 per thousand board feet, what is the cost of 560 boards 10 feet long, 4 inches wide, and 1 inch thick? $121.33

3. A water tank is 7 feet 6 inches long, 3 feet 4 inches wide, and 1 foot 10 inches deep. How many cubic feet does it hold? How many gallons of water does it hold? $45\frac{5}{6}$ cu. ft.; 342.9 gal.

4. Find the number of board feet in 60 planks $2\frac{1}{2}$ inches thick, 10 inches wide, and 14 feet long. 1750

5. A steel rail expands $\frac{1}{3}$ of an inch in length when the temperature changes from our lowest to our highest. What per cent of the total length does such a rail expand if it is 30 feet long? .0926%

6. A circle is constructed inside a square to touch all four of its sides. Find the area of both the square and the circle if the diameter is 12 inches. 144 sq. in. $113\frac{1}{7}$ sq. in.

7. A farmer selling his milk is paid at the rate of $18\frac{1}{2}$ cents a gallon. How much does he get for the milk from one cow that yields 8940 pounds of milk for the year? (One gallon of milk weighs 8.6 pounds.) $192.31

8. How long will it take a man to dig 80 rods of ditch 2 feet wide and $3\frac{1}{2}$ feet deep, if he removes 320 cubic feet of earth a day? $28\frac{7}{8}$ days.

9. If it costs $960 to pave a block of street 40 feet wide and 480 feet long, how much will it cost to pave a street 45 feet wide and 1800 feet long? $4050

10. Find the number of bricks required to build a 17-inch wall, 160 feet long and 7 feet high. (See page 240.) 31,360

11. Loads of coal weighing 5760 lbs., 6050 lbs., 5810 lbs., 5370 lbs., 5040 lbs. are delivered. At $7.40 a ton, what is the value of this coal? How much would be saved by buying the coal three months earlier at $7.10 a ton? $103.71; $4.20

1. A lot is 175 feet deep. How wide is it if it contains 5337.5 square feet? **30.5 ft.**

2. In a certain town the tax on real estate is 1.06%. How much tax must a man pay on a lot valued at $2400 and a house valued at $11,500? What is the tax on a factory valued at $145,000? **$147.34; $1537**

3. What is the circumference of a wagon wheel 4 feet 4 inches in diameter? How many revolutions does this wheel make in going 1 mile? **$13\frac{13}{21}$ ft.; 387.7 revs.**

4. How many bushels of potatoes will a bin hold that is 12 feet wide, $14\frac{1}{2}$ feet long, and $5\frac{1}{2}$ feet deep, if one cubic foot is .63 bushels of potatoes? (Bushels of different products vary, depending on how much the measure is to be heaped up.) **602.9 bu.**

5. An automobile bought for $2150 was sold a year later for $1750. At what rate did it depreciate? **18.6%**

Suggestion: $400 is how many % of $2150?

6. A lady bought $12\frac{3}{4}$ yards of cloth marked $3.50 a yard. How much did the lady pay if she received a discount of 15%? **$37.93**

7. How many heat units are required to melt 15 pounds of ice and then raise the water to 80 degrees. (It requires 144 heat units to melt one pound of ice. See page 242.) **2880**

8. Which requires more heat, to melt 35 pounds of ice or to raise 30 pounds of water from 60 degrees to the boiling point? **Former**

9. Using a tape measure, a boy finds that the circumference of a round log at its middle is 85 inches. Find the diameter of the log. **27 in**

10. If it costs $56 to carpet a room 16 feet wide and 18 feet long, how much will it cost to carpet a room 25 feet wide and 36 feet long? **$175**

1. A brick barge on the Hudson River carries 650 tons of brick. What is the value of this barge load of brick at $6.75 a thousand if bricks weigh on an average $4\frac{1}{2}$ pounds apiece? **$1950**
2. The total area of the State of Illinois is 56,650 square miles. How many acres is this? In 1909 the total area used for raising grain in this state was 16,536,500 acres. What per cent of the area of Illinois was used for raising grain? **36,256,000 a.; 45.6%**
3. What is the circumference of a wagon wheel 4 feet in diameter? How many revolutions will this wheel make in going one hundred miles? **$12\frac{4}{7}$; 42,000 rev.**
4. A train travels 100 miles in 2 hours and 45 minutes. How many miles an hour is it going? How long will it take the train to go from St. Paul to Chicago, a distance of 420 miles? **$36\frac{4}{11}$ mi.; 11 hr. 33 min.**
5. In a certain country school the salary of the teacher is $450 for the year. Other expenses for the school are $245 for the year. The building cost $3500, on which the school district is paying interest at the rate of $5\frac{1}{2}$%. Find the total expense per year of this school. What is the average expense per pupil if there are 28 pupils in the school? **$887.50; $31.70**
6. If it takes 4 days to plow a piece of land 160 rods long and $10\frac{1}{2}$ rods wide, how long should it take to plow a piece of land 160 rods long and 54 rods wide? **$20\frac{4}{7}$ days.**
7. The contract price of a house is $8640. What is the cost of this house, including architect's fees, which are 7% of the contract price? **$9244.80**
8. Give convenient dimensions for a coal bin to hold 20 tons of soft coal. (One ton of soft coal measures about 36 cubic feet.) **12′ × 10′ × 6′**
9. A man invested $12,500 and received a net annual income of $1050. What rate per cent did he receive? Find result to the nearest tenth of one per cent. **8.4%**
10. The value of a certain forest increases 4% each year. If it is worth $150,000 now, how much will it be worth in 3 years? **$168,729.60**

1. A new automobile costs \$1560. If it depreciates 35% in value the first year and 25% the second year, for how much could it be sold at the end of the second year? **\$760.50**

Suggestion: First deduct 35% of the original cost to find the value at the end of the first year, and then deduct 25% of this value to find the value at the end of the second year.

2. If a farmer gets $18\frac{1}{4}$¢ a gallon for milk, what is the value of the milk from one cow which yields 10,435 pounds in one year? (See page 245.) **\$221.44**

3. A school district bought text-books as follows: 45 books at 64 cents, discount 20%; 60 books at 56 cents, discount 15%; 80 books at 50 cents, discount 20%. How much did the books cost? **\$83.60**

4. At 35¢ a square yard, how much does it cost to plaster the walls and ceiling of a room 18 feet long, $15\frac{1}{2}$ feet wide, and 9 feet high, making no allowance for doors or windows? **\$34.30**

5. A school district bought 64 desks at \$5.25; 2 map cases at \$12.50; 2 teachers' desks at \$25; 2 office chairs at \$4.50; 2 globes at \$35; 1 reading chart \$17.50. What was the total if a discount of $12\frac{1}{2}$% was given? **\$443.00**

6. At \$84 per thousand, what is the cost of 2460 linear feet of flooring $2\frac{1}{2}$ inches wide and $\frac{3}{4}$ inches thick? (A board less than 1 inch thick is always counted as 1 inch.) By linear feet is meant the sum of the lengths of all the boards. The length is given in this way because the pieces of flooring are of different lengths. **\$43.05**

7. A grain dealer bought 45,000 bushels of oats at 37¢ a bushel, and sold them two months later at 41¢ a bushel. If the oats shrank 2%, and if his expenses were \$340, did he gain or lose, and how much? **\$1091 gain**

8. See who can bring in the most interesting problems for the class to solve.

148. Discount Series. Sometimes two or more discounts are given on the same goods.

Thus, goods may be sold at a 25% discount because the market price is lower than the list price. Another discount of 10% may be given because of an unusually large order, and still another discount of 5% because of prompt payment.

Example. Find the selling price of an article listed at $100.00 if discounts of 25% and 10% are allowed:

$100 less 25% of itself equals $75.
$75 less 10% of itself equals $75 − $7.50 = $67.50, which is the selling price.

The first discount is computed on the list price as a base; the second discount is computed on the amount left after the first discount has been deducted.

Solve this problem by first deducting 10% of the list price and then 25% of the remainder.

Notice that the *order* in which the discounts are deducted makes no difference.

WRITTEN EXERCISES

Find the selling price in each of the following examples. In the first four get the results in two ways:

	List price	Discounts			List price	Discounts	
1.	$48.50	20%, 15%	$32.98	9.	$248.00	25%, 15%	$158.10
2.	$760.00	15%, 8%	$594.32	10.	$8670.75	30%, 20%	$4855.62
3.	$240.00	20%, 12%	$168.96	11.	$1450	10%, 5%	$1239.75
4.	$94.50	15%, 5%	$76.31	12.	$4600	15%, 10%	$3519.00
5.	$345.00	30%, 10%	$217.35	13.	$890	20%, 5%	$676.40
6.	$18.60	20%, 10%	$13.39	14.	$1860	25%, 15%	$1185.75
7.	$360.00	15%, 10%	$275.40	15.	$2470	30%, 20%	$1383.20
8.	$125.00	20%, 12½%	$87.50	16.	$6150	35%, 15%	$3397.88

WRITTEN EXERCISES

Find the selling price in the following. See how many you can do in 6 minutes:

	List price	Discounts			List price	Discounts	
1.	$45.00	25%	$33.75	7.	$250.00	$12\frac{1}{2}$%	$218.75
2.	$60.00	30%	$42	8.	$28.00	$16\frac{2}{3}$%	$23.33
3.	$1.40	40%	$.84	9.	$35.00	$8\frac{1}{3}$%	$32.08
4.	$18.00	10%	$16.20	10.	$75.00	45%	$41.25
5.	$160.00	$33\frac{1}{3}$%	$106.67	11.	$125.00	15%	$106.25
6.	$24.00	45%	$13.20	12.	$64.00	$37\frac{1}{2}$%	$40

Find the selling price of goods reduced as follows:

	List price	Discounts			List price	Discounts	
13.	$80.00	15%, 10%	$61.20	19.	$1.75	15%, 5%	$1.41
14.	$45.00	10%, 5%	$38.48	20.	$3.50	40%, 10%	$1.89
15.	$18.00	20%, 15%	$12.24	21.	$2.75	60%, 5%	$1.05
16.	$60.00	30%, 10%	$37.80	22.	$12.50	40%, 30%	$5.25
17.	$45.00	25%, 10%	$30.38	23.	$65.00	15%, 8%	$50.83
18.	$85.00	10%, 5%	$72.68	24.	$750.00	25%, 20%	$450

Find the rates of discount in the following to the nearest per cent.

	List price	Selling price			List price	Selling price	
25.	$45.00	$35.00	22%	31.	$40.00	$28.00	30%
26.	$2.40	$1.80	25%	32.	$50.00	$32.00	36%
27.	$6.50	$4.50	31%	33.	$125.00	$85.00	32%
28.	$8.25	$6.00	27%	34.	$12.50	$8.75	30%
29.	$5.00	$2.40	52%	35.	$62.50	$37.50	40%
30.	$0.75	$0.40	47%	36.	$90.00	$55.00	39%

149. **Why Taxes are Collected.** Money derived from taxes is needed for many purposes. The teachers in your school, county officials such as judges and other officers of the courts, state officials such as the Governor and members of the legislature, are all paid with money obtained from taxes. If you live in a city, you have a police department and a fire department. There are schoolhouses, and maybe a courthouse. The streets are paved and sewers built. All these things are paid for with money obtained from some kind of taxation or with borrowed money, which is to be paid back from taxes.

150. **Kinds of Taxes.** There are many ways of levying taxes. One of the most common ways is to make the owners of houses and lands (called *real estate*) pay a certain per cent of their value each year.

151. **Tax Rate, Valuation.** The rate per cent of the value of property which is paid as taxes is called the *tax rate.*

WRITTEN EXERCISES

1. If the tax rate on real estate is $\frac{3}{4}$ of one per cent, what is the tax on a house and lot valued at \$12,000? *Suggestion:* $\frac{3}{4}\% = .0075$. **\$90**

2. If the tax rate is $\frac{1}{2}$ of one per cent, what is the tax on a farm containing 140 acres valued at \$65 per acre, and buildings valued at \$3400? **\$62.50**

3. If the tax rate is $1\frac{1}{2}\%$, what is the tax on a city property valued at \$45,000? ($1\frac{1}{2}\% = .015$.) **\$675**

4. If the tax rate is $\frac{7}{8}$ of one per cent, what is the tax on a lot 50 by 120, valued at 45¢ a square foot, and a house valued at \$8500? ($\frac{7}{8}\% = .00875$.) **\$98**

5. If the tax rate is $\frac{4}{5}$ of one per cent, what is the tax on vacant city lots 150 by 140 feet, valued at 25¢ a square foot? **\$42**

Sometimes the rate is most conveniently expressed in decimals, Thus, a tax rate of .00894 on the dollar means that the number of dollars' valuation of the property is multiplied by .00894 to obtain the amount of the tax.

In each of the following find the result correct to the nearest cent:

1. If the tax rate is .00927 on the dollar, what is the tax on property valued at $4750? **$44.03**

2. If the tax rate is .0103 on the dollar, what is the tax on a house and lot valued at $16,000? **$164.80**

3. A man owns two lots, each 25 feet by 120 feet, valued at 55¢ a square foot. On one of them stands a building valued at $7000. What is the tax on this property, if the rate is .00734 on the dollar? **$75.60**

4. A farm contains 350 acres, valued at $85 an acre. The buildings are valued at $6400. What is the tax on this property if the rate is .00825 on the dollar? **$298.24**

Personal property is any property not real estate. The tax rate on personal property is different from that on real estate.

5. A man has personal property valued at $12,400, and real estate valued at $10,000. What is his tax if the rate on personal property is .0054 on the dollar, and on real estate .0075 on the dollar? **$141.96**

6. Find the tax of a man whose personal property is valued at $8600 and real estate at $14,000, if the rate on personal property is .00925 on the dollar, and on real estate .0125 on the dollar. **$254.55**

7. What is the tax of a man who pays a personal tax (poll tax) of $3.00 and a rate of .00945 on property valued at $24,800? **$237.36**

8. Find the tax rate in your own town, and the valuation of some building, and then find how much tax is paid on it.

152. Fire Insurance Policy. A contract in which a company agrees to pay a certain amount in case a property is destroyed or damaged by fire is called *a fire insurance policy.*

153. Premium Rate. The amount paid by the insured to the company for insuring his property is called the *premium.* The premium is usually given as a certain rate per cent of the amount insured. This rate is called the *insurance rate.*

The insurance rate depends on a large number of conditions. The rate is higher on a wooden house than on a brick or stone house. It is higher where the houses are crowded close together, or where the protection against fire is poor.

The rate is lower when the insurance is taken out for a long time, say three years, than it is for a shorter period.

Problem. Find the annual premium on a fire insurance policy for \$3500 if the rate is .6%.

$$\begin{array}{r} \$3500 \\ \underline{.006} \\ 21.000 \end{array}$$

Multiplying \$3500 by .006, we get \$21, which is the yearly premium.

WRITTEN EXERCISES

Find the annual premiums on the following:

Amount of insurance	Rate		Amount of insurance	Rate	
1. \$5000	.5%	\$25	3. 1500	.7%	\$10.50
2. \$2000	.6%	\$12	4. \$32,000	.3%	\$96

5. A man carried \$8000 insurance on his home and \$5000 on personal property. What was his yearly premium, if the rate on the house was .4% and on the personal property .5%? \$57

6. The insurance rate on a factory building was decreased from .45% to .275% by the installation of automatic sprinklers. How much was saved in yearly premium if the factory was insured for \$225,000? \$393.75

154. Long and Short Term Rates. The standard period for which fire insurance rates are quoted is one year. Three-year policies are usually issued for a premium equal to $2\frac{1}{2}$ times the yearly premium, and 5-year policies for a premium equal to 4 times the yearly premium. For terms less than one year higher rates are charged.

1. Find the three-year premium on a policy for $4000, its yearly rate being .5%, if the three-year premium equals $2\frac{1}{2}$ times the yearly premium. **$50**

2. Find the five-year premium on a policy for $8500, the yearly premium being .45%, if the five-year premium equals 4 times the yearly premium. How much is saved by taking out 1 five-year policy instead of 5 one-year policies? **$153; $38.25**

3. Household goods are placed in storage, and insured at $2500 for 90 days. What is the premium if the yearly rate is .56%, and the rate for 90 days is 40% of the yearly premium? **$5.60**

4. A building worth $16,000 is insured for 80% of its value. What is the yearly premium if the rate is .375%? **$48**

5. Furniture is insured for $3400, the annual rate being .6%. What is the premium for 6 months if it is 70% of the annual rate? **$14.28**

6. Find the premium on a five-year policy for $25,000, the annual rate being .35%, if the premium for the five years is 4 times the annual premium? **$350**

7. A building is insured for $12,000. What is the total of premiums for 20 years if five-year policies are used, the annual rate being .45%, the premium for 5 years being 4 times the annual premium? **$864**

8. Find the insurance rates on different kinds of property in your locality, and then find how much is paid in premium on certain buildings.

9. Is your schoolhouse insured? for how much? What is the premium?

155. **Life Insurance Policy.** A life insurance policy is a contract whereby a company agrees to pay a certain sum at a certain time, or on the death of the person insured. There are several kinds of personal insurance, such as accident insurance, health insurance, and so on. Only regular life insurance is considered in this book.

156. **Kinds of Life Insurance.** There is a large variety of life insurance policies, such as *straight life, limited payment life,* and *endowment policies.*

157. **Annual Premiums.** The premium paid per thousand dollars of life insurance varies with the age of the person insured, and the kind of policy. (See next page.)

158. **Straight Life.** Thus a healthy person 21 years of age may take out a straight life one-thousand-dollar policy for a yearly premium of $19.62. This means that the insured is to pay $19.62 to the company every year as long as he lives. In consideration of this, the company agrees to pay $1000 to the heirs of the insured at the time of his death, whether his death occurs immediately after the policy is taken out or 60 years later.

159. **Twenty Year Endowment.** For a yearly premium of $48.63 a person 21 years of age may obtain a $1000 policy by which the company agrees to pay $1000 at the expiration of 20 years or earlier in case the insured dies.

160. **Twenty Payment Life.** In the same company a person 21 years of age may get a life policy for $1000 on the payment of $29.84 for 20 years. On this policy, the insurance company will pay $1000 on the death of the insured, but his payments cease at the end of 20 years.

161. **Table of Rates.** The table on the next page shows the yearly premium per thousand dollars. Note how it increases with the age of the insured at the time the policy is taken out.

Age	Straight life	20-payment life	20 years endowment
21	$19.62	$29.84	$48.63
25	21.49	31.83	49.33
30	24.38	34.76	50.43
35	28.11	38.34	51.94
40	33.01	42.79	54.06
45	39.55	48.52	57.34
50	48.48	56.17	62.25
55	60.72	66.69	70.81
60	77.69	81.60	83.82

Use this table in solving the following problems:

1. At 30 years of age, what is the yearly premium on a straight life policy for $2500? **$60.95**

2. At 25 years of age, what is the yearly premium on a $2000 twenty-payment life policy? **$63.66**

3. At 35 years of age, what is the yearly premium on a $5000 twenty-year endowment policy? **$259.70**

4. A man takes out a $5000 straight life policy at the age of 35. If he dies after making 17 payments how much more will he get from the company than he paid to it? **$2610.65**

5. At 40 years of age a man takes out a $4000 twenty-year endowment. If he lives twenty years and received the $4000 from the company, will he receive more or less than the sum of the premiums paid in? **Less ($324.80)**

6. At 45 years of age a man takes out a straight life policy for $10,000. If he dies after making 28 payments, by how much will the total premiums exceed the amount paid at the time of his death, not counting interest on the premiums? **$1047**

7. Make and solve other problems on life insurance, using the above table.

162. Promissory Note. When a man borrows money from a bank, he makes out a note, payable to the bank. This note is a promise to pay so much money to the bank on a certain day. The form of such a note is as follows:

> $200.00 New York, Dec. 15, 1916.
> March 3, 1917. .... I PROMISE
> TO PAY TO THE ORDER OF............Myself.................
> TWO HUNDRED..............................DOLLARS ($200.00)
> AT THE CORN EXCHANGE BANK, UNIVERSITY BRANCH,
> BROADWAY AND 113TH STREET, NEW YORK CITY, N. Y.
> VALUE RECEIVED
> No. 1034 DUE March 3, 1917. *James B. Noble,*
> 542 West 124th St.
> (Residence or place of business.)

The amount written in the note, viz., $200.00, is called the *face* of the note. James B. Noble is called the *maker* of the note.

The bank computes interest on this note from Dec. 15, 1916, to March 3, 1917, at some agreed rate of interest, say 6% per annum, deducts this from the face of the note, and credits James B. Noble with the difference. In this case the time is 78 days, and the interest amounts to $200 \times .06 \times \frac{13}{60} = 2.60$ dollars.

163. Bank Discount, Proceeds. The remainder, when the $2.60 is deducted from the face of the note, is called the *proceeds* of the note. The interest deducted is called the *bank discount.*

Find the bank discount and the proceeds of each of the following:

	Face of note		Rate of discount		Time
1.	$500	$7.50	6%	$492.50	90 days
2.	$450	$5.63	6%	$444.37	75 days
3.	$1280	$15.65	$5\frac{1}{2}$%	$1264.35	80 days
4.	$2740	$43.12	$5\frac{1}{2}$%	$2696.88	103 days
5.	$7860	$49.13	5%	$7810.87	45 days

164. Discounting Bills. If you owe a debt due two months from now, the man to whom you owe it may need the money, and hence may be willing to take a little less than the full amount if you pay it at once. The amount by which the bill is reduced on account of immediate payment is called *discount.*

In discounting a bill in this manner the discount is stated as a certain per cent of the amount or *face* of the bill, and is not computed as interest for a certain time, as in the case of bank discount.

Find the proceeds in each of the following:

	Amount or face of bill	Rate of discount			Amount or face of bill	Rate of discount	
1.	$260.00	2%	$254.80	4.	$48.20	3%	$46.75
2.	$356.70	3%	$346	5.	$86,140	$2\frac{1}{4}\%$	$84,201.85
3.	$160.25	$2\frac{1}{2}\%$	$156.24	6.	$4370	$2\frac{3}{4}\%$	$4,249.83

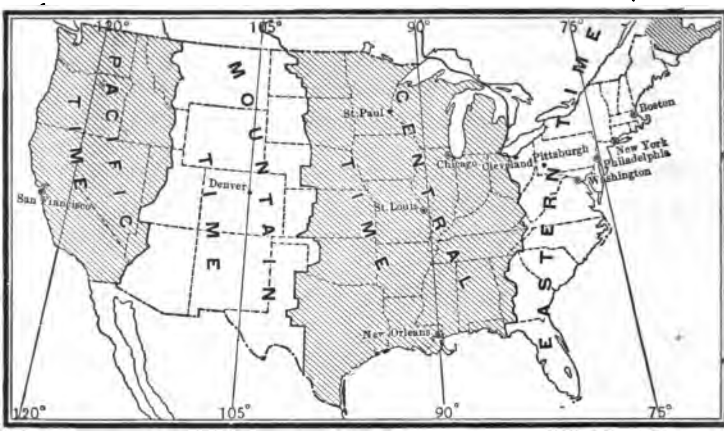
Problem. A debt of $8500 is settled for $8000 on account of immediate payment. What was the rate of discount?

$$\begin{array}{r@{\,}l} & .0588 = 5.88\% \\ 8500\,) & 5.0000 \\ & .425 \\ \hline & \;\;750 \\ & \;\;680 \\ \hline & \;\;\;700 \end{array}$$

500 is how many per cent of 8500? The result to the nearest tenth of 1% is 5.9%.

Find the rate of discount in the following:

	Face of bill	Proceeds			Face of bill	Proceeds	
7.	$4000	$3800	5%	12.	$560	$530	5.4%
8.	$12,000	$11,600	$3\frac{1}{3}\%$	13.	$2800	$2700	3.6%
9.	$840	$800	4.8%	14.	$4300	$4100	4.7%
10.	$375	$350	$6\frac{2}{3}\%$	15.	$5760	$5400	$6\frac{1}{4}\%$
11.	$1800	$1700	5.6%	16.	$3940	$3700	6.1%



165. Standard Time. If the difference in the longitude of two places is 15°, the time of day differs by one hour. Thus, noon comes about an hour earlier in Philadelphia than in St. Louis, and one hour earlier in St. Louis than in Denver. For convenience the United States has been divided into four regions, in each of which the same time is used throughout the region. Thus, Boston, New York, and Philadelphia all have the same time. Similarly, Cleveland, Chicago, and St. Paul have the same time.

1. When it is 6 P. M. in Chicago, what time is it in Boston? in Denver? in San Francisco? 7 p. m.; 5 p. m.; 4 p. m.

2. When it is noon at your home, what time is it in Philadelphia? in Los Angeles?

3. A newspaper in San Francisco prints a story at 1 P. M. of an event which occurrred in New York at 3 P. M. Explain.

4. From what part of the country do election returns come in first? Why?

ORAL WORK

1. Bring any book or magazine to class in which there are graphs. Get the teacher to help you find the meaning of such graphs.

2. What is meant by discounting a bill? If you know the full amount of a bill and the rate of discount, how do you find the proceeds?

3. What is meant by the *proceeds*?

4. If you know the face of a bill and the proceeds, how do you find the rate of discount?

5. What is a promissory note? What is the face of a note? Who is the maker of a note?

6. What is meant by bank discount? Give an example to show how a bank discounts a note.

7. What is a life insurance policy? Describe several different kinds of life insurance policies.

8. What is a fire insurance policy? Describe several different kinds of fire insurance policies.

9. What is meant by premium? premium rates? Give examples.

10. What are taxes? Give several reasons why taxes are collected. Describe two kinds of taxes.

11. What is meant by tax rate? Give two different forms of expressing tax rates.

12. Make and solve a problem on taxation. If possible, get the material from your neighborhood.

13. Do New York and San Francisco have noon at the same time? If not, why not?

14. Which has noon earlier, and how much, Boston or Seattle?

15. Which have noon earlier, people in your home, or people living in St. Louis? in Denver?

Lineal Measure

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards	= 1 rod (rd.)
$16\frac{1}{2}$ feet	= 1 rod
40 rods	= 1 furlong (fur.)
8 furlongs	= 1 statute mile
5280 feet	= 1 statute mile
6080 feet	= 1 nautical mile (knot)

Liquid Measure

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)
$31\frac{1}{2}$ gallons	= 1 barrel (bbl.)
2 barrels	= 1 hogshead (hd.)
1 gallon	= 231 cubic inches

1 gallon of water or petroleum weighs 8.355 pounds.
1 gal. of milk weighs 8.62 lb.

Square Measure

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.) or one section (sec.)
36 square miles	= 1 township

Cubic Measure

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
128 cubic feet	= 1 cord (cd.)

1 cubic foot of water weighs 62.5 pounds
1 cubic foot of ice weighs 57.5 pounds

Dry Measure

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)
11 pecks	= 1 barrel of apples
1 bushel	= 2150.42 cubic inches

1 English bushel = 2,218.92 cubic inches
1 English bushel of water weighs 80 pounds
1 bushel of wheat weighs 60 pounds
1 bushel of corn weighs 56 pounds
1 bushel of potatoes weighs 60 pounds

Time

60 seconds (sec.)	= 1 minute (min.)	365 days	= 1 year (yr.)
60 minutes	= 1 hour (hr.)	12 units	= 1 dozen (doz.)
24 hours	= 1 day (da.)	12 dozen	= 1 gross (gr.)
7 days	= 1 week (wk.)	12 gross	= 1 great gross

Avoirdupois Weight

$437\frac{1}{2}$ grains (gr)	= 1 ounce (oz.)
16 ounces	= 1 pound (lb.)
25 pounds	= 1 quarter (qut.)
4 quarters	= 1 hundredweight (cwt.)
20 hundredweights	= 1 ton (T.)
2000 pounds	= 1 ton (short)
2240 pounds	= 1 ton (long)

Money

10 mills	= 1 cent	25 cents	= 1 quarter
10 cents	= 1 dime	50 cents	= 1 half-dollar
10 dimes	= 1 dollar	5 dollars	= 1 half-eagle
10 dollars	= 1 eagle	20 dollars	= 1 double eagle

There are other tables, which are of little general use, and which are not given here. The so-called Apothecaries' table of weights is used in weighing drugs, and the *Troy* table of weights is used to some extent in weighing precious metals.

Both the Troy and the Apothecaries' ounce contains 480 grains, and the pound 12 ounces.

The Avoirdupois pound contains 7000 grains, and the Troy and Apothecaries' pound contains 5750 grains.

The same table of time measure is used nearly the world over. In England the measures of length and weights are practically the same as those used in the United States. The English ton is 2240 pounds, the same as our long ton. In the United States the long ton is used for weighing coal at the mines, for shipping purposes, and in the United States Customs House.

The meter, which is about 39.37 inches, is used in France, Germany, Russia and many other countries.

INDEX

PREFACE TO NOTES IN TEACHER'S EDITION

ONE purpose of the teacher's edition of these books is to provide detailed suggestions to the teacher. Many inexperienced teachers have had little or no normal training in the art of teaching, and even those who have had such training frequently find it difficult to make practical application of general principles to each day's work. For all such teachers these notes should be of decided value.

A second purpose is to provide in convenient form much of the material necessary for the best kind of teaching, thus setting the teacher free from a large share of the drudgery that has too often been regarded as inseparable from her work.

This material includes:

(*a*) A list of local material for supplementary work.

(*b*) Material and suggestion for drills in the fundamentals at frequent intervals throughout the book. Answers are given to all proposed exercises, thus relieving the teacher of unnecessary work in performing the computations.

(*c*) A series of standardized tests in the fundamentals which will enable the teacher to decide whether or not her classes are up to the normal standards in the four fundamental operations.

(*d*) Six sets of material for examinations are provided for each year's work. This will relieve the teacher of the trouble of finding suitable material for examinations and the labor of performing the computations.

(*e*) All problems involving two or more steps are solved, showing the result of each step. This has been done not because there may be some exceptionally weak teachers who do not know how to solve the problems, but because it is desired to relieve all teachers of the work of performing the computations. In checking papers the teacher needs to know in what step an error may have been made. Hence the result of each step is given.

(*f*) The answers to all problems and exercises are given in the

text in distinctive type and on the same page with the problem and exercises themselves. This saves the teacher the trouble of looking in the answer book to verify the answers.

(*g*) Except for the answers just mentioned, the page of the main text in the teacher's edition is exactly like that of the pupil's book. It is therefore unnecessary for the teacher to ever use the pupil's book.

It has been customary to regard a teacher as lazy or indifferent if she seeks to avoid doing any of the work that she asks her pupils to do. Nothing could be more unfair and misleading. There are good reasons why young children should spend much time on drills in the fundamentals, but there is no reason why a competent teacher should spend any time whatever on such drills. Teachers, supervising officers, and makers of text books should constantly bear in mind that whenever the teacher is relieved of unnecessary drudgery she is thereby set free for the more essential parts of her extremely difficult work of leading onward in the best way the young minds entrusted to her care. To this end the authors of these books have worked with singleness of purpose.

LOCAL DATA FOR SUPPLEMENTARY MATTER

It is generally agreed that the best work in Arithmetic is possible only when considerable use is made of local material. Such material may be collected with very little trouble by the teacher or the pupils, or both, if it is known some time in advance just what will be needed, whereas it might be practically impossible to obtain it on the spur of the moment at the time it is to be used. For this reason a list is given here of local material suggested in the text or in the notes together with the pages in the text in connection with which it will be needed. In this list frequent reference is made to pages in the text and the notes, where a more detailed description may be found.

Page 2. Large numbers in practical use, such as numbers representing populations, areas, agricultural and mining products. See note to page 2.

Page 8. Cancelled bank checks and bank's list of withdrawals.

Pages 14–15. Data on canning and preserving for the winter. See notes to these pages.

Page 50. See note to this page.

Page 62. Get a real screw and show what is meant by "pitch." A large screw will be best.

Page 81. See note to this page. Such data will also be needed on pages 210, 211.

Page 93. Get a number of local bills. Endeavor to get some on which "extending" the items really amounts to something.

Page 103. See note to this page. Try to get blue prints of simple plans.

Page 106. Local facts about farming similar to those used on this page in the text.

Page 113. Have the boys bring in interesting cyclometer readings.

Page 126. Ask for local milk records (if the school is in or near the country) such as are given on this page. Similar material will be needed on page 169.

Page 133. Get actual records of base-ball teams that are of special interest to the children. See note to this page.

Page 136. Get real bills with discount deducted. Discounts stated as rates per cent should be shown on such bills. Also get advertisements showing discount. Similar material will be needed on page 178.

Pages 150–151. Records of expenditures of Boy Scout troops. See note to these pages.

Page 174. Miscellaneous practical uses of percentage, such as charges for collecting bills, etc. See problems in text on this page.

Page 178. Get additional material showing practical uses of discounts, such as price lists showing discounts, catalogs, etc.

Page 185. See note to this page.

Page 187. See note to this page.

Page 190. See note to this page.

Page 192. See note to this page.

Page 194. See note to this page.

Page 220. See note to this page.

Page 222. If the school is near a creamery find weight of gallon of milk used for practical purposes. Also find weight of a bushel of various kinds of grain

Page 223. If the school is located in the country find amount of various kinds of seed used for an acre.

Page 224. The weight of normal loads of coal and hay.

Page 225. The market prices of cattle, hogs, and sheep. These may always be found in the daily papers.

Page 229. Cylinders in such shape that the diameters and circumferences may both be measured. Tin cans will do well.

Pages 230–231. The distance actually traveled in a season or on a special trip by certain automobiles and the diameters of their wheels. Then find the number of revolutions made by their wheels.

Page 233. See page in text for material to be used.

Page 234. Get actual bills for lumber.

Page 236. Get data like those used on this page from a house in the neighborhood. Also find local cost of painting.

Page 237. Estimate wall paper for rooms actually measured by the pupils.

Page 238. Estimate carpet for room actually measured by the pupils. In what kind of rooms are carpets in use?

Page 239. Estimate amount of lumber in floor actually measured by the pupils.

Page 240. Measure walls of some building and find number of bricks in it.

See notes to pages 248, 250, 252–255, 256–259, 264–267, 276, 278, 280, 282, for material to be used in connection with these pages.

INTRODUCTION TO NOTES

In these notes, each book of this series is treated as a separate unit. Thus a topic introduced for the first time in book two is recorded as "new matter," though it may be treated in book one, and may not be new to the child. However, in teaching book two, book one should be on the teacher's desk for reference.

The general functions of a review in arithmetic are: (a) to review the fundamental number facts and to fix them more firmly in the memory; (b) to gain a little deeper insight into the nature of the operations of arithmetic and the applications of these operations to the solution of problems. Effort to gain deeper insight should be made only insofar as this promises results. One of the most prevalent errors of teachers is that they explain too much. Frequently we hear a student in an upper class exclaim, when for the first time he has really learned to understand an operation or an application: "Why did they not explain that to me this way a long time ago, so I could have understood it from the first?" The child is likely mistaken. The chances are he could not have understood it "from the first." He does not realize how the years have brought him maturity and power.

Page 1

New Matter. The system of numerals. The principle of place value.

Remarks. Endeavor at this point to explain more fully the principle of place value. Thus a digit, as 4, may represent 4 ones, 4 tens, 4 hundreds, 4 thousands, 4 tenths, 4 hundredths, and so on, depending entirely upon the place in which it stands.

Also discuss at this point the function of zero in a place as meaning *none* of the unit represented by numerals in that place. The children will be interested to know that the zero was the last of the numerals to be invented and used, and that when the zero was once invented the system of numerals became effective and flexible, and has served the world without material change ever since.

Page 2

New Matter. Reading numbers.

Supplementary Matter. At this point have the children bring in large numbers which they have encountered in their other school work, such as in the study of geography. The teacher may also bring in large numbers, especially numbers of local significance such as the population of the state, the number of bushels of wheat or some other cereal raised in the state, or the number of bales of cotton produced, and so on, depending upon the locality.

Some work should also be done in developing an understanding of the meaning of large numbers. Thus, if one state has a population of five million, while another state has a population of two and a half million, it may be pointed out that one state is twice as great in population as the other. Many similar comparisons between large numbers should be made. The teacher should realize that none of us **image** such large numbers. We acquire an understanding of their meaning by comparing them with other large numbers, and ultimately by comparing them with smaller numbers. Thus, we define a million as one thousand thousands; which, of course, is nothing but a comparison of the million with the thousand. The important point is that the child should learn the uses of large numbers. The operations with these numbers will cause no serious difficulty. "Four times two millions equals eight millions" is no more difficult than $4\times2=8$.

Page 3

New Matter. The Roman numerals. Definition of addition.

Remarks. Do not stress particularly the Roman numerals, nor the definition of addition. As a matter of fact, in advanced work in mathematics, addition is not defined formally. The statements made in the text should be regarded as descriptive, rather than as formal definitions. If the teacher wishes to convince herself or others of the practical usefulness of our system of numerals let her write 697 and 842 in the Roman notation and multiply them.

Supplementary Matter. Let each child rule a sheet of paper into 45 rectangles and write one of the 45 addition combinations in each rectangle as follows:

1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9
2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9	3+3
3+4	3+5 /	3+6	3+7	3+8	3+9	4+4	4+5	4+6
4+7	4+8	4+9	5+5	5+6	5+7 ////	5+8	5+9	6+6
6+7	6+8	6+9	7+7	7+8	7+9 ~~////~~ /	8+8	8+9	9+9

On this paper make a record of the errors in the tests given below. The marks in the illustration above show 1 error in 3+5, 4 errors in 5+7, and 6 errors in 7+9.

The purpose of these tests is to review once more the addition and multiplication combinations. The teacher may read one test in addition on each of 5 consecutive days. Have the children mark and record their own errors. These tests are as much for the information of the children as of the teacher. Let the children show the teacher their record of errors.

First test in addition.

7	7	4	5	6	2	8	4	4	8	5	3	8	5	2	9	5	5	8	5
9	3	4	7	9	3	9	8	9	3	9	6	7	3	7	9	6	7	2	8

4	6	6	3	2	8	4	2	1	7	1	8	5	7	6	5	3	4	2	7
9	9	9	9	8	8	7	7	7	9	9	5	8	5	7	5	9	6	1	2

Second test in addition.

9	9	9	9	9	9	8	8	5	8	7	9	9	8	6	6	8	4	8	3
7	8	4	5	6	7	6	4	5	7	3	9	1	5	2	3	4	4	2	3

1	4	1	2	2	4	3	8	2	6	5	3	6	6	7	6	5	4	8	7
1	6	5	9	5	7	5	8	8	6	7	6	7	8	8	9	9	9	9	9

Third test in addition.

8	4	5	6	7	7	4	6	5	2	3	9	1	5	2	3	1	4	1	1
9	9	9	9	8	9	5	7	8	7	9	9	8	6	6	8	6	8	4	2

7	3	5	8	1	4	2	2	4	1	7	7	8	4	5	6	7	6	6	5
7	9	5	8	9	7	5	9	4	6	2	9	9	9	9	9	8	8	7	8

Fourth test in addition.

9	1	5	2	3	1	4	1	1	6	5	5	3	6	7	6	5	4	8	7
9	8	6	6	8	6	8	4	3	6	7	8	6	8	8	9	9	9	9	9

7	4	6	5	7	7	8	4	5	6	3	1	8	1	4	1	4	3	2	3
8	5	7	8	7	9	9	9	9	9	9	7	8	9	7	3	6	3	9	4

Fifth test in addition.

3	2	2	3	2	4	1	8	2	6	5	2	6	6	7	6	5	4	8	7
7	2	4	9	4	7	9	8	8	6	7	7	7	8	8	9	9	9	9	9

1	4	5	6	4	6	3	7	5	5	3	9	5	5	3	3	3	4	2	3
7	9	9	9	5	8	6	9	8	7	9	9	5	6	4	8	3	8	2	7

Page 4

New Matter. Addition with carrying.

Remarks. Copy the columns of figures on this page, and add them. At this point have each child make a note of the time required to do this work, but for the present make no effort to bring the children up to standard speed. Children should realize the importance of copying figures well. The work in adding should be checked by adding each column both ways. The numbers to be carried may be written below the numbers in the sums, in small pencil figures, as the work proceeds. This is valuable because it preserves a record of the numbers carried, and thus facilitates checking when the column is added in the opposite direction.

The standard of accuracy should be made very high. In practical computing, the work must be absolutely accurate, and that is the aim we should have in the schoolroom. Speed is of secondary importance as compared with accuracy. We should adopt a system of checking and a standard of accuracy which will bring the latter practically up to one hundred per cent.

Page 5

New Matter. The decimal number system.

Review and Drill. Addition.

Remarks. Add the numbers on this page without copying them. Emphasize absolute accuracy.

Discuss the decimal number system. Point out that the decimal number system is a very different matter from our system of numerals. For one thing it is very much older than these numerals. Thus, the Romans used the decimal number system, but their system of numerals was very different from our present system. Indeed, the Romans did not use the principle of place value at all in their numerals. This discussion should not aim to bring this distinction fully home to all the pupils. Nevertheless it will start some of them thinking, and the brighter ones will get something of real value out of it.

Page 6

New Matter. Subtraction. Definitions of terms used in subtraction.

Remarks. The oral exercises on this page indicate the sort of questions that may be asked to bring out what kind of situations in practical life lead to the process of subtraction. The children should be led to ask many questions of this sort. It will serve greatly to clarify a troublesome matter if the child is led to describe situations that lead to each operation.

Page 7

New Matter. Subtraction. Borrowing or carrying.

Remarks. Select the method of subtraction with which the child is already familiar. If the additive method is used, be sure to proceed as indicated on this page. Thus: Say $14=6+8$, not $6+8=14$. The additive method has the advantage that the subtraction combinations are exactly in the form in which the child has learned the addition combinations. The only difference is that the sum is named first, then that one of the addends which is given, and finally the addend to be found.

If the teacher thinks best, the following explanation of subtraction may be given. The teacher should go through this form on the board, but the children themselves should not be required to repeat it.

Additive Method.

$$\begin{array}{r}
3204+1000+100+10=3000+1200+100+14 \\
1846+1000+100+10=\underline{2000+900+50+6} \\
1000+300+50+8
\end{array}$$

Taking Away Method.

$$\begin{array}{r}
3204=2000+1100+90+14 \\
\underline{1846}=\underline{1000+800+40+6} \\
1000+300+50+8
\end{array}$$

Pages 8 and 9

Review and Drill. Addition.

Applications. Bank deposits and withdrawals.

Remarks. Show the children a check and tell them how it is made out. There is no need at this time to have them make out checks. Also show them a bundle of cancelled checks and a bank's list of withdrawals. Tell the children that people make checks only when they want to take out money on deposit in the bank. Also explain the convenience of paying bills with checks.

Page 10

New Matter. Multiplication. Definition of multiplier, multiplicand, product. Sign of multiplication, the multiplication table.

Remarks. The first sentence in section 10 should be regarded as descriptive matter to be read and not as a definition to be memorized. Proceed with multiplication combinations, as suggested in the note to page 3 on addition combinations. The combinations used in the addition tests may be used here, and also the form suggested on page 301 for recording errors. The only difference is that the products instead of the sums are to be written.

The relation between addition and multiplication should be brought out clearly, as suggested in the example on this page.

Page 11

New Matter. Definitions of a unit, concrete and abstract numbers. Statement about numbers to be used as multipliers and multiplicands.

Remarks. Do no such impractical thing as multiplying 4 by 274 to get the result in the example cited on this page. The simple fact is that both the 4 and the 274 are concrete numbers. The most sensible thing to do is to regard both numbers as abstract for the purpose of computation, and then to call the result dollars for reasons that we all know. In this case the problem specifies that the result is to be in terms of dollars, and there is no need for elaborate reasoning to determine what is already given.

Page 12

New Matter. Multiplication by such numbers as 27, 400, and so on.

Remarks. It should be understood that the work on this page is preparatory to multiplying by numbers containing two or more figures. In multiplying, for instance, by 264, we multiply by 4, then by 60, and then by 200. Note that the explanation goes a little more into detail here than in book one. Thus the process is illustrated by showing that instead of multiplying by 6 we multiply by 3 and then by 2. This intensification of explanation as the work goes on from year to year, is one of the cardinal elements in a good course of arithmetic. To deepen the understanding of the work already done is one of the essential elements of every review.

Page 13

Review and Drill. Ordinary multiplication.

Remarks. If the teacher thinks best, the following explanation may now be made of long multiplication. The explanation should be treated precisely as the explanation suggested in the notes to pages 6 and 7 on subtraction.

$$\begin{array}{rr} & 6492 \\ & \underline{32} \\ 2\times6492= & 12984 \\ 30\times6492= & \underline{194760} \\ & 207744 \end{array} \qquad \begin{array}{rr} & 4502 \\ & \underline{207} \\ 7\times4502= & 31514 \\ 200\times4502= & \underline{900400} \\ & 931914 \end{array}$$

Pages 14 and 15

Applications. Canning fruit and vegetables, and preserving eggs.

Remarks. These pages were taken from actual cases, including the prices and amounts.

Supplementary Matter. Get local data and make problems of this kind. The problems need not, of course, be confined to the articles given on these pages. Make problems on things which are of interest in the community and which come within the general idea of preserving for the winter.

Page 16

New Matter. Definition of division, dividend, divisor, quotient.

Review and Drill. Short division.

Remarks. Go over again parts of the multiplication table, leaving out one of the factors, but giving the products. In this way connect division with multiplication. Note that the definition of division given here applies directly to this work. It is interesting that in more advanced mathematics this definition of division is practically always used, and that at the same time it fits in most closely with the needs of the small learning child.

Page 17

New Matter. Dividing by such numbers as 80, 600, and so on. Estimating quotients.

Remarks. Example 2 may be explained by saying that we divide 600 by dividing by 100 and then by 6. The process may be made clear by using cancelling as shown on page 239 of book one. In this process care must always be taken to get the right remainder. That is the most serious objection to this short method.

Emphasis should be placed on the process of estimating the quotient. It may be necessary to give more examples than are furnished in the text. In this case the teacher may make selections from among the following and write them on the board. In explaining the process the teacher may make use of page 234, book one.

$$\begin{array}{rrrrrrrr}
8 & 7 & 5 & 5 & 6 & 2 & 8 & 7 \\
47\overline{)398}, & 54\overline{)416}, & 57\overline{)324}, & 83\overline{)418}, & 76\overline{)476}, & 94\overline{)274}, & 73\overline{)614}, & 67\overline{)487}, \\
6 & 4 & 7 & 4 & 4 & 9 & 5 & 5 \\
88\overline{)575}, & 38\overline{)189}, & 75\overline{)589}, & 45\overline{)189}, & 49\overline{)244}, & 73\overline{)667}, & 58\overline{)335}, & 87\overline{)448}, \\
6 & 7 & 5 & 6 & 3 & 8 & 9 & 5 \\
74\overline{)467}, & 78\overline{)546}, & 37\overline{)193}, & 46\overline{)312}, & 85\overline{)286}, & 77\overline{)664}, & 53\overline{)514}, & 34\overline{)187}, \\
6 & 8 & 5 & 5 & 7 & 8 & 8 & 9 \\
43\overline{)297}, & 84\overline{)675}, & 93\overline{)472}, & 47\overline{)276}, & 78\overline{)576}, & 58\overline{)476}, & 47\overline{)398}, & 64\overline{)593}.
\end{array}$$

Page 18

New Matter. Division by three-figure numbers.

Remarks. Again practice finding quotient figures, by using examples like those at the bottom of the page. The teacher may select further examples from those given here. In explaining the process, the teacher may make use of page 252 in book one.

$\begin{array}{r}8\\574\overline{)4978}\end{array}$,	$\begin{array}{r}7\\693\overline{)5476}\end{array}$,	$\begin{array}{r}4\\519\overline{)2567}\end{array}$,	$\begin{array}{r}9\\837\overline{)7546}\end{array}$,	$\begin{array}{r}6\\657\overline{)4193}\end{array}$,
$\begin{array}{r}5\\435\overline{)2197}\end{array}$,	$\begin{array}{r}7\\374\overline{)2937}\end{array}$,	$\begin{array}{r}5\\478\overline{)2542}\end{array}$,	$\begin{array}{r}9\\748\overline{)6934}\end{array}$,	$\begin{array}{r}8\\852\overline{)6938}\end{array}$,
$\begin{array}{r}6\\391\overline{)2487}\end{array}$,	$\begin{array}{r}8\\584\overline{)4762}\end{array}$,	$\begin{array}{r}8\\752\overline{)6475}\end{array}$,	$\begin{array}{r}5\\674\overline{)3469}\end{array}$,	$\begin{array}{r}6\\841\overline{)5200}\end{array}$,
$\begin{array}{r}9\\216\overline{)1962}\end{array}$,	$\begin{array}{r}8\\639\overline{)5467}\end{array}$,	$\begin{array}{r}8\\246\overline{)2124}\end{array}$,	$\begin{array}{r}8\\852\overline{)6934}\end{array}$,	$\begin{array}{r}6\\679\overline{)4560}\end{array}$.

Page 19

Review and Drill. Long division of numbers representing money.

Remarks. Make no effort to explain decimals here. Go over again the long division steps as enumerated on page 232 of book one. The child should now be able to explain long division to the extent of describing the steps, one by one, as he goes through the work.

Standard Tests in Speed and Accuracy

During the last two decades many tests have been made by different investigators to ascertain the standard speed at which children in each grade can perform the fundamental operations of arithmetic. The speed tests suggested here are based on the general conclusions thus far obtained.

Some recent investigators have urged that efforts be made to bring all the children up to the mean standard of speed, and that no effort be made to raise a child above that standard. This is certainly questionable advice. It is believed that at a given stage

of development there is a speed at which each child does his best work. This speed may be called the optimum speed for that child. If he works below that speed he not only wastes time, but is more liable to error, whereas if, at this stage of development, he tries to work at a higher speed he again makes a larger number of errors. Two children of the same age and of normal intelligence may have very different optimum speeds. If the policy is adopted of letting children with high optimum speeds work at the so-called standard speed, they will be left far below their optimum speed. In this way the child with a natural high speed may actually do less accurate work than the child whose optimum speed coincides with the standard speed. On the other hand, it is practically impossible to bring the naturally slow child up to standard speed.

These tests are so timed that working at the standard speed half of a normal class should finish correctly half or more of the work assigned.

The tests given here for the beginning of the fifth grade are the standard for the fourth grade. The tests given for the middle of the fifth grade are the standard for that grade, and tests given for the end of the fifth grade are the standard tests for the sixth grade.

Similarly the tests given for the beginning of the sixth grade are the standard for the fifth grade, the tests for the middle of the sixth grade are the standard for that grade, and the tests for the end of the sixth grade are the standard tests for the seventh grade.

The child slows down considerably during the vacation, and should attain a standard somewhat beyond his grade by the end of the year.

The tests should be mimeographed or copied on sheets in such a manner so that the children can perform all the operations on the one sheet and without copying.

Place these sheets face down on the desks, and have the children turn them up and go to work at a given signal. Time the children carefully, seeing to it that they all stop promptly and turn down their papers. In scoring count the correct results only.

Test in addition. First test, grade five. Time 6 minutes.
Half of the class should have 10 or more correct answers.

1.	2.	3.	4.	5.
32	61	531	73	891
48	473	21	453	435
672	19	49	19	75
89	64	67	28	23
347	478	134	268	57
48	12	57	15	47
1236	1107	859	856	1528

6.	7.	8.	9.	10.
892	691	765	612	543
24	61	37	92	75
78	374	146	37	89
319	14	23	169	26
78	67	69	45	325
25	95	51	18	24
1416	1302	1091	973	1082

11.	12.	13.	14.	15.
798	357	43	298	960
28	46	639	24	24
570	95	28	16	18
68	16	59	38	69
34	81	46	395	373
53	269	523	15	98
1551	864	1338	786	1542

16.	17.	18.	19.	20.
32	801	645	571	476
18	36	92	364	39
69	87	346	22	40
268	51	23	48	36
55	495	48	36	792
893	63	75	73	75
1335	1533	1229	1114	1458

Test in subtraction. Time 6 minutes.

Half of the class should have 20 or more correct answers.

1. 6984	2. 7696	3. 4086	4. 6491	5. 2936
2537	3478	2832	2357	1852
4447	4218	1254	4134	1084
6. 7045	7. 6451	8. 3527	9. 3261	10. 6957
5832	2380	1465	1930	2738
1213	4071	2062	1331	4219
11. 2481	12. 6271	13. 3192	14. 4138	15. 7468
1279	3186	2931	1427	3286
1202	3085	261	2711	4182
16. 8765	17. 7392	18. 4056	19. 8219	20. 4265
2537	2721	3049	3172	3641
6228	4671	1007	5047	624
21. 5376	22. 8932	23. 4867	24. 4217	25. 6738
2734	3791	3548	2806	3419
2642	5141	1319	1411	3319
26. 4625	27. 3761	28. 1982	29. 5967	30. 9241
2374	2459	1734	3783	5720
2251	1302	248	2184	3521
31. 9216	32. 9298	33. 6721	34. 1745	35. 6089
3804	5139	2571	1594	4573
5412	4159	4150	151	1516
36. 3745	37. 3264	38. 5939	39. 1827	40. 5473
2561	3072	2684	1518	3541
1184	192	3255	309	1932

Test in multiplication. Time 6 minutes.

Half of the class should have 10 or more correct answers.

1. 92	2. 348	3. 72	4. 759	5. 34
37	47	75	16	52
3404	16356	5400	12144	1768
6. 628	7. 45	8. 316	9. 57	10. 209
27	93	57	34	73
16956	4185	18012	1938	15257
11. 75	12. 546	13. 92	14. 283	15. 42
98	37	16	54	67
7350	20202	1472	15282	2814
16. 605	17. 54	18. 378	19. 32	20. 469
37	39	57	17	48
22385	2106	21546	544	22512

Test in division. Time, 6 minutes.

Half of the class should have 5 or more correct answers.

60–15	90–2	87–17
1. 31)1875	2. 21)1892	3. 41)3584
62–2	46–43	48–38
4. 61)3784	5. 51)2389	6. 81)3926
64–52	114–14	76–11
7. 71)4596	8. 31)3548	9. 71)5407
87–69		
10. 91)7986		

Page 20

Applications. The cost of heating and lighting a house.

Remarks. Actual data should be obtained from your own neighborhood, as suggested in example 8.

Page 21

Applications. Buying things for boys and girls.

This again affords a good opportunity for local problems, and the children will take great interest in bringing in data and in helping to formulate the problems. Opportunities of this sort should never be permitted to pass without being used to their fullest possibility.

Page 22

New Matter. Definition of even and odd numbers. The divisibility of numbers.

Remarks. The children will like to play with the tests given on this page. Give them a number which is divisible by 3, for instance; ask them to find whether or not it is divisible by 3, and then have them divide to see whether or not the test really works.

Page 23

New Matter. Comparison of multiplication and division. Grouping and partitioning.

Remarks. This connecting of the processes of multiplication and division is of fundamental importance, both as a matter of pure arithmetic and for the purpose of applying the arithmetic to the solution of problems. The definition of division must be made a real living thing to the child.

There is no need of emphasizing the distinction given in section 22.

Pages 24 and 25

Review and Drill. The area of a rectangle.

Remarks. Pages 24 to 29 inclusive contain what is perhaps the most far-reaching idea connected with the solution of problems in arithmetic. The foundation for the development of this idea has been laid all through book one. The development of the multiplication table, and the division within the table

has been made precisely with this final purpose in mind. If the teacher sees fit she may draw on the board a rectangle 5 units long and 3 wide, and then show that by taking one-third of the area we have 5 squares, and by taking one-fifth of the area we have three squares. At this point read pages 52, 99, 123, 133 of book one.

One important point to bring home is the one on page 25, namely, that the length and width must be given in the same unit, and that if that is done the area will be found in terms of a square unit whose side is the unit of length used in measuring the dimensions of the rectangle. Anything beyond this in endeavoring to logicalize the situation is vicious and leads to nonsensical performances which people forget as soon as they are outside the schoolroom.

It is perfectly possible to introduce the algebraic symbol x at this point, and to make more formal use of the algebraic equation throughout the course wherever the principle of product and factors is now used.

Pages 26 and 27

Remarks. Remarks made about pages 24 and 25 apply here. Whenever trouble arises with problems of this kind, refer to the fundamental statement that "price multiplied by number of things bought equals cost." Then the solution of the problem should follow directly.

Pages 28 and 29

Applications. Speeds and distances.

Remarks. The idea of speed and distance and their relation to time is usually hazy in the child's mind, and frequently in the mind of the adult. The idea is, however, essentially simple, and if developed in some such way as suggested on page 28 will present no serious difficulty. The haziness is due to the fact that the attention has never been focused directly on the problem. Without doubt, it is the duty of the school to develop in the child's mind

definite ideas on this subject, and there would seem to be no more opportune point at which to do it than precisely at this stage in arithmetic.

Instead of organizing a separate analysis for each problem as it arises, it is proposed that they be grouped together under the one dominating idea that each one involves three numbers, one of which is the product of the other two. It is a fundamental law that whenever an idea is applied to many and diverse situations it is thereby clarified and the memory of it made much more permanent. The teacher and the supervising officers should conceive it as a part of their duty to so arrange the course in arithmetic that it shall be grouped about a small number of fundamental ideas. In this way each of these fundamental ideas will become clear and definite, and the child will have some milestones to which he can refer with confidence and certainty.

Note that the essentially arithmetic difficulty in all of the problems on these pages is precisely that which arises when any one of the three numbers in $3 \times 4 = 12$ is omitted.

If the teacher prefers, however, she may give an analysis of the problems on pages 26 to 29 in such manner as the following:

Problem: A man bought 175 head of sheep for $2450. How much did he pay per head for the sheep?

Analysis: If he paid $2450 for 175 sheep, then one sheep cost as many dollars as $175 is contained times in $2450. Hence to find the cost per sheep in dollars divide 2450 by 175.

At $56 per acre how many acres can be bought for $7840?

Analysis: If one acre cost $56, then the number of acres which can be bought for $7840 is equal to the number of times $56 is contained in $7850. Hence, to find the number of acres, divide 7840 by 56.

How long does it require a train going 45 miles an hour to go 270 miles?

Analysis: If a train goes 45 miles in one hour, then the number of hours in which it will go 270 miles is equal to the number of times 45 is contained in 270. Hence divide 270 by 45.

The Pennsylvania limited goes from New York to Chicago in 20 hours. What is its average speed if the distance is 912 miles?

Analysis: If the train goes 912 miles in 20 hours then its average speed in miles per hour is equal to the number of times 20 is contained in 912. Hence, divide 912 by 20.

Such analyses are usually given of problems like these. One cannot escape the conclusion that these analyses contain little more than directions for solving the problem. To say that the average speed in miles per hour is equal to the number of times 20 is contained in 912 is simply a repetition in different words of the conclusion that follows it, namely: "divide 912 by 20."

It is believed that instead of really clarifying the situation such formal sounding statements usually bring confusion. Too often they are but as "sounding brass and a tinkling cymbal."

Periodical Examinations.

In many grade schools examinations are given periodically. Perhaps the most usual period between such examinations is six weeks. For the convenience of teachers material for examinations is given at appropriate intervals in these notes. The purpose in putting in this material is to save the teacher the labor of arranging balanced material and the work of obtaining the results of the examples. The teacher who spends her time in unnecessary labor is, in part, robbing the school of that freshness and energy for essential work, to which it is entitled.

The concensus of opinion now seems to be that in any examination enough material should be provided to keep the fastest child at work during the whole period.

The amount of material given here is of course much greater than can be used in a single examination. By marking the examples used in any one year, a variety of examinations may be provided from year to year.

The teacher should of course select from this material examples for each test so as to make it cover the proper topics.

Material for First Examination, Fifth Year.

Add:

1. 876	2. 391	3. 519	4. 607	5. 715	6. 493
498	648	648	194	913	286
265	550	710	842	778	936
937	739	434	956	546	274
910	439	567	104	217	516
146	218	198	326	486	128
3632	2985	3076	3029	3655	2633

Subtract:

7. 56014	8. 310916	9. 45315	10. 62210	11. 24819	12. 71531
23728	122348	27128	29654	16074	48779
32286	188568	18187	32556	8745	22752

Multiply:

13. 1849	14. 5683	15. 6094	16. 7845	17. 9124	18. 8593
631	217	832	476	715	487
1166719	1233211	5070208	3734220	6523660	4184791

Find Quotients and Remainders:

60–744	33–490	96–18	78–398
19. 817)49764	20. 624)21082	21. 408)39186	22. 819)64280

23. A man had \$206.40 in the bank and drew out \$41.20, \$19.75 and \$56.80. How much did he have in the bank then? **\$88.65**

24. A boy bought a pair of skates for \$2.75, a sweater for \$5.50 and a cap for \$1.25. What was his change from a ten-dollar bill? **\$.50**

25. A school board bought 45½ tons of coal at \$6.30 a ton. How much did this coal cost? **\$286.65**

26. A farmer harvested 1426 bu. of corn from a field containing 23 acres. What was the yield per acre? **62 (bu. per acre)**

27. A train goes 918 miles in 18 hours. What is its average speed in miles per hour? **51 (miles per hour)**

28. A fast steamship sailed 1175 miles in 47 hours. What was its average speed per hour? **25 (miles per hour)**

29. A lot 45 feet wide contains 4320 square feet. What is its depth in feet? **96 (feet)**

30. At 35 cents a square foot what is the value of a lot 30 feet wide and 115 feet deep? **$1207.50**

31. How many acres are there in a farm 180 rods by 240 rods? **270 (A)**

32. A farmer sold 46 head of cattle for $4370. What was the average price per head? **$95**

33. At 18 miles per hour how many hours of actual driving will be required to go 2682 miles? **149 (hrs.)**

34. If a boy spends 5 hours per day in school how many hours will he spend in school in 6 school years of 195 days each? **5850 (hrs.)**

Page 30

New Matter. Description of games which may be used in drills in fundamentals.

Remarks. It is believed that the games described on this page, and especially the last two, are as nearly ideal number games as can be made. They keep all the children busy with arithmetic all the time that is devoted to the drill; they bring into definite play the great motivating forces of competition, of loyalty to a group, and of the interest which one individual of a group will take in other individuals of the same group.

Page 31

Review and Drills. Addition, multiplication, and long division.

Pages 32 and 33

New Matter. Material for drills in the fundamentals.

Remarks. Nothing should be done with the material on these pages at this point. Suggestions will be made throughout the book for drills to be given from time to time in which these numbers will be used.

Page 34

New Matter. Simple fractions of a unit.

Remarks. The children, of course, know everything that is given on this page. The purpose here is to recall these matters in order to make them stand out more clearly, and thus to have the material well in hand for the work that is to follow. One of the points to be attended to is that the general idea of a fraction should be obtained from fractions of many different concrete things. Here, as elsewhere, simple illustrations are the best.

Page 35

Review and Drill. Fractions of a group of things.

Remarks. The child experiences no difficulty at this stage in understanding the meaning of a unit fraction, either of one whole or of a group. The ideas of one-half, one-third, one-fourth, and so on, are perfectly clear to him by this time. There is no need to distinguish sharply between a fraction of a single object or unit and a fraction of a group of things. The child is perfectly acquainted with both, and attempts to analyze and classify will simply muddle him.

A different situation arises, however, when we come to consider such fractions as two-thirds of a number. Such a fraction involves essentially a two-step idea; namely, first, one-third of a thing or number, then twice one-third. The child knows perfectly well that one-third of twelve is four, but it is much more difficult for him to see that two-thirds of twelve equals eight. The trouble arises from the fact that he does not know the way his thought should go. He tries to get two-thirds of twelve directly, instead of going by the way of one-third first, and then two-thirds. A little suggestion and drill will clear up the whole matter.

Pages 36 and 37

New Matter Definition of a fraction, numerator and denominator.

Review and Drill. One-half, one-fourth, one-eighth, one-third, one-sixth, and one-twelfth.

Remarks. Supply other material upon which to base the concept of a fraction. The teacher may draw a certain number of marks on the board and draw a line through one-half of them, or one-third of them, or one-fourth of them, and so on. She may direct that the children draw, say, twelve marks on a piece of paper, and then draw a line through a certain fraction of them, such as one-half, one-third, and so on. Such marks may also be made on the board as a matter of instruction to prepare for drawing them on the paper. The point is that a variety of things should be made the basis of the concept of a fraction. The children may point out one-half of a group of children, one-third of them, and so on; one-half of a group of books, one-third of a group, one-fourth, and so on.

In actual practice only very simple fractions are used, but these should be grasped thoroughly.

Pages 38 and 39

New Matter. The terms of a fraction may be multiplied or divided by the same number without changing the value of the fraction.

Remarks. This fundamental property of fractions should be developed thoroughly. The order of procedure is the following: First, a body of individual concrete facts, such as, one-half equals two-fourths, one-third equals two-sixths, or three-fourths equals six-eighths. Next, the generalization that the terms of any fraction may be multiplied by 2, without changing its value. Then we learn in the same way that the terms of a fraction may be multiplied by 2 without changing its value, and so on; and finally, that the terms of a fraction may be multiplied by any number whatsoever without changing its value.

The next step is to formulate this general truth into a verbal statement. That is the final achievement and the most difficult.

Page 40

New Matter. Factors of numbers. Common factors. Fractions in lowest terms.

Review and Drill. Reduction of fractions to their lowest terms.

Supplementary Matter. Drill in Fundamentals. Multiply A I, II by E I, II, page 32. The products in order, are:

1,464,892; 1,285,268; 703,710; 2,974,678; 1,645,300; 822,984; 1,054,248; 3,921,944; 5,383,396; 4,977,553.

Page 41

New Matter. Proper and improper fractions. Mixed numbers.

Remarks. In reducing an improper fraction such as $\frac{72}{4}$ to an integer two explanations may be used: First one may think of $\frac{72}{4}$ as 72 divided by 4, in which case the reduction is performed by simply carrying out the indicated division. A second explanation is the following: Since 1 contains 4 fourths, there are as many ones in 72 fourths as 4 is contained times in 72; hence divide 72 by 4.

Page 42

New Matter. Addition of fractions with a common denominator, and of two fractions such that one of them can be reduced to the same denominator as the other.

Remarks. The adding of fractions having the same denominator is very simple, and the child will not have the least trouble doing it. It should be pointed out that we cannot add such fractions as $\frac{1}{2}$ and $\frac{1}{4}$ directly. The child will see that quickly enough. The only difficulty that he has in adding these fractions is in getting the sequence of the steps. He knows well enough that $\frac{1}{2}$ is equal to $\frac{2}{4}$, and the only trouble is to get the idea that the half must be reduced to fourths. However, he will get on easily with problems of the kind given on this page. A rather slight hint should be sufficient. Do not over-explain. Give the child a chance to do something for himself. The child is not a very good spectator, but he is a very good participant where he is able to participate.

Page 43

New Matter. Addition of fractions whose common denominator is different from the denominator of any one of the fractions to be added.

Remarks. In introducing this page, ask questions like the following: Can we reduce one-half to thirds? To fourths? To fifths? To sixths? To sevenths? Eighths? and so on. Can we reduce one-third to fourths? To fifths? To sixths? and so on. Do not at this stage attempt to develop the whole machinery of multiples. The child will learn quickly to what kind of fractions a given fraction may be reduced, and that one-third and one-fourth can both be reduced to twelfths, but not to a fraction having a lower denominator. The child should be allowed to work himself into the situation by trying various denominators to see if they will work, and then by making an effort to try the right one first. Thus some children may use 48 as the common denominator of $\frac{1}{6}$ and $\frac{1}{8}$, while others will use 24, which, of course, is better. Do not attempt to force the whole situation on the children. Let them do a little work on their own responsibility. They will like it, and they will be learning a lot about fractions as they do it.

Page 44

Applications. Parts of a foot. An hour, a day, a year, a school year.

Supplementary Matter. Try to get the children to bring in other material that can be treated in this manner, such as: the number of quarts in a peck, the number of pecks in a bushel, and in fractions of a bushel, the number of cents in a dollar and in fractions of a dollar, the number of inches in a yard and in fractions of a yard, and so on.

For the drill in fundamentals, divide the numbers in B I, II by the numbers in E I, II, page 32. The results, in order, are:

1675–21; 871–20; 297–37; 1486–23; 1812–2; 1358–23; 485–11; 457–22; 1330–36; 734–43.

Page 45

Applications. Miscellaneous problems. Addition of fractions.

Remarks. Note problems 3 to 8. The check suggested by solving the problems differently will prove of real interest. The children will like to see that they get the same answer in two entirely different ways.

Supplementary Matter. The children will be interested in trying to make other problems like these. They may be asked to try to find entirely different situations where problems of this sort may be made, though they are not likely to succeed in doing so. It will, however, be of real educational value to try it. It is also likely that they will suggest the adding of one-half and, say, one-fifth of a foot. That, of course, will get them into difficulty when they try to verify by the method used in example 5. That, too, will be instructive, as it will again raise the question as to what denominators one-half and one-fifth may be reduced. Do not forget that we often learn more by trying to do things which will not come out the way we expect them to than we do by solving simple problems that come out without any difficulty whatever.

Pages 46 and 47

Remarks. These pages contain material which will be used for drills in fundamentals from time to time throughout the book. At this time do nothing whatever with this material. Page 47 contains decimals, but these will not be used until the decimals have been formally introduced. The material is placed here for the sake of convenience in arranging the pages.

Page 48

New Matter. Multiples of a number. The least common multiples of numbers.

Remarks. The work on fractions up to this point forms a background for the work on this page, and may have created a need for it. Note the method used in the example solved on this page.

The great majority of cases in actual practice where it is necessary to find the least common multiple are so simple that the multiple may be found directly by inspection, or informally by the method suggested in this example. After some practice the series of multiples of 20 would not be written down. One thinks of 40 and sees that that is not a multiple of 6 or 8; then 60, then 80, and so on, and finally one reaches 120, which is at once seen to be a multiple of both 8 and 6. The lowest common multiple is, of course, needed in working with fractions, but nearly all the problems that occur can be solved mentally. Many children are taught in such a way that they find the common denominator of one-half, one-third, and one-fourth by an elaborate and cumbersome method, which is needed only at the very rarest intervals, and then only by persons who are much more skillful in arithmetic than ordinary people.

Page 49

New Matter. The least common denominator.

Remarks. The example solved on this page shows the method which may be used, but even this formal method will soon be discarded. It is very seldom that an example as complicated as this one occurs in practice. In exercises like the first one at the bottom of the page, the child will see readily that the common denominator is 24: that $\frac{5}{6}$ equals $\frac{20}{24}$, that $\frac{3}{8}$ equals $\frac{9}{24}$, and that $\frac{7}{12}$ equals $\frac{14}{24}$. These should then be written down directly, as follows:

$$\begin{aligned} \tfrac{5}{6} &= \tfrac{20}{24} \\ \tfrac{3}{8} &= \tfrac{9}{24} \\ \tfrac{7}{12} &= \underline{\tfrac{14}{24}} \\ & \ \tfrac{43}{24} = 1\tfrac{19}{24} \end{aligned}$$

Our aim should be to get along with the least formal machinery that will serve us. The less formal machinery we have, the more will our thoughts be engaged on the process, and the more rapidly we shall get on.

Supplementary Matter. Drill in Fundamentals. Add A, B, C, D of I, and E, F, G of I, page 46.

The first example is $\frac{7}{8}+\frac{3}{4}+\frac{1}{2}+\frac{2}{5}$. The sums, in order, are:

$2\frac{21}{40}$; 2; $2\frac{1}{24}$; $3\frac{7}{48}$; $3\frac{1}{24}$; $\frac{17}{24}$; $1\frac{5}{16}$; $1\frac{31}{144}$; $\frac{185}{288}$; $1\frac{87}{288}$.

Page 50

New Matter. Addition of mixed integers and fractions.

Supplementary Matter. If there is a class in sewing in which there are members of the fifth grade, they may be able to find some good examples that the class can solve. Either have the pupils bring in the material from their work or get it directly from the sewing teacher.

Remarks. The addition of mixed numbers is very easy and requires no particular explanation. Note the form of the example on this page. Instead of writing that $1\frac{1}{2}=1\frac{5}{10}$ simply write the fractions reduced to the common denominator in a column without using the equality sign. That amounts simply to the separate consideration of the fractions. After they have been added the integers are added independently and the two are taken together.

Solution of Problems

Example 12. Add $1\frac{1}{4}+1\frac{1}{4}+1\frac{1}{3}+1\frac{1}{3}$. Avoid multiplication of fractions in solving this problem.

Pages 51 and 52

New Matter. Subtraction of simple fractions.

Remarks. Note the sequence of difficulties on these pages as exemplified by the following: $\frac{3}{8}-\frac{1}{8}$; $\frac{1}{2}-\frac{1}{4}$; $\frac{1}{2}-\frac{1}{3}$. In the first example the two fractions have the same denominator. In the second example one fraction may be reduced to the same denominator as that of the other fraction. In the third the common denominator is different from either of the given denominators. There is only a very slight amount of difficulty here, inasmuch as similar problems have already been solved in addition.

Page 53

New Matter. Reduction to improper fractions.

Supplementary Matter. Play game No. 3, page 30. For the first event add II, III A to E and IV, V A to E, page 32. The sums, in order, are:

536,856; 459,945; 43,984; 4,676; 548; 685,309; 511,294; 59,437; 5550; 655.

For the second event subtract II, III B from II, III A, page 32. The results in order, are:

1676; 8106; 6280; 12,738; 4978; 4442; 14,761; 802; 15,418; 7710.

For the third event multiply II, III C by II, III D, page 32. The products, in order, are:

1,492,020; 371,664; 3,402,720; 2,552,352; 2,133,912; 2,875,194; 905,412; 2,073,280; 923,738; 1,535,739.

Pages 54 and 55

New Matter. Subtraction of mixed numbers.

Remarks. Note the sequence of difficulties as they are introduced on these two pages. Let the child face each difficulty separately. These difficulties are typified by the examples: $12\frac{3}{8}-8$; $2-\frac{3}{4}$; $3\frac{3}{4}-1\frac{1}{4}$; $4-1\frac{4}{5}$; $5\frac{1}{3}-2\frac{1}{4}$; $3\frac{1}{3}-1\frac{1}{2}$.

Note the two methods used in solving example 2 on page 54 and example 2 on page 56. These two methods are, of course, analogous to the two methods of subtraction used throughout these books. The first is the additive or Austrian method, and the second is the usual taking-away-borrowing method.

Material for Second Examination, Fifth Year.

Add:

1.	2.	3.	4.	5.
8176	9786	7137	5810	6182
4391	3940	2648	9726	2139
6580	1725	3917	3192	1913
9834	9753	2120	2256	7121
1810	3246	3716	4161	2350
4916	3908	1327	1902	7136
35707	32358	20865	27047	26841

Subtract:

6.	7.	8.	9.	10.
191406	410641	67246	520461	719360
63782	127319	32892	247839	367829
127624	283322	34354	272622	351531

Multiply:

11.	12.	13.	14.	15.
4932	2864	1392	5240	7193
876	372	468	761	678
4,320,432	1,065,408	651,456	3,987,640	4,876,854

Divide:

16.	17.	18.
650–200	584–94	849–348
$288\overline{)187400}$	$739\overline{)431670}$	$684\overline{)581064}$

Add:

19.	20.	21.	22.	23.
$8\frac{1}{2}$	$6\frac{3}{4}$	$24\frac{1}{5}$	$14\frac{3}{8}$	$67\frac{1}{2}$
$4\frac{2}{3}$	$5\frac{1}{2}$	$17\frac{1}{2}$	$47\frac{3}{4}$	$49\frac{1}{5}$
$5\frac{1}{4}$	$8\frac{1}{8}$	$14\frac{1}{4}$	$18\frac{1}{2}$	$27\frac{1}{3}$
$18\frac{5}{12}$	$20\frac{3}{8}$	$55\frac{19}{20}$	$80\frac{5}{8}$	$144\frac{1}{30}$

Subtract:

24.	25.	26.	27.	28.
15	$12\frac{1}{3}$	$8\frac{1}{4}$	$16\frac{1}{2}$	$47\frac{3}{8}$
$3\frac{1}{2}$	$6\frac{1}{4}$	$4\frac{1}{3}$	$12\frac{2}{3}$	$24\frac{3}{4}$
$11\frac{1}{2}$	$6\frac{1}{12}$	$3\frac{11}{12}$	$3\frac{5}{6}$	$22\frac{5}{8}$

29. A storekeeper had $38\frac{3}{8}$ yards of cloth in a bolt, and sold $17\frac{3}{4}$ yards of it. How much did he have left? $20\frac{5}{8}$ (yds.)

30. A storekeeper sold $8\frac{1}{4}$ yards, $3\frac{3}{8}$ yards, and $10\frac{1}{2}$ yards of cloth from the same piece. How many yards did he sell? $22\frac{1}{8}$ (yds.)

31. The distance between two stations on a railway is $18\frac{7}{8}$ miles. A station situated between them on this railway is $11\frac{1}{4}$ miles from one of these stations. How far is it from the other? $7\frac{5}{8}$ (miles)

32. A boy grew $6\frac{3}{8}$ inches in three years. The first year he grew $2\frac{1}{4}$ inches, the second $1\frac{7}{8}$ inches. How much did he grow the third year? $2\frac{1}{4}$ (ins.)

33. A man bought a house for $3200, spent $360 for improvements on it, and sold it for $4000. What was his gain? $440

34. A man bought an automobile for $1450, spent $280 on it, and sold it for $1525. What was his loss? **$205**

35. A man had $84.60 in the bank, put in $250 more, and then drew out $181.80. How much did he have in the bank then? **$152.80**

36. A boy has $32.60 in his savings bank. He then puts in $1.20, $2.45, and $3.15. How much does he have in the bank then? **$39.40**

37. A girl earns $1.20, $1.45, $2.10, $3.75, and $2.85 during vacation. She spends $4.65 of this money and puts the rest of it into the bank. How much does she put into the bank? **$6.70**

38. A boy had $47.25 in the Savings Bank, and drew out enough to pay for the following: an outing suit for $6.50, a pair of shoes for $4.75, and a hat for $1.85. How much did he have left in the bank? **$34.15**

39. A girl spent $\frac{3}{4}$ hour each day on language, $\frac{2}{3}$ hour on arithmetic, and $\frac{1}{2}$ hour on reading. How many hours did she spend on those three subjects in one day? **$1\frac{11}{12}$ (hrs.)**

Page 56

Review and Drill. Addition and subtraction of mixed numbers.

Supplementary Matter. For the drill in fundamentals, questions such as the following may be asked:

1 ft.	= how many in.?	1 acre	= how many sq. rds.?
1 yd.	= how many ft.?	1 mile	= how many rds.?
1 yd.	= how many in.?	1 dollar	= how many cents?
1 wk.	= how many da.?	1 dollar	= how many dimes?
1 da.	= how many hrs.?	1 sq. yd.	= how many sq. ft.?
1 hr.	= how many min.?	1 dollar	= how many quarters?
1 min.	= how many sec.?	1 year	= how many days?
1 lb.	= how many oz.?	1 pk.	= how many qt.?
1 qt.	= how many pt.?	1 bu.	= how many pk.?
1 gal.	= how many qt.?	1 bu.	= how many qt.
1 doz.	= how many units?	1 yr.	= how many months?
1 ton	= how many lbs.?	1 quarter	= how many cents?

Page 57

Applications. Miscellaneous problems.

Supplementary Matter. Find what fractions of a yard of cloth are usually sold in your town and then make problems like this: In a bolt of cloth there were $54\frac{1}{2}$ yards, and $1\frac{1}{4}$ yards, $3\frac{1}{2}$ yards, $4\frac{7}{8}$ yards were sold. How many yards were left?

Many problems of this sort may be made.

Solution of Problems

Problem 1. To find the length of the picture including the frame, add $\frac{5}{8}+\frac{5}{8}=1\frac{1}{4}$ to the width. Hence, width $=6\frac{1}{4}+1\frac{1}{4}=7\frac{1}{2}$ (inches).

Similarly, length $=8\frac{3}{4}+1\frac{1}{4}=10$ (inches).

The amount added to the length is $\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$ (in.) Hence the length is $6\frac{1}{4}+\frac{1}{4}=6\frac{1}{2}$ (in.)

$3\frac{1}{2}+3\frac{3}{4}=6\frac{5}{4}=7\frac{1}{4}$ (yards used).

$8\frac{1}{2}-7\frac{1}{4}=1\frac{1}{4}$ (yards left). This is one yard and nine inches.

$\frac{7}{8}+3\frac{1}{4}=3\frac{9}{8}=4\frac{1}{8}$ (thickness in inches). $4\frac{1}{8}-2\frac{1}{2}=3\frac{9}{8}-2\frac{4}{8}=1\frac{5}{8}$ (inches)

Pages 58 and 59

New Matter. Multiplication by integers. No cancelling.

Supplementary Matter. Drill in Fundamentals. The following exercises in multiplying a fraction by an integer may be used:

$7\times\frac{3}{4}$; $13\times\frac{5}{6}$; $9\times\frac{3}{4}$; $3\times\frac{4}{5}$; $5\times\frac{5}{6}$; $6\times\frac{3}{5}$; $7\times\frac{5}{8}$; $3\times\frac{7}{8}$; $8\times\frac{4}{9}$; $9\times\frac{7}{16}$; $12\times\frac{3}{5}$; $10\times\frac{3}{7}$; $15\times\frac{3}{4}$; $20\times\frac{2}{3}$; $10\times\frac{3}{11}$; $8\times\frac{4}{5}$; $12\times\frac{3}{5}$; $14\times\frac{2}{3}$; $4\times\frac{3}{5}$; $14\times\frac{2}{5}$; $7\times\frac{5}{6}$; $25\times\frac{1}{3}$; $18\times\frac{2}{5}$; $27\times\frac{1}{4}$; $36\times\frac{1}{5}$; $14\times\frac{1}{3}$; $50\times\frac{2}{3}$; $40\times\frac{4}{9}$; $11\times\frac{3}{4}$; $13\times\frac{7}{8}$.

Remarks. Note that the rule at the bottom of the page is developed by induction. That is, a few examples are given, the reason for the method used in solving them is shown; and then the general principle is stated without further detailed discussion. This is exactly the way that the young child learns. We must be

careful, however, to go on far enough to actually make the induction and to state the result in a definite formal manner. What is most of all to be avoided is to learn rules without the foundation which is supplied by the individual examples upon which they are based. In example 23, on page 59, there is a case where cancellation may be used, namely, in multiplying $2\frac{1}{8}$ by 6, but this need not worry us now. We shall learn about this later.

Pages 60 and 61

New Matter. Cancellation. Multiplying a mixed number by an integer.

Remarks. The fundamental property of a fraction is, that both its terms may be multiplied or divided by the same number, without changing its value. As we proceed with the various applications of this principle it is referred to repeatedly to make sure that it will be made to stand out as the essential principle in each case. Too often this simple principle is not brought out adequately.

Pages 62 and 63

Applications. Miscellaneous problems.

Supplementary Matter. If there is much canning or preserving of vegetables for the winter in the community, material for additional problems like those on page 63 may be obtained. Do not bother with such problems unless they are actually of local significance.

Page 62

Solution of Problems

1. Turning the screw around drives it $\frac{1}{8}$ of an inch into the wood. Hence in turning it 12 times it goes into the wood $12\times\frac{1}{8}=1\frac{1}{2}$ (inches).
3. $1000\times\frac{3}{5}=600$ (ounces).
   $600\div16=37\frac{1}{2}$ (pounds).

4. $120{,}000 \times \frac{13}{16} = 97500$ (pounds).
$97500 \div 2000 = 48.75$ (tons).

7. $30 \times 1\frac{5}{8} = 48.75$ (pounds).
$24 \times 48.75 = 1170$ (cost in cents). Hence, cost = \$11.70.

Page 63

Solution of Problems

6. 48 ft. = 16 yards, 38 ft. = $12\frac{2}{3}$ yards.
$16 \times 12\frac{2}{3} = 202\frac{2}{3}$ (sq. yd.).
Hence, cost is $202\frac{2}{3}$ dollars or \$202.67.

Pages 64 to 67 (inclusive)

New Matter. The products of two fractions.

Supplementary Matter. Drill in fundamentals. For the first event in the drill in fundamentals on page 65 add A, B, C of III, IV, V, page 46. The sums in order are:

$20\frac{49}{120}$; $16\frac{23}{24}$; $16\frac{1}{10}$; $17\frac{1}{24}$; $26\frac{13}{24}$; $37\frac{5}{6}$; $40\frac{5}{8}$; $17\frac{11}{24}$; $18\frac{23}{24}$; $19\frac{23}{36}$; $12\frac{31}{40}$; $13\frac{7}{24}$; $13\frac{1}{18}$; 17; $14\frac{17}{24}$.

For the second event, multiply G I, II, III, IV by K I, II, III, IV, page 46. The products, in order, are:

$\frac{1}{4}$; $1\frac{5}{16}$; $1\frac{7}{8}$; $1\frac{3}{4}$; $2\frac{2}{3}$; $4\frac{1}{2}$; $7\frac{1}{2}$; $4\frac{1}{2}$; $2\frac{2}{5}$; $5\frac{5}{8}$; $5\frac{1}{4}$; 3; 3; $1\frac{3}{4}$; $1\frac{1}{10}$; $2\frac{1}{2}$; 3; $3\frac{3}{4}$; $1\frac{1}{2}$; $2\frac{1}{2}$.

Remarks. Note the sequence of difficulties on these pages. They are typified by the following: $\frac{1}{2}$ of $\frac{1}{3}$. $\frac{1}{2}$ of $\frac{2}{3}$, problems of this sort by cancellation, $\frac{2}{3}$ of $\frac{4}{5}$, the final rule. Again, the rule is justified by a process of induction. The various possible cases are given and developed in such a way that the child will understand them separately. Then he will learn to understand that all of these culminate in the general rule. This basis for the rule is supplied, not because it is believed that the child will remember this inductive process, but because it is desirable that the child shall always feel that every rule is based upon a reasonable course of thought and not thrown at him out of the sky. Let us help the youngster keep his feet on the ground. In that way we may have some hope of developing sensible citizens and not merely dreamers.

Pages 68 and 69

Applications. Miscellaneous problems.

Remarks. In finding the distance between two points on the map of the township, measure to the nearest sixteenth of an inch, write down this distance as found on the map, and then by multiplying by the proper number find the actual distance.

Supplementary Matter. By the method suggested here it should now be possible to find very accurately the distances between any two points on a local map.

Page 70

New Matter. Products of mixed numbers.

Remarks. Note the solution of example 2 by the so-called four-step process. While this process is omitted in some texts it is used almost exclusively by business men when the numbers involved are fairly large. It is easy to give practical cases in which it would be hopelessly silly to reduce both multiplier and multiplicand to a proper fraction and then multiply. Thus, suppose we are expected to find the area of a lot $72\frac{3}{4}$ feet long by $52\frac{7}{8}$ feet wide. No sane person would use any other method than the four-step process in this case, provided he knows it.

Page 71

Review and Drill. For the game at the bottom of the page, multiply B I, II by D I, II, page 32.

The products, in order, are:

13,724,865; 3,230,106; 5,494,820; 5,762,082; 25,369,120; 10,567,260; 2,195,416; 16,398,108; 27,634,112; 41,084,532

Page 71

Solution of Problems

13. $6\times4\frac{2}{7}=24\frac{12}{7}=25\frac{5}{7}$ or 25.71 (dollars).
    31 days in $4\frac{3}{7}$ weeks.
    $6\times4\frac{3}{7}=24\frac{18}{7}=26\frac{4}{7}$ or 26.57 (dollars).

Pages 72 and 73

New Matter. Division of fractions.

Remarks. The method used in solving example 2 near the bottom of page 72 will serve to solve every problem in division of fractions. In common practice, however, the rule on page 73 is used, and what precedes is to be regarded as merely forming an inductive basis for it. To test the rule, solve several problems by means of it, and also by the method of example 2, page 72, to see that they yield the same results. This method for verifying results is perfectly conclusive to the child.

Page 74

New Matter. Special methods for dividing a fraction by an integer.

Review and Drill. Exercises in dividing fractions.

Supplementary Matter. For the drill in fundamentals, divide the numbers in D I, II, III, IV by E I, II, III, IV page 46. The results, in order, are:

$1\frac{1}{5}$; $1\frac{1}{3}$; 15; $5\frac{1}{4}$; 10; 12; $3\frac{3}{5}$; 8; $1\frac{2}{5}$; $1\frac{1}{8}$; $7\frac{7}{8}$; $4\frac{2}{5}$; $10\frac{5}{8}$; $6\frac{23}{24}$; $9\frac{3}{4}$; $6\frac{5}{6}$; $30\frac{6}{25}$; $7\frac{4}{7}$; $25\frac{1}{15}$; $25\frac{1}{6}$.

Page 75

New Matter. Dividing by a mixed number. Special methods for dividing a mixed number by an integer.

Remarks. A convenient method for dividing in examples such as 1 and 2, on top of page 75, is shown in the following:

$$
\begin{array}{r}
3\frac{4}{5})\overline{\;34\;} \\
5 \\
19)\overline{170}(8\frac{18}{19} \\
\underline{152} \\
18
\end{array}
\qquad\qquad
\begin{array}{r}
2\frac{1}{3})\overline{18\frac{3}{4}} \\
12 \\
28)\overline{225}(8\frac{1}{28} \\
\underline{224} \\
1
\end{array}
$$

In the first example multiply dividend and divisor by 5. In the second example multiply dividend and divisor by 12. This reduces each example to a simple division of integers.

Pages 76 and 77

New Matter. The relation between division and multiplication of fractions.

Remarks. The principle developed here has arisen several times before (see pages 25–26). The teacher should realize that one element of excellence in a course in Mathematics is that the same idea is made to serve in a great many different situations.

Nearly all the problems in fractions that cause trouble are of the kind given on these pages. The pupil should read and should be led to believe the statements given near the bottom of page 77. If he does, he is likely to take hold of these pages more earnestly, and thus try to clear up the whole matter at once.

Supplementary Matter. Drill in Fundamentals. For the first event multiply B III, IV by D III, IV, page 32. The products, in order, are:

9,442,278; 32,118,486; 25,568,862; 15,023,898; 54,338,035; 9,241,448; 27,061,800; 8,986,341; 33,562,048; 70,640,288.

For the second event, divide A I, II by D I, II, page 32. The quotients and remainders, in order, are:

178–272; 346–88; 38–58; 767–47; 117–292; 105–271; 233–4; 85–159; 259–76; 88–703.

Page 78

Review and Drill. Addition and subtraction of fractions.

Remarks. Note the simplicity of these oral exercises. In every case one of the denominators is a multiple of the other denominator. In fact, the combinations of denominators, 2 and 4, 4 and 8, 4 and 16, 2 and 16, and 8 and 16, are the only ones which occur on this page.

On inquiry, one finds that the fractions which are actually used in business are surprisingly simple. Following are two lists of fractions in practical use.

Diameters in inches of standard sizes of wire manufactured in the United States:

$\frac{1}{2}$; $\frac{15}{32}$; $\frac{7}{16}$; $\frac{13}{32}$; $\frac{3}{8}$; $\frac{11}{32}$; $\frac{5}{16}$; $\frac{9}{32}$; $\frac{17}{64}$; $\frac{1}{4}$; $\frac{15}{64}$; $\frac{7}{32}$; $\frac{13}{64}$; $\frac{3}{16}$; $\frac{11}{64}$; $\frac{5}{32}$; $\frac{9}{64}$; $\frac{1}{8}$; $\frac{7}{64}$; $\frac{3}{32}$; $\frac{15}{64}$; $\frac{9}{128}$; $\frac{1}{16}$; $\frac{9}{160}$; $\frac{1}{20}$; $\frac{7}{160}$; $\frac{3}{80}$; $\frac{11}{320}$; $\frac{1}{32}$; $\frac{9}{320}$; $\frac{1}{40}$; $\frac{7}{320}$; $\frac{3}{160}$; $\frac{11}{640}$; $\frac{1}{64}$; $\frac{9}{640}$; $\frac{1}{80}$; $\frac{7}{640}$; $\frac{13}{1280}$; $\frac{3}{320}$; $\frac{11}{1280}$; $\frac{5}{640}$; $\frac{9}{1280}$; $\frac{17}{2560}$; $\frac{1}{160}$.

The thickness in inches of commercial iron and steel sheets:

$\frac{1}{500}$; $\frac{1}{250}$; $\frac{3}{500}$; $\frac{1}{125}$; $\frac{1}{100}$; $\frac{3}{250}$; $\frac{7}{500}$; $\frac{1}{64}$; $\frac{9}{500}$; $\frac{1}{50}$; $\frac{11}{500}$; $\frac{1}{40}$; $\frac{7}{250}$; $\frac{1}{32}$; $\frac{9}{250}$; $\frac{1}{25}$; $\frac{9}{200}$; $\frac{1}{20}$; $\frac{11}{200}$; $\frac{1}{16}$; $\frac{13}{200}$; $\frac{7}{100}$; $\frac{3}{40}$; $\frac{2}{25}$; $\frac{17}{200}$; $\frac{9}{100}$; $\frac{19}{200}$; $\frac{1}{10}$; $\frac{11}{100}$; $\frac{1}{8}$; $\frac{27}{200}$; $\frac{3}{20}$; $\frac{32}{200}$; $\frac{9}{50}$; $\frac{1}{5}$; $\frac{11}{50}$; $\frac{6}{25}$; $\frac{1}{4}$.

The fractions used in other practical applications are nearly all included in these two series. In the great majority of practical applications the fractions used are only the simplest among these.

Page 79

Review and Drill. Multiplication and division of complex fractions.

Remarks. The compound fractions at the bottom of the page are, of course, equivalent to division problems such as are given on this page. To reduce example No. 43, we may, if we wish, proceed as follows:

$$\frac{8\times 4\frac{3}{4}}{8\times 2\frac{3}{8}}=\frac{38}{19}=2.$$

In this case we multiply the numerator and denominator by the least common multiple of the minor fractions. This at once reduces the problem to the division of two integers. This, of course, amounts to precisely the same as the suggestion in the note to page 75.

This method of dividing fractions is frequently used by business people. The principle involved is, of course, that the dividend and divisor may be multiplied by the same member without changing the quotient.

Standard Tests

See note to page 308.

Test in Addition. Second Test, Grade Five. Time, 6 minutes. Half of the class should have 10 or more correct answers.

1. 391	2. 473	3. 958	4. 453	5. 543
487	927	41	347	847
265	48	768	965	698
930	532	357	37	260
137	175	451	457	465
93	619	201	101	81
2303	2774	2776	2360	2894

6. 118	7. 457	8. 912	9. 537	10. 439
960	365	336	286	102
63	118	76	592	836
352	416	528	798	697
467	35	416	56	46
238	497	357	896	132
2198	1888	2625	3165	2252

11. 49	12. 729	13. 437	14. 614	15. 978
715	173	321	321	34
903	418	69	389	365
562	457	287	501	571
133	57	155	23	719
246	346	368	459	964
2608	2180	1637	2307	3631

16. 426	17. 561	18. 846	19. 376	20. 670
237	84	426	134	458
528	164	65	69	67
76	215	429	516	463
120	942	191	406	238
382	571	843	612	219
1769	2537	2800	2113	2115

Test in Subtraction. Time, 6 minutes.

Half of the class should have 20 or more correct answers.

1.	2.	3.	4.	5.
19734	62892	31975	54628	61473
10682	35560	20784	28417	50960
9052	27332	11191	26211	10513

6.	7.	8.	9.	10.
92148	54356	91789	78494	62512
31725	23425	58274	35823	31481
60423	30931	33515	42671	31031

11.	12.	13.	14.	15.
61238	30797	74632	91383	18369
20524	30658	32721	20762	15275
40714	139	41911	70621	3094

16.	17.	18.	19.	20.
63841	46758	82137	62543	49176
21790	25824	37021	20921	37843
42051	20934	45116	41622	11333

21.	22.	23.	24.	25.
68221	42753	92733	64716	97622
25310	30821	51562	32572	52710
42911	11932	41171	32144	44912

26.	27.	28.	29.	30.
72693	52019	51373	76885	64793
50547	31075	20842	59743	28352
22146	20944	30531	17142	36441

31.	32.	33.	34.	35.
56425	19703	42831	62548	34981
32714	17251	21750	38427	31790
23711	2452	21081	24121	3191

36.	37.	38.	39.	40.
39076	86032	41932	36715	14876
24524	37021	39720	35209	12593
14552	49011	2212	1506	2283

Test in Multiplication. Time, 6 minutes.

Half of the class should have 10 or more correct answers.

1. 854	2. 943	3. 321	4. 276	5. 386
14	57	89	54	74
11956	53751	28569	14904	28564
6. 519	7. 214	8. 527	9. 896	10. 453
68	93	13	74	86
35292	19902	6851	66304	38958
11. 468	12. 518	13. 123	14. 735	15. 827
79	46	59	17	46
36972	23828	7257	12495	38042
16. 635	17. 197	18. 146	19. 597	20. 813
48	85	32	92	72
30480	16745	4672	54924	58536

Test in Division. Time, 6 minutes.

Half of the class should have 5 or more correct answers.

440,–118	44,–252	49,–123
1. 173)76238	2. 561)24936	3. 728)35795
321,–24	130,–273	166,–193
4. 169)54273	5. 586)76453	6. 287)47835
42,–574	233,–155	91,–161
7. 745)31864	8. 213)49784	9. 426)38927
197,–178		
10. 318)62824		

Page 80

Applications. Speeds and distances.

Remarks. The subject of speeds and distances should receive attention for its own sake. This idea is important, and there is no more opportune place in the school curriculum to build it up than in arithmetic. Review again pp. 24 to 29.

Supplementary Matter. Bring in time tables from a local railroad showing distances and time required for certain trains. Compute the speeds in miles per hour. Reports of recent interesting races will be found in the last edition of great newspaper almanacs, such as the *Daily News Almanac,* or the *World Almanac.* One of these compendia should be on the teacher's desk constantly. A boy will take any amount more interest in computing speeds if the question at issue is the velocity, say, of an aeroplane in a recent record performance, or the speed of an automobile in a recent record-making race. Teachers cannot afford to lose any opportunity of this sort to develop interest.

Solution of Problems

1. 3 hours 45 minutes $=3\frac{3}{4}$ hours.
   $3\frac{3}{4}\times 46\frac{1}{2}=174\frac{3}{8}$ (miles).
2. 1 hour 24 minutes $=1\frac{2}{5}$ hours.
   $1\frac{2}{5}\times 84\frac{3}{5}=118\frac{11}{25}$ (miles).

Page 81

Applications. Miscellaneous problems.

Supplementary Matter. By keeping her eyes open, the teacher can find local situations which will give rise to problems like those given on this page. The children, especially the boys, may also be able to bring in such material. If the boys are working in a manual training shop, they may be led to see numerical relations which will form the basis of problems. Coöperation with the teacher of manual training is excellent for the class in arithmetic.

Drill in Fundamentals. For the drill in fundamentals indicated at the bottom of the page add V, VI A to E, page 32, XI, XII A to E, page 33, for the first event. The sums, in order, are:

741,555; 601,597; 57,036; 5412; 645; 524,196; 521,529; 50,312; 6318; 569.

For the second event, add VII, VIII A to E, page 33, and IX, X A to E, page 33. The sums, in order, are:

675,933; 496,330; 59,671; 7180; 635; 553,396; 537,887; 56,783; 5880; 608.

Solution of Problems

1. $22\frac{1}{2}'' \div 6 = 3\frac{3}{4}'' =$ width.
   $28\frac{1}{2}'' \div 5 = 5\frac{7}{10}'' =$ length.
2. $24 \times 36 \times 5 = 4320$ (cu. ft.).
   $4320 \div 27 = 160$ (cu. yds.).
4. $480 \div 1\frac{1}{3} = 360$ (loads).
5. $10 \times 18 \times \frac{7}{3} = 420$ (cu. yds.).
6. $\frac{18}{12} \times \frac{16}{12} \times \frac{8}{12} = \frac{3}{2} \times \frac{4}{3} \times \frac{2}{3} = 1\frac{1}{3}$ (cu. ft.).
8. $2\frac{3}{4} \div \frac{1}{18} = 49\frac{1}{2}$ (times).

Material for Third Examination. Fifth Grade.

Add:

1.	2.	3.	4.	5.
1217	4128	8074	4898	7670
1416	9326	2368	2710	6380
2827	2453	2259	1232	7958
4390	7892	2198	7659	9211
6780	6024	3820	1320	3842
3958	8217	8431	8498	7964
4693	1890	4927	6483	6345
5476	2388	6812	8222	4211
30,757	42,318	38,889	41,022	53,581

Subtract:

6.	7.	8.	9.	10.
590912	2910406	1416287	8739120	1417600
387678	1412837	684390	1428642	1143902
203234	1497569	731897	7310478	273698

Multiply:

11.	12.	13.	14.	15.
35109	82176	219378	61417	54693
7847	5234	8642	3958	5476
275,500,323	430,109,184	1,895,864,676	243,088,486	299,498,868

Divide:

16. 876)59100 — 67–408
17. 954)787000 — 824–904
18. 638)519000 — 813–304

Add:

19.	20.	21.	22.	23.
$1\frac{1}{2}$	$8\frac{4}{5}$	$6\frac{3}{8}$	$12\frac{5}{8}$	$16\frac{7}{8}$
$3\frac{1}{3}$	$5\frac{3}{4}$	$7\frac{3}{4}$	$7\frac{1}{4}$	$5\frac{5}{12}$
$6\frac{1}{4}$	$6\frac{1}{2}$	$5\frac{1}{2}$	$5\frac{1}{2}$	$8\frac{1}{3}$
$7\frac{1}{6}$	$8\frac{1}{4}$	$8\frac{1}{6}$	$8\frac{1}{6}$	$7\frac{3}{4}$
$18\frac{1}{4}$	$29\frac{3}{10}$	$27\frac{19}{24}$	$33\frac{13}{24}$	$38\frac{3}{8}$

Subtract:

24.	25.	26.	27.	28.
$78\frac{3}{5}$	$41\frac{3}{4}$	$52\frac{3}{8}$	$104\frac{1}{2}$	$14\frac{3}{16}$
$43\frac{2}{3}$	$29\frac{7}{8}$	$31\frac{3}{4}$	$78\frac{5}{8}$	$7\frac{5}{32}$
$34\frac{14}{15}$	$11\frac{7}{8}$	$20\frac{5}{8}$	$25\frac{7}{8}$	$7\frac{1}{32}$

Multiply:

29.	30.	31.	32.	33.
$12\frac{1}{2}$	$1\frac{3}{4}$	$6\frac{1}{4}$	$69\frac{1}{2}$	$102\frac{1}{4}$
$7\frac{1}{3}$	$2\frac{1}{2}$	$3\frac{2}{3}$	$84\frac{1}{3}$	$67\frac{2}{3}$
$91\frac{2}{3}$	$4\frac{3}{8}$	$22\frac{11}{12}$	$5861\frac{1}{6}$	$6918\frac{11}{12}$

Divide:

34. $12\frac{1}{2} \div 2\frac{1}{3} = 5\frac{5}{14}$ 35. $61\frac{1}{4} \div 3\frac{1}{2} = 17\frac{1}{2}$

36. $14\frac{3}{4} \div 4\frac{2}{3} = 3\frac{9}{56}$ 37. $26\frac{2}{3} \div 3\frac{3}{4} = 7\frac{1}{9}$

38. $74\frac{1}{2} \div 4\frac{1}{3} = 17\frac{5}{26}$ 39. $76\frac{1}{3} \div 12\frac{2}{3} = 6\frac{1}{38}$

40. A boy spends $\frac{5}{6}$ hour on arithmetic each school day. How many hours does he spend on arithmetic in 185 school days? $154\frac{1}{6}$ (hrs.)

41. How many square inches are there in a sheet of paper $8\frac{1}{2}$ inches by $6\frac{3}{4}$ inches? $57\frac{3}{8}$ (sq. in.)

42. If you step $2\frac{3}{5}$ feet, how far will you go in taking 1640 steps? 4264 (ft.)

43. A boy steps $2\frac{2}{5}$ feet. How many steps must he take to go one mile (5280 feet)? 2200 (steps)

44. An auto wheel goes $9\frac{3}{7}$ feet in making one revolution. How far does it go in making 7460 revolutions? $70,337\frac{1}{7}$ (ft.)

45. Find the number of square yards in a plot of ground 38 feet by 46 feet. $194\frac{2}{9}$ (sq. yd.)

46. Find the number of square feet in a table $2\frac{3}{4}$ feet by $3\frac{3}{8}$ feet. $9\frac{9}{32}$ (sq. ft.)

47. A man drives a car 97 miles in $3\frac{3}{4}$ hours. What is his average speed in miles per hour? $25\frac{13}{15}$ (miles per hour)

48. How many acres are there in a field 58 rods by 90 rods? $32\frac{5}{8}$ (acres)

49. A girl had the following grades in spelling: 89, 91, 93, 90, 86, 97, 95. What is the exact average of these grades? $91\frac{4}{7}$

50. On a certain map 1 inch represents 50 miles. Find the distance between two points which are $3\frac{7}{16}$ inches apart on this map. $171\frac{7}{8}$ (miles)

Pages 82 and 83

New Matter. Quotients to the nearest integer.

Remarks. This kind of division is frequently very useful. In any event nearly all quotients are to the nearest "something or other," inasmuch as most examples in division fail to come out exactly either in terms of integers or decimals. The subject must be faced somewhere, and this is the simplest case with which to start. Be sure to refer to the fact that the rule given on page 83 is only a special case of the general rule that the numerator and denominator of a fraction may be multiplied or divided by the same number without changing its value.

Pages 84 and 85

New Matter. The use of cancellation in solving problems.

Remarks. This topic is sometimes introduced early in the study of fractions, or even before fractions are studied at all. However, this is the most difficult case of cancellation that occurs, and consequently the proper place for it is here, after cancellation has been studied in connection with the reduction of fractions. Such examples as those on page 84, which occur readily in practice, are more complicated than any that we have studied heretofore. There has been a misconception of this whole matter, namely, that cancellation is something apart from fractions, whereas it is an application of the fundamental property of fractions.

Supplementary Matter. Drill in Fundamentals. For the first event in the drill in fundamentals suggested on page 85 subtract B III, IV, V, VI from A III, IV, V, VI, page 32. The results, in order, are:

4442; 14,761; 802; 15,418; 7710; 18,903; 46,176; 31,918; 20,862; 13,493; 1656; 13,188; 21,861; 5874; 84; 7854; 11,898; 54,514; 17,090; 5939.

For the second event, multiply A V, VI by D V, VI, page 32. The results, in order, are:

19,315,254; 18,797,568; 20,993,512; 68,093,544; 36,604,224; 52,997,571; 64,482,822; 72,863,328; 17,656,947; 35,734,314.

For the third event, divide A III, IV by D III, IV, page 32. The results, in order, are:

93–245; 116–551; 56–101; 198–46; 88–639; 144–307; 81–507; 237–134; 86–530; 95–395.

Page 86

Applications. Volume and areas.

Remarks. The impropriety of trying to write into solutions the names of the various units involved is nowhere more apparent than in problems solved by means of cancellation, as on this page.

Page 87

New Matter. Work preparatory to solving problems.

Remarks. Equations of the type given on this page occur very frequently in the solution of problems. They lead directly to cancellation, and the work given here should be understood fully.

Pages 88 and 89

New Matter. The equation used in the solution of problems.

Remarks. These problems should be solved by cancellation, whether the explanations in the book are adopted or not. If the teacher prefers, the following explanation may be given of example 6 on page 89:

One layer of inch cubes in this box contains $8 \times 16 = 128$ cubic inches. Hence the number of cubic inches in a foot, divided by 128, gives the number of layers of inch cubes, or the depth of the box.

Page 89

Solution of Problems

1. To find the volume of a rectangular box multiply length by width by depth. The three dimensions must be reduced to the same unit before multiplying.

4. $3'' \times 4'' \times 10''$; $3'' \times 5'' \times 8''$; $6'' \times 5'' \times 4''$; $12'' \times 5'' \times 2''$; $12'' \times 10'' \times 1''$; $24'' \times 5'' \times 1''$.

7. $\dfrac{864}{36 \times 8} = 3$ (inches).

8. $\dfrac{720}{10 \times 8} = 9$ (feet).

Pages 90 and 91

Applications. The solution of problems by means of cancellation.

Remarks. These problems were taken from a bulletin of the Agricultural Experiment Station at Urbana, Illinois, and are thus certainly practical. Note that the idea involved in each example is "the number of units multiplied by price equals the cost."

Supplementary Matter. For the drill on page 91, use the following combinations for addition and then for multiplication.:

8	7	9	5	4	6	9	4	3	8	4	9	8	7	9	6	5	3	2
6	3	9	7	9	7	5	7	4	7	5	8	8	7	7	9	8	5	7

4	6	3	2	4	6	4	9	8	6	2	5
4	3	8	9	6	6	8	3	2	5	6	5

Tell the class that this drill is to be on combinations which they should surely know, and that we now want to find out if they have forgotten any of them. Then have the papers marked as usual by the pupils themselves, and a list of errors made. The combinations missed now should be studied earnestly by each child. The children should be made to feel that it is *their* business to see to it that they know these combinations.

Solution of Problems (page 91)

1.	1912 pounds at 83¢ a bu.	$28.34
	30 pounds at $27 a ton.	.40
	2075 pounds at $13 a ton.	13.49
		$42.23
2.	1600 pounds at 83¢ a bu.	$23.71
	533 pounds oats at 45¢ a bu.	7.49
	35 pounds oil meal at $27 a ton.	.47
	2162 pounds hay at $13 a ton.	14.05
		$45.72
3.	1077 pounds oats at 83¢ a bu.	$15.96
	1077 pounds oats at 45¢ a bu.	15.15
	34 pounds oil meal at $27 a ton.	.46
	2194 pounds hay at $13 a ton.	14.26
		$45.83
4.	1808 pounds corn at 83¢ a bu.	$26 80
	352 pounds bran at $20 a ton.	3.52
	35 pounds oil meal at $27 a ton.	.47
	2081 pounds hay at $13 a ton.	13.53
		$44.32

Page 92

New Matter. The items of a bill.

Remarks. The small letter @ or the word "at" written after the name of each item on a bill is now frequently, or even usually, omitted. The tendency is not to write more words than are absolutely necessary.

Page 93

Applications. Making out bills.

Supplementary Matter. Be sure to get some real local bills and extend and foot them. Also make out bills from new material as suggested in the last example on this page.

Solution of Problems

2.	42 chests tea at $31.60	$1327.20
	20 chests tea at $24.60	492.00
	10 chests tea at $51.60	516.00
	20 boxes lemons $5.30	106.00
	30 boxes oranges $3.40	102.00
	Total	$2543.20
3.	12 chairs at $22.50	$270.00
	1 table	85.50
	4 book cases at $5.10	20.40
	1 table	110.00
	4 chairs at $50.00	200.00
	Total	$685.90
4.	40 pens at $0.45	$18.00
	30 pencils at $0.72	21.60
	25 pens at $0.76	19.00
	30 compasses at $0.65	19.50
	15 compasses at $1.60	24.00
	Total	$102.10
6.	35 suits at $15.50	$542.50
	45 suits at $17.50	787.50
	60 suits at $20.00	1200.00
	25 suits at $25.00	625.00
	20 suits at $30.00	600.00
	Total	$3755.00

Pages 94 and 95

Review and Drill. Oral review of fractions.

Remarks. This page calls for verbal statement of definitions and rules. The real purpose of such statements is sometimes misunderstood. They are one of the two final results that we seek to achieve. They are built upon a large number of simple, special cases, and are the culmination of our work on any particular topic. The sequence of events is:

(*a*) Simple individual examples.

(*b*) The solving of more complicated examples, involving the same principle.

(*c*) The summarizing of the principle involved into a general comprehensive statement.

These remarks apply also to the problems without numbers on page 95.

Pages 96 and 97

Review and Drill. Reduction of fractions. Addition and subtraction of simple fractions.

Supplementary Matter. Drill in Fundamentals. For the drill repeat the combinations given in the note to page 91.

In the drill in fundamentals on page 97, add B, C, D of III, IV, page 46.

$20\frac{33}{40}$; 17; $17\frac{41}{60}$; $14\frac{3}{4}$; $22\frac{13}{24}$; $30\frac{7}{24}$; $44\frac{19}{40}$; $19\frac{1}{6}$; $27\frac{1}{2}$; $34\frac{49}{72}$.

Pages 98 and 99

Review and Drill. Multiplication of fractions.

Supplementary Matter. In the drill in fundamentals on page 99, multiply A III, IV by B III, IV, page 46. The products in order, are:

$25\frac{47}{48}$; $16\frac{11}{12}$; $32\frac{1}{16}$; $19\frac{1}{6}$; $45\frac{41}{48}$; $236\frac{4}{9}$; $211\frac{3}{32}$; $18\frac{35}{48}$; $47\frac{1}{2}$; $13\frac{17}{27}$.

Remarks. These reviews and drills are purposely confined to simple fractions and mixed numbers. In practical work nearly all the fractions that occur are of this kind. We should develop the ability to handle examples of this sort orally with very considerable speed and with absolute accuracy.

Pages 100 and 101

Review and Drill. Multiplication and division of fractions and mixed numbers.

Material for Fourth Examination. Fifth Grade.

Add:

1.	2.	3.	4.	5.
2489	8140	9007	9694	8319
3176	6375	9350	6210	6986
8489	5913	2426	8789	1520
7194	1829	2174	7631	5120
1876	4567	7105	4141	6745
4562	5020	8931	6579	3173
6731	6140	3654	2387	9747
1910	3729	1568	1461	6811
36427	41713	44215	46892	48421

Subtract:

6.	7.	8.	9.	10.
814700	129782	53702	459082	8910430
426918	83968	27964	173748	3543687
387782	45814	25738	285334	5366743

Multiply:

11.	12.	13.	14.	15.
6718	5178	1492	9054	8410
2694	7802	7348	7482	3687
18,098,292	40,398,756	10,963,216	67,742,028	31,007,670

Divide: (Find quotients to nearest integer).

16. $827\overline{)72894}$ Answer: 89

17. $563\overline{)483720}$ Answer: 859

18. $764\overline{)476400}$ Answer: 624

Add:

19.	20.	21.	22.	23.
$41\frac{1}{3}$	$8\frac{7}{8}$	$21\frac{1}{5}$	$12\frac{3}{4}$	$67\frac{1}{6}$
$71\frac{1}{2}$	$5\frac{3}{4}$	$6\frac{2}{3}$	$7\frac{5}{8}$	$43\frac{1}{2}$
$14\frac{1}{4}$	$6\frac{1}{2}$	$7\frac{1}{6}$	$8\frac{2}{3}$	$17\frac{3}{4}$
$12\frac{1}{6}$	$4\frac{1}{3}$	$4\frac{3}{4}$	$6\frac{1}{6}$	$14\frac{5}{8}$
$139\frac{1}{12}$	$25\frac{11}{24}$	$39\frac{47}{60}$	$35\frac{5}{24}$	$143\frac{1}{24}$

Subtract:

24. $41\frac{5}{6}$ $19\frac{3}{4}$ — $22\frac{1}{12}$

25. $31\frac{4}{5}$ $17\frac{2}{3}$ — $14\frac{2}{15}$

26. $67\frac{3}{8}$ $18\frac{3}{5}$ — $48\frac{31}{40}$

27. $49\frac{1}{2}$ $27\frac{5}{6}$ — $21\frac{2}{3}$

28. $81\frac{3}{5}$ $42\frac{7}{8}$ — $38\frac{29}{40}$

Multiply:

29. $47\frac{1}{3}$ $12\frac{1}{2}$ — $591\frac{2}{3}$

30. $71\frac{1}{4}$ $82\frac{1}{3}$ — $5866\frac{1}{4}$

31. $62\frac{1}{4}$ $18\frac{1}{8}$ — $1128\frac{9}{32}$

32. $1\frac{1}{8}$ $2\frac{1}{4}$ — $2\frac{17}{32}$

33. $2\frac{1}{3}$ $3\frac{1}{8}$ — $7\frac{7}{24}$

Divide:

34. $42\frac{1}{3} \div 17\frac{1}{2} = 2\frac{44}{105}$

35. $72\frac{1}{3} \div 18\frac{1}{4} = 3\frac{211}{219}$

36. $68\frac{1}{4} \div 12\frac{1}{8} = 5\frac{61}{97}$

37. $31\frac{4}{5} \div 17\frac{2}{3} = 1\frac{212}{265}$

38. $67\frac{3}{8} \div 18\frac{3}{5} = 3\frac{463}{744}$

39. $49\frac{1}{2} \div 27\frac{5}{6} = 1\frac{130}{167}$

Reduce by Cancellation:

40. $\frac{84\times76\times32}{94\times64\times38} = \frac{42}{47}$

41. $\frac{49\times85\times66}{42\times55\times12} = 9\frac{11}{12}$

42. $\frac{54\times38\times96}{63\times56\times81} = \frac{304}{441}$

43. $\frac{34\times108\times6}{240\times45\times8} = \frac{51}{200}$

44. $\frac{18\times8\times14}{24\times36\times42} = \frac{1}{18}$

45. $\frac{8\times9\times15\times26}{45\times18\times16\times4} = \frac{13}{24}$

46. At an average speed of $16\frac{1}{2}$ miles an hour how far will an auto go in $4\frac{1}{4}$ hours? $70\frac{1}{8}$ (miles)

47. How many cubic feet does a box hold if it is $1\frac{1}{3}$ feet deep, $2\frac{1}{4}$ feet wide and $6\frac{1}{2}$ feet long? $19\frac{1}{2}$ (cu. ft.)

48. How many cubic yards are there in an excavation 38 feet long, 26 feet wide and 5 feet deep? $182\frac{26}{27}$ (cu. yd.)

49. A screw goes into the wood $\frac{3}{16}$ inches when turned completely around once. How many times must this screw be turned around to enter the wood $2\frac{3}{8}$ inches? $12\frac{2}{3}$

50. At $6.75 a ton what is the value of a load of coal containing 5460 pounds? $18.43

51. At $1.65 a bushel what is the value of a load of wheat weighing 4360 pounds? (1 bushel = 60 pounds.) $119.90

52. At $1.14 a bushel what is the value of 15640 pounds of corn on the ear? (72 lb. = 1 bushel.) **$247.63**

53. A farmer delivered 11860 lbs. of hay. At $21.70 a ton what is the value of this hay? **$128.68**

54. A housekeeper bought 680 pounds of potatoes at $1.25 a bushel. How much did they cost her? (60 lb. = one bushel.) **$14.17**

55. How many cubic feet does a box hold if it is 21 inches deep, 40 inches wide and 65 inches long? **$31\frac{43}{72}$ (cu. ft.)**

56. At 36 cubic feet to the ton, how many tons of coal can be put into a bin 12 feet by 16 feet by 6 feet? **32 (tons)**

Page 102

New Matter. Extension of the idea of drawing to scale.

Remarks. The most difficult part of drawing simple objects to scale is to get a real idea of what is meant by the scale, and then to select a scale which is appropriate for the problem at hand. For this reason examples like those under No. 1 on this page should be used freely.

Page 103

New Matter. Working drawings.

Supplementary Matter. Have the children make drawings of this kind on stiff manilla paper. Then cut out and fold to see if the box and the cover actually fit together. Actual trying out of the work in practice is the very best criticism of that work, because if the drawings are made right the boxes will be what they should be; otherwise they will not.

Some of the boys may bring in stories about working drawings which they have seen where buildings are being put up, machinery made, etc. The point is that we are trying to make the child acquainted with the world around him, and not only to teach pure arithmetic. These things are not brought in wholly for the sake of the arithmetic, but partly because this point in the school curriculum furnishes a convenient place to introduce them.

Pages 104 and 105

Applications. Computing the distances between points on a map, when the scale of the map is given.

Remarks. One of the principal uses of maps is to indicate the distances between places. That is the reason that a scale is used, and we have here an excellent opportunity to give drill in this work, and at the same time get good problems in multiplying fractions by integers. Frequently the notion of distances between places is that of railway distances. The idea of air-line distances should also be developed. Thus, the railway distance from Chicago to New York is over nine hundred miles, whereas the first non-stop aeroplane flight covered the space in a little over seven hundred miles.

Page 106

Application. Farm problems.

Supplementary Matter. If the school is in the country or in a small village bring in other problems like these, to fit the particular conditions in the neighborhood. If it is in a large city, there may be no good reason for making additional problems of this kind.

Remarks. While a boy in the north may have little practical interest in the cost of cotton, he should know something about it. He should know about the weight of a bale, the cost of a pound of cotton, and so on. If he wants to be an intelligent person he must be able to pick up newspapers and magazines and read them understandingly, and such information is required to do that. Conditions vary widely over the country; the number of tons of hay to the acre, the number of tons fed to one cow during the year, the price per bushel of corn, the yield of corn to the acre, etc. It will be well worth while to have the girls and boys find what these data are for their neighborhood.

Solution of Problems

4. $83 \div 2\frac{1}{4} = 36\frac{8}{9}$ (acres). 5. $\frac{1190}{1000} \times \$17.50 = \$20.83$.

Page 107

Applications. Military problems. A number of problems relating to military matters are here grouped together for the sake of a unity of interest. Note that questions on speed recur from time to time in this book. There are certain ideas which are especially important, and the aim is to develop a clear understanding of them. Speed is one of these ideas. It is surprising how many intelligent people there are who do not fully comprehend that time multiplied by speed equals distance, and the other rules which come directly from this one.

Many pupils grow up and get into Physics in the high school, and even in the university, without getting this idea thoroughly through their heads. The reason is not that the ideas are difficult but that the attention has never been directed definitely to them. By such repetitions as we have in these books, these fundamental ideas should be cleared up fully.

Solution of Problems

4. Time = distance ÷ speed.

Hence, time $= 48 \div 160 = .3$ hours.

But .3 hours = 18 minutes.

Page 108

Applications. The composition and weights of United States coins.

Remarks. Much of the information given on this page is such as most intelligent citizens should possess. At least, they should know where to find the information if they desire it.

Solution of Problems

5. $5000 \times 25\frac{4}{5} = 129{,}000$ (grams in $5000 gold).

$\frac{9}{10}$ of $124000 = 116{,}100$ (grams pure gold).

Page 109

Applications. The lighting space in proportion to the floor space in various kinds of rooms.

Remarks. The children should be made acquainted with the normal standards of lighting, inasmuch as their health and welfare is directly affected by such matters. Again they should know where to find the normal standards even if they do not remember them permanently. A part of the function of a course in arithmetic is to make people intelligent about the number phases of their environment.

Solution of Problems

1. $13 \times 17 = 221$ (area of room in sq. ft.).
   $2 \times 3\frac{1}{2} \times 5\frac{3}{4} = 40\frac{1}{4}$ (area of windows in sq. ft.).
   $40\frac{1}{4} \div 221 = .182$, which is more than $\frac{1}{6}$.

2. $27 \times 33 = 891$ (floor space).
   $7 \times 3 \times 8\frac{3}{4} = 183\frac{3}{4}$ (window space).
   But $183\frac{3}{4} \div 891 = .206$, which is more than $\frac{1}{5}$.

3. $12 \times 14 = 168$ (floor space).
   $3\frac{1}{4} \times 5\frac{3}{4} = 18\frac{11}{16}$ (window space).

But $18\frac{11}{16}$ is clearly more than one-tenth of 168. Hence the window space is more than $\frac{1}{10}$ of the floor space.

Page 110

Applications. Miscellaneous problems.

Remarks. Problem No. 6 may require explanation, but it is worth while to explain it, because this is the sort of thing that boys, especially, will run across in their reading and thinking if they are at all bright. Example 9 suggests that the population of the city where the child lives, and the number of registered voters, may be obtained to see whether or not that number is more or less than two-fifths of the whole population. This fraction will be found approximately correct where both men and women vote.

Solution of Problems

6. $1\frac{1}{8} \times 1\frac{3}{4} = \frac{63}{32}$ (area of cross-section).
$40{,}000 \times \frac{63}{32} = 1250 \times 63 = 78{,}750$ (pounds).
7. $4500 \times \frac{33}{4} \times \frac{33}{8} = 153{,}140\frac{5}{8}$ (pounds pressure).

Page 111

New Matter. Formal treatment of decimal fractions.

Remarks. The form of the decimal fractions is already known from writing numbers representing money. The child should be led to realize clearly that, for instance, seven-tenths is the same in decimal form as in the common fractional form, and that the decimal form is adopted only for the sake of simplicity. No consistent attempt should be made, at this stage, to develop the operations on decimals from the operations on common fractions. Just go ahead using the decimal fractions as has been already done in the case where such fractions are used to represent money. It may be pointed out, however, as is done on this page, that decimal fractions are reduced to common denominators without trouble. This is really the fact which makes them of such great convenience.

Page 112

New Matter. Development of the idea of decimal fractions and of place value in decimals. Do not make any attempt at a more detailed explanation at this stage than that suggested on this page.

Page 113

New Matter. Reading and writing decimals.

Supplementary Matter. Suggest that the children note the distances shown by cyclometers on automobiles and bring the numbers to school, especially if such cyclometers register one-tenth of a mile. Some books show cyclometers that register up to trillions in one direction and down to millionths in the other. This, of course, is silly. No such cyclometers were ever constructed.

Pages 114 and 115

New Matter. Addition and subtraction of decimals.

Remarks. Make it clear that the methods of adding and subtracting decimals are precisely the same as those for integers. Insist that the decimal point must be written in a column.

Supplementary Matter, Drills in Fundamentals. Play game No. 3 on page 30. For the first event add A, B, C, D of II, III, page 46. The sums, in order, are:

$3\frac{1}{30}$; $2\frac{1}{15}$; $2\frac{223}{240}$; $2\frac{143}{160}$; $2\frac{11}{16}$; $25\frac{79}{120}$; $20\frac{5}{8}$; $22\frac{13}{30}$; $22\frac{5}{12}$; $31\frac{5}{12}$.

For the second event, multiply B III, IV by C III, IV, page 46. The products, in order, are:

$54\frac{33}{40}$; $40\frac{4}{9}$; $31\frac{1}{20}$; $17\frac{3}{16}$; $64\frac{7}{12}$; $121\frac{1}{3}$; $186\frac{31}{32}$; $31\frac{5}{12}$; $34\frac{4}{9}$; $43\frac{5}{9}$.

For the third event, divide C III, IV by D III, IV, page 46. The quotients, in order, are:

$1\frac{33}{35}$; $2\frac{4}{11}$; $\frac{69}{95}$; $1\frac{12}{43}$; $2\frac{22}{39}$; $1\frac{11}{41}$; $\frac{155}{252}$; $1\frac{48}{165}$; $\frac{31}{94}$; $\frac{98}{151}$.

Pages 116 and 117

New Matter. Multiplication of decimals.

Remarks. Note the only new element in the multiplication of decimals, namely, the rule for placing the decimal point in the product. This is obtained by solving a number of examples and then formulating the general statement from the individual cases.

Supplementary Matter. Drill in Fundamentals. Play game No. 3, page 30. For the first event add I, II A to E, and III, IV A to E, page 47. The sums, in order, are:

48,253.54; 2611.794; 440.751; 14.060; 16.67; 34,342.77; 6538.962; 202.297; 17.492; 27.00.

For the second event multiply B V, VI by D V, VI, page 32. The products, in order, are:

18,417,702; 13,733,376; 15,134,764; 63,911,256; 36,567,936; 45,999,657; 56,475,468; 28,325,390; 13,709,157; 32,990,496.

For the third event, divide A V, VI by D V, VI page 32. The results, in order, are:

65–407; 127–184; 292–78; 134–229; 196–60; 66–675; 142–248; 109–131; 330–207; 167–193.

Page 118

Review and Drill. Multiplication of decimals.

Remarks. Note how the difficulties on this page are brought in one by one. In the first group are examples which may be solved orally, and where the attention therefore is mainly on the process. Then comes a simple example with one whole number and one decimal number, and then another example with a three-place decimal and a whole number. "One difficulty at a time" is the slogan.

Page 119

New Matter. Products to the nearest tenth; the nearest hundredth; and so on.

Remarks. The finding of these products is simple enough, so that it may be disposed of definitely at this time. The question of shortening the multiplication in case there are many decimal places in one of the factors had better be postponed until the seventh grade, when it can be explained to better advantage.

Page 120

Applications. Miscellaneous problems.

Remarks. Bring out as clearly as possible the comparative convenience of common fractions and decimal fractions. This page offers a very good opportunity to do this, inasmuch as the same material has been used in the form of common fractions on page 108.

Page 121

Applications. Miscellaneous problems.

Remarks. See the remark in the note to page 120. In these problems data are used which are in themselves valuable, and which many of the pupils would have to learn in some other connection if they were not given here. It is hoped that by using them repeatedly in problems they will tend to become fastened permanently in the memory.

Solution of Problems

2. $5.5 \times 5.5 = 30.25$ (sq. yd. in one sq. rd.).
   $160 \times 30.25 = 4{,}840$ (sq. yd. in 1 acre).

3. $16.5 \times 16.5 = 272.25$ (sq. ft. in sq. yd.).
   $160 \times 272.25 = 43{,}560$ (sq. ft. in 1 acre).

8. $62\frac{1}{2} \times .92 = 57.5$ (wt. in pounds of 1 cu. ft.).
   $35 \times 57.5 = 2012.5$ (lbs.); $2012.5 - 2000 = 12.5$ (lbs.).

Page 122

Supplementary Matter, Drill in Fundamentals. Play game No. 3 on page 30. For the first event add C, D, E, F of III and IV, page 46. The sums, in order, are:

$16\frac{11}{30}$; $13\frac{19}{24}$; $12\frac{1}{5}$; $13\frac{17}{30}$; $18\frac{7}{24}$; $13\frac{1}{24}$; $21\frac{3}{5}$; $17\frac{7}{24}$; $22\frac{7}{24}$; $32\frac{8}{15}$.

For the second event, multiply A III, IV by B III, IV, page 46. The products, in order, are:

$25\frac{47}{48}$; $16\frac{11}{12}$; $32\frac{1}{16}$; $19\frac{1}{6}$; $45\frac{41}{48}$; $236\frac{4}{9}$; $211\frac{8}{35}$; $18\frac{35}{48}$; $47\frac{1}{2}$; $13\frac{17}{27}$.

Page 123

Review and Drill. Dividing a decimal by an integer.

Remarks. Note the rule for placing the decimal point in the quotient, namely, that it should be placed directly above the decimal point in the dividend. This rule is the one which is finally remembered, and we may as well introduce it now as later. The real understanding of the process will come later on.

In reality, of course, the remainder is not 29, but $\frac{29}{1000}$. Consequently the statement that the remainder is greater than half the divisor is, strictly speaking, not true. This, however, should be ignored at this time. For the purpose of comparison, the remainder should be regarded as a whole number, and the argument for this is very simple, namely, that it does get the right answer. The real explanation is complicated for a child at this age, and had better be omitted altogether.

Pages 124 and 125

Review and Drill. Division of decimals.

Remarks. Note the method of placing the decimal point in the quotient when the divisor is a decimal. This method is believed to be the simplest inasmuch as it requires only the application of the now well-known principle that dividend or divisor may be multiplied or divided by the same number without changing the value of the quotient. This principle, once applied, the placing of the decimal point in the quotient takes care of itself. Notice how on these pages the difficulties that arise in the division of decimals are introduced gradually. First a decimal is divided by an integer, and then by a decimal in the ordinary way. Then comes the problem in which zeros must be added to the remainder to carry the division to the required extent.

Supplementary Matter. Play game No. 1 on page 30. Divide A III, IV, V by B III, IV, V, page 46. The quotients, in order, are:

$\frac{116}{129}$; $\frac{87}{112}$; $\frac{19}{27}$; $3\frac{1}{15}$; $1\frac{89}{124}$; $\frac{19}{28}$; $\frac{70}{193}$; $1\frac{37}{87}$; $1\frac{11}{160}$; $1\frac{5}{64}$; $3\frac{69}{95}$; $1\frac{28}{57}$; $\frac{37}{47}$; $\frac{102}{115}$; $1\frac{61}{219}$.

Pages 126 and 127

Applications. Miscellaneous problems.

Remarks. Note the method used in examples No. 4 and 5 on page 126, in connection with ordinary measuring. If the school is in the country, find how milk and cream are measured in the creamery.

Solution of Problems (page 126)

1. $.041 \times 9840 = 403.44$ (lb. butter fat).
   $403.44 \times \$.43 = \$173.48$ (value of butter fat).
2. $17 \times 2.16 = 36.72$ (no. tons hay).
   $36.72 \times \$14.50 = \$532.44$ (value of hay).
3. $9840 \div 2.15 = 4576.7$ (quarts).
   $4576.7 \times \$.042 = \$192.22$.
5. $14.3 \times \$.186 = \$2.66$; $31 \times \$2.66 = \$82.46$.

Solution of Problems (page 127)

3. $24.6 \times 61.3 = 1507.98$ (area in sq. rods).
   $1507.98 \div 160 = 9.42$ (area in acres).
5. $118 \times .62 = 73.16$ (value in dollars).
6. $\frac{3240}{70} \times .94 = 43.52$ (value in dollars).
7. $\frac{2480}{2000} \times 18.45 = 22.88$ (value in dollars).
8. $22.7 \times 87.3 = 1981.71$ (area in sq. ft.).
   $.28 \times 1981.71 = 554.88$ (value in dollars).

Material for Fifth Examination. Fifth Grade.

Add.

1.	2.	3.
5164	3168	1456
2417	2930	1927
8246	6418	3890
8904	4234	4246
1605	8765	8600
3900	9102	5325
7287	6301	7901
2930	4917	6246
40453	45835	39591

Subtract:

4.	5.	6.	7.
719184	3918470	471060	974560
283728	1743846	327848	439778
435456	2174624	143212	534782

Multiply:

8.	9.	10.
78412	84567	53764
7319	5398	9708
573,897,428	456,492,666	521,940,912

Find Quotients to Nearest Thousandths:

11. $931\overline{)8412}$ 9.035
12. $573\overline{)4819}$ 8.410
13. $846\overline{)49857}$ 58.933

Add:

14.	15.	16.
$12\frac{1}{6}$	$7\frac{1}{8}$	$52\frac{1}{5}$
$17\frac{1}{4}$	$5\frac{1}{3}$	$14\frac{3}{8}$
$10\frac{1}{3}$	$9\frac{1}{2}$	$5\frac{3}{4}$
$6\frac{1}{2}$	$8\frac{2}{3}$	$7\frac{5}{16}$
$46\frac{1}{4}$	$30\frac{5}{8}$	$79\frac{51}{80}$

Subtract:

17.	18.	19.	20.
$49\frac{5}{8}$	$94\frac{1}{3}$	$32\frac{1}{2}$	$124\frac{3}{8}$
$34\frac{13}{16}$	$76\frac{5}{6}$	$24\frac{3}{4}$	$108\frac{3}{4}$
$14\frac{13}{16}$	$17\frac{1}{2}$	$7\frac{3}{4}$	$15\frac{5}{8}$

Multiply:

21. $84\frac{1}{2}$	22. $4\frac{1}{5}$	23. $7\frac{1}{8}$	24. $67\frac{1}{4}$	25. $140\frac{1}{2}$
$67\frac{1}{4}$	$2\frac{1}{3}$	$2\frac{1}{3}$	$42\frac{2}{3}$	$64\frac{1}{8}$
$5682\frac{5}{8}$	$9\frac{4}{5}$	$16\frac{5}{8}$	$2869\frac{1}{3}$	$9009\frac{9}{16}$

Divide:

26. $85\frac{1}{2} \div 3\frac{1}{3} = 25\frac{13}{20}$. 27. $64\frac{2}{3} \div 5\frac{1}{2} = 11\frac{25}{33}$. 28. $18\frac{5}{6} \div 4\frac{3}{5} = 4\frac{13}{138}$

Find the missing numbers, using cancellation first, then reduce to the nearest hundredth:

29. $36 \times ? = 48 \times 6 \times 8$ 64. 30. $18 \times 5 \times 10 = ? \times 6 \times 8$ $18\frac{3}{4}$.
31. $12 \times 48 \times 64 = ? \times 7 \times 18$ $292\frac{4}{7}$.
32. If one nautical mile is 1.15 statute miles, how many statute miles are there in 3210 nautical miles? 3691.5
33. If one nautical mile equals 1.15 statute miles, how many nautical miles are there in 3560 statute miles? 3095.7 (nautical miles)
34. At 65¢ a square foot what is the value of a lot $36\frac{1}{2}$ feet by $108\frac{3}{4}$ feet? $2580.09
35. A man paid $1500 for a lot, paid $480 for improvements on it, put up a house costing $4200 and paid $240 for other expenses. He then sold the house and lot for $7500. Did he gain or lose and how much? $1080 (gain)
36. At $7.45 a ton, what is the cost of 15.5 tons of coal? $115.48
37. How many rods are there in 845 feet? (One rod is 16.5 feet.) $51\frac{7}{33}$ (rods)
38. The circumference of an auto wheel is 9.3 feet. How many times must this wheel revolve in going 74 miles. (1 mile = 5280 feet.) 42,012.9 (revolutions)
39. Cast iron weighs 7.9 times as much as water. What is the weight of a cubic foot of cast iron if one cubic foot of water weighs 62.5 pounds? 493.75 (lbs.)
40. A milkcan weighs 18.7 pounds when empty and 103 when filled with milk. How many gallons does the can hold if one gallon of milk weighs 8.6 pounds? 9.8 (gallons)

Pages 128 and 129

New Matter. Reducing fractions to decimals; reducing to nearest hundredths or nearest thousandths; combination of decimal and common fraction.

Remarks. Note that a fraction is here regarded as an indicated division. The required reduction is then obtained by carrying out this division.

Page 130

New Matter. Fractions exactly reducible to decimals. Fractions and decimal equivalents to be memorized.

Remarks. The children may make a list of fractions which can be reduced exactly to decimals, and another list of fractions which cannot be so reduced.

Page 131

Review and Drill. Problems without numbers.

Remarks. If the child is unable to answer one of the questions on this page, try to get him to work a simple concrete problem involving the principle whose statement is required. Then try to get him to state the principle itself. All through, the process is one of inductions.

Pages 132 and 133

New Matter. Reduction of unusual common fractions.

Remarks. In the spring, after the baseball session begins, and in the fall before it ends, get one or more baseball scores to be kept on the board permanently, changing them from day to day to keep them up to date. This will afford the very best practice in the reduction of fractions to decimals.

Supplementary Matter, Drill in Fundamentals. Play game No. 4, page 30. Reduce the fractions in A I, II, page 46, to 4-place decimals. The results, in order, are:

.8750; 0.8333; .7500; 0.9375; 0.9167; .9333; 0.6111; 0.8125; 0.7813; 0.8333.

Standard Tests. See notes to page 308.

Test in Addition. Third test, grade five. Time, 6 minutes. Half of the class should have 10 or more correct answers.

1.	2.	3.	4.	5.
257	359	239	145	135
123	478	352	231	456
476	256	173	534	487
389	390	619	347	269
247	371	895	965	268
812	347	768	457	674
367	379	357	273	308
2671	2580	3403	2952	2597

6.	7.	8.	9.	10.
128	574	479	357	394
639	639	218	473	810
238	245	336	268	201
354	611	617	259	837
467	462	528	897	796
328	453	416	560	645
634	216	357	896	213
2788	3200	2951	3710	3896

11.	12.	13.	14.	15.
349	629	473	698	312
517	497	416	160	431
238	317	213	312	978
903	185	962	893	564
256	483	728	515	175
331	357	524	327	719
624	462	368	495	965
3218	2930	3684	3400	4144

16.	17.	18.	19.	20.
426	238	863	459	569
313	165	470	736	697
237	483	289	413	584
285	460	506	569	107
671	152	249	316	364
125	249	911	457	802
132	715	438	816	319
2189	2462	3726	3766	3442

Test in Subtraction. Time, 6 minutes.

Half of the class should have 20 or more correct answers.

1. 5435	2. 7624	3. 3128	4. 8263	5. 6882
2576	2756	1939	2587	3997
2859	4868	1189	5676	2885
6. 7367	7. 8564	8. 6524	9. 4820	10. 2462
3589	2787	3647	2945	1596
3778	5777	2877	1875	866
11. 7183	12. 7452	13. 3162	14. 7283	15. 8326
5495	2679	1975	3596	3749
1688	4773	1187	3687	4577
16. 4813	17. 7134	18. 7578	19. 3482	20. 3826
2947	5856	3689	2793	1958
1866	1278	3889	689	1868
21. 5648	22. 8632	23. 5173	24. 4823	25. 6231
2939	3758	4685	1958	3576
2709	4874	488	2865	2655
26. 7438	27. 5764	28. 3876	29. 4861	30. 8642
2959	3876	2988	1973	3783
4479	1888	888	2888	4859
31. 4735	32. 9628	33. 6874	34. 3621	35. 7638
2847	3949	3986	1754	3549
1888	5679	2888	1867	4089
36. 2537	37. 2634	38. 4370	39. 9516	40. 2436
1958	1358	2883	5639	1958
579	1276	1487	3877	478

Test in Multiplication. Time, 6 minutes.

Half of the class should have 10 or more correct answers.

1. 412 × 47 = 19364	2. 5326 × 27 = 143802	3. 972 × 64 = 62208	4. 356 × 48 = 17088	5. 8915 × 67 = 597305
6. 249 × 37 = 9213	7. 479 × 35 = 16765	8. 5673 × 89 = 504897	9. 2347 × 59 = 138473	10. 587 × 34 = 19958
11. 5196 × 27 = 140292	12. 419 × 28 = 11732	13 467 × 35 = 16345	14. 6847 × 19 = 130093	15. 896 × 42 = 37632
16. 934 × 57 = 53238	17. 3196 × 38 = 121448	18. 456 × 37 = 16872	19. 4897 × 15 = 73455	20. 684 × 12 = 8208

Test in Division. Time, $6\frac{1}{2}$ minutes.

Half of the class should have 5 or more correct answers.

1. 568)34698 — 61–50	2. 459)63932 — 139–131	3. 389)64057 — 164–261
4. 347)90426 — 260–206	5. 576)34908 — 60–348	6. 652)54978 — 84–210
7. 873)71937 — 82–351	8. 934)62894 — 67–316	9. 426)77702 — 182–170
10. 289)38746 — 134–20		

Pages 134 and 135

New Matter. Percentage.

Remarks. The less explaining that is done here, the better. The one thing that the child needs to get thoroughly is the meaning of the words "per cent." When that is once clear it is obvious that ordinary decimals can be changed to per cents by moving the decimal points two places to the right. Since we now know how to change a fraction to a decimal, and how to change an ordinary decimal to per cents, we should know perfectly how to express a fraction as per cents. No further explanation should be given. The more talking we do here, the more we shall befog the issue.

Page 136

New Matter. Simple discount given in per cents.

Remarks. If the work up to this point has been done well, there will be nothing difficult about this page. If the child has trouble, it may be that he is deficient in decimals, or in some other material that precedes. It would be unfortunate in the extreme if we were to explain this matter without reference to what has already been done. The best explanation will contain very little that is not in the nature of review.

Supplementary Matter. Get real bills with discounts given in per cents.

Page 137

New Matter. Formal use of the word "discount."

Remarks. No explanation whatsoever should be needed besides what is given in the text. The point is that the child should be led to see that the only new element introduced is the word *discount*. The arithmetic involved has already been learned.

Page 138

Applications. Miscellaneous problems involving discount.

Solution of Problems

2.

6 chairs at $4.75 . . .	$28.50
3 chairs at $7.80 . . .	23.40
2 rugs at $45.00	90.00
3 rugs at $7.50	22.50
1 table	30.00
Sideboard	60.00
Dining table	45.00
	$299.40
10% discount	29.94
	$269.46

3.

Dishes	$35.00
Fireless cooker	15.00
3 pans at 75¢	2.25
Bread-box	1.25
Utensils	12.40
	$65.90
Discount	3.29
	$62.61

4.	16 sheets at 45¢....	$7.20
	24 pillow-cases at 25¢	6.00
	5 doz. towels at $3.40	17.00
	8 towels at 32¢....	2.56
	18 towels at 12¢...	2.16
		$34.92
	Discount	5.24
		$29.68

5.	Suit...............	$27.50
	1 pr. shoes..........	6.50
	1 pr. shoes..........	8.00
	1 coat..............	35.00
	6 shirts at $2.25.....	13.50
	4 underwear at $1.75	7.00
	1 hat..............	5.00
	8 pr. socks at 45¢...	3.60
		$106.10
	Discount.........	21.22
		$84.88

Page 139

Review and Drill. The decimal number system. The system of notation for numbers.

Page 140

Review and Drill. The principle of place value. Problems on multiplying and dividing numbers by 10. Note the last lines on page 140. The statement made here should be brought home to the child. He may as well begin to learn something of the practical value of our system of numerals.

Page 141

Review and Drill. Adding integers; writing decimals in a column.

Supplementary Matter, Drill in Fundamentals. At the end of page 141, play game No. 3, page 30. For the first event add II, III A to E, and IV, V A to E, page 32. The sums, in order, are:

526,856; 459,945; 43,984; 4676; 548; 685,309; 511,294; 59,437; 5550; 65.5.

Page 142

For the second event, add II, III A to E, and IV, V A to E, page 47. The sums, in order, are:

38,366.74; 3463.212; 275.553; 13.627; 18.88; 29,473.734; 10,738.449; 358.886; 24.034; 20.74.

Page 143

Review and Drill. Subtraction of integers and of decimals.

Supplementary Matter. Play game No. 2 on page 30. Subtract B I, II, III from A I, II, III, page 47. The remainders, in order, are:

2409.856; 1102.83; 5314.37; 6456.60; 6548.937; 8713.04; 141.535; 5551.21; 5290.36; 4113.008; 5217.16; 110.085; 71.02; 211.46; 5484.65.

Page 144

Applications. Weights of boys and girls at different ages.

Page 145

Review and Drill. Multiplication of integers, especially multiplication by the numbers **25**, 50 and 125.

Page 146

Review and Drill. Multiplication of decimals.

Applications. Miscellaneous problems involving decimals.

Supplementary Matter. Play game No. 3 on page 30. For the first event, multiply A I, II, III by E I, II, III, page 32. The products, in order, are:

1,464,892; 1,285,268; 703,710; 2,974,678; 1,645,300; 822,984; 1,054,248; 3,061,944; 5,383,396; 4,977,553; 2,019,213; 4,975,704; 650,981; 1,633,112; 6,093,445.

For the second event, multiply A I, II by E I, II, page 47. The products, in order, are:

608.635; 342.312; 35,966.471; 39,320.40; 6363.7575; 6678.944; 187.584; 1927.84; 2495.548; 1079.078.

Solution of Problems

17. $9 \times 3.25 = 29.25$ (cords), $29.25 \times \$4.25 = \$124.31$.

19. $6.25 \times 9.75 = 60.9375$ (area in sq. ft.).
$60.9375 \times \$1.60 = \$97.50$ (value).

20. $24.37 \times 81.64 = 1989.57$ (area in sq. ft.).
$1989.58 \times \$1.45 = \$2884.89$.

Page 147

Review and Drill. Long division of integers and of decimals.

Supplementary Matter. Play game No. 3 on page 30, on three successive days. For the first event, add VIII, IX A to E and X, XI A to E, page 33. The sums, in order, are:

662,343; 497,931; 67,072; 6516; 561; 553,714; 567,982; 52,316; 6740; 594.

For the second event, multiply A VII, VIII by D VII, VIII, page 33. The products, in order, are:

10,724,994; 45,428,721; 53,930,400; 55,550,978; 25,068,015; 49,403,508; 36,341,865; 80,614,170; 80,874,101; 61,054,352.

For the third event, divide A VII, VIII by D VII, VIII, page 33. Find the quotients to the nearest tenth. The quotients, in order, are:

189.3; 176.7; 63.7; 95.4; 59.9; 69.7; 113.0; 90.3; 112.7; 74.7.

Page 148

Review and Drill. Problems without numbers.

Standard Tests. See note to page 308.

Test in Addition. First Test, Grade Six. Time, 6 minutes.

Half of the class should have 10 or more correct answers.

1.	2.	3.	4.	5.
$2.65	$4.85	$7.14	$6.71	$1.69
2.30	6.02	.49	1.49	4.80
.52	.15	6.21	.50	78
2.39	.27	9.03	.25	8.90
.76	2.18	1.30	2.46	.25
4.57	4.12	.89	1.98	2.45
$13.19	$17.59	$25.06	$13.39	$18.87

6.	7.	8.	9.	10.
$7.90	$7.23	$4.58	$7.14	$4.19
2.79	4.78	2.06	.49	9.05
.96	2.91	8.21	6.23	4.62
.74	.82	.42	4.09	.89
5.48	.95	.39	1.03	.67
3.92	3.46	6.17	.89	1.40
$21.79	$20.15	$21.83	$19.87	$20.82

11.	12.	13.	14.	15.
$1.29	$6.38	$2.86	$ .47	$7.08
4.16	2.37	5.71	3.25	8.21
6.13	.76	.35	5.47	.79
.38	.15	.13	6.25	3.68
.21	2.94	8.67	2.38	.47
9.37	6.90	9.08	.72	1.60
$21.54	$19.50	$26.80	$18.54	$21.83

16.	17.	18.	19.	20.
$2.48	$2.90	$4.37	$2.30	$5.48
4.86	6.14	1.65	3.98	8.42
2.15	3.50	4.03	.36	.38
.87	.38	.96	8.72	.13
.61	.49	.89	6.38	2.50
3.50	5.36	2.15	.27	1.63
$14.47	$18.77	$14.05	$22.01	$18.54

Test in Subtraction. Time, 6 minutes.

Half of the class should have 20 or more correct answers.

1.	2.	3.	4.	5.
73931	96283	91375	45862	41673
15420	73831	79204	32670	20581
58511	22452	12171	13192	21092

6.	7.	8.	9.	10.
12948	28316	83641	86291	92763
10753	15293	27230	32258	59321
2195	13023	56411	54033	33442

11.	12.	13.	14.	15.
56542	93076	56243	73521	69251
25191	32634	32821	39410	35720
31351	60442	23422	34111	33531

16.	17.	18.	19.	20.
34596	70378	72345	92501	37319
28432	16235	21523	70490	24167
6164	54143	50822	22011	13152

21.	22.	23.	24.	25.
68230	34734	96542	91785	64932
35720	23472	34191	75634	23571
32510	11262	62351	16151	41361

26. 31872	27. 72933	28. 73651	29. 82431	30. 93142
19451	61752	51437	61329	72831
12421	11181	22214	21102	20312
31. 28735	32. 59864	33. 84796	34. 31982	35. 52643
17392	27382	38254	27431	28421
11343	32482	46542	4551	24222
36. 86745	37. 92315	38. 92741	39. 62149	40. 41853
35271	71412	50830	41825	30547
51474	20903	41911	20324	11306

Test in Multiplication. Time, 6 minutes.

Half of the class should have 10 or more correct answers.

1. 568	2. 392	3. 713	4. 627	5. 836
24	47	52	46	54
13632	18424	37076	28842	45144
6. 519	7. 421	8. 257	9. 789	10. 345
68	39	34	45	67
35292	16419	8738	35505	23115
11. 584	12. 815	13. 123	14. 327	15. 278
29	46	57	69	46
16936	37490	7011	22563	12788
16. 635	17. 791	18. 164	19. 395	20. 382
94	58	38	47	54
59690	45878	6232	18565	20628

Test in Division. Time, 6½ minutes.

Half of the class should have 5 or more correct answers.

247-4	65-81	286-26
1. 273)67435	2. 651)42396	3. 278)79534
434-79	127-9	309-155
4. 169)73425	5. 287)36458	6. 187)57938
155-36	148-173	210-63
7. 318)72842	8. 237)35849	9. 256)79423
144-78		
10. 342)49326		

Page 149

Applications. Miscellaneous problems.

Supplementary Matter. The problems on this page suggest possible additional problems, the material for which may be found, for instance, in one of the newspaper almanacs. The teacher will, of course, use her judgment as to the amount of additional material that she will use. It should be adapted to local conditions.

Solution of Problems

1. $69600 \div 567 = 123$ (correct to nearest unit).
5. $27 \times 2\frac{3}{5} = 70.2$ (tons); $70.2 \times \$17.40 = \$1221.48$ (value).
6. $\begin{array}{r} 2218192 \\ \underline{2150420} \\ 67772 \end{array}$ (difference in cu. in.).

$67,772 \div 2150.42 = 31.5$ (American bushels).

Pages 150 and 151

Applications. Miscellaneous problems. Boy Scouts' camping expedition.

Supplementary Matter. If there are members of the Boy Scouts in the class, get them to bring a set of problems of this kind. If possible, such material should be obtained from one of their outings. It may be well in this connection to arrange the expenditures in a column as in an account.

Solution of Problems

Page 151

7. (For each boy)

Food	\$6.00
Transportation	.74
Outfit	12.00
Total	\$18.74

$32 \times \$18.74 = \$599.68$. (Total expense for the troop.)

Page 152

New Matter. The meaning of a fraction.

Remarks. The two meanings of a fraction given on this page should be understood fully; that is, the child should comprehend fully that a fraction may always be regarded as representing a certain number of the equal parts of a whole, and also as an indicated division in which the numerator is to be divided by the denominator.

Page 153

Review and Drill. Reduction of fractions.

Remarks. When reducing improper fractions to integers or mixed numbers, point out that a fraction is simply an indicated division, and carry out the division. Any attempt at a more detailed explanation is likely to befog the whole thing.

Page 154

Review and Drill. Addition of simple fractions.

Supplementary Matter. Play game No. 2, page 30. Divide B I, II by D I, II, page 47. Find quotients to the nearest tenth. The quotients, in order, are:

146.9; 261.3; 14.4; 214.8; 374.8; 456.3; 616.0; 114.6; 812.1; 1285.9.

Page 155

Review and Drill. Subtraction of fractions.

Supplementary Matter. Play game No. 4, page 30. Add C, D, E, F of II and III, page 46. The sums, in order, are:

$2\frac{1}{4}$; $1\frac{49}{360}$; $1\frac{158}{240}$; $2\frac{3}{8}$; $2\frac{5}{16}$; $16\frac{11}{30}$; $13\frac{19}{24}$; $12\frac{1}{5}$; $13\frac{17}{40}$; $18\frac{7}{24}$.

If additional drill in subtracting fractions is needed, use the following: $8\frac{1}{2}-3\frac{1}{4}$; $21\frac{3}{4}-17\frac{1}{2}$; $8\frac{3}{4}-4\frac{7}{8}$; $14\frac{1}{2}-6\frac{2}{3}$; $15-3\frac{4}{5}$; $6\frac{1}{3}-4\frac{1}{2}$; $7\frac{3}{8}-3\frac{3}{4}$; $1\frac{5}{32}-\frac{9}{16}$; $3\frac{1}{3}-1\frac{1}{2}$; $4\frac{1}{4}-1\frac{1}{8}$; $9\frac{1}{4}-3\frac{1}{3}$; $6\frac{1}{3}-2\frac{1}{2}$; $4\frac{1}{2}-1\frac{3}{4}$; $7\frac{1}{5}-2\frac{5}{6}$; $14\frac{3}{4}-8\frac{4}{5}$.

The results, in order, are:

$5\frac{1}{4}$; $4\frac{1}{4}$; $3\frac{7}{8}$; $7\frac{5}{6}$; $11\frac{1}{5}$; $1\frac{5}{6}$; $3\frac{5}{8}$; $\frac{19}{32}$; $1\frac{5}{6}$; $3\frac{1}{8}$; $5\frac{11}{12}$; $3\frac{5}{6}$; $2\frac{3}{4}$; $4\frac{11}{30}$; $5\frac{19}{20}$.

Page 156

New Matter. Definition of fractions and of prime and composite numbers.

Review and Drill. Addition and subtraction of fractions.

Remarks. As far as possible, get the children to find the lowest common denominator by proceeding as on page 42. In case some children cannot do this, study again page 49. The fractions to be added on this page are as complicated as any that are likely to occur in practice.

Page 157

Review and Drill. Multiplication of fractions and of mixed numbers.

Remarks. Study again, if necessary, the four-step method given on page 70. Two numbers such as those given in example 23 should never be multiplied by the ordinary method. Business men would always use the four-step method here.

Page 158

Review and Drill. Division of fractions and mixed numbers.

Page 159

Review and Drill. Reduction of fractions to decimals. Practice in the four fundamental operations on fractions and mixed numbers.

Material for First Examination, Sixth Grade.

Add:

1.	2.	3.	4.	5.
$419.60	$456.70	$416.09	$657.60	$438.91
197.50	451.93	312.00	543.50	577.95
678.75	272.80	855.35	442.56	887.75
18.64	540.26	179.10	568.19	116.25
931.50	645.34	971.35	644.75	912.86
356.42	567.50	582.95	561.20	519.31
159.36	697.85	788.16	577.65	300.90
282.54	816.90	796.50	985.32	505.56
$3044.31	$4449.28	$4901.50	$4980.77	$4259.49

Subtract:

6.	7.	8.	9.	10.
\$4191.78	\$4567.08	\$6756.45	\$67875.18	\$18491.67
1628.42	1978.43	4367.82	64935.00	13974.28
\$2563.36	\$2588.65	\$2388.63	\$2940.18	\$4517.39

Multiply:

11.	12.	13.	14.	15.
796.5	582.95	6829.32	54676.3	674.91
678.8	39.86	846.7	62.82	86.48
540,664.2	23,236.387	5,782,385.244	3,434,765.166	58,366.2168

Find quotients to the nearest hundredth:

16. $59.32\overline{)471.678}$ 7.95 17. $28.47\overline{)436.800}$ 15.34 18. $64.62\overline{)479.284}$ 7.42

Add:

19.	20.	21.	22.	23.
$3\frac{1}{5}$	$71\frac{1}{2}$	$16\frac{3}{4}$	$43\frac{3}{5}$	$81\frac{1}{6}$
$2\frac{1}{3}$	$31\frac{3}{4}$	$42\frac{1}{2}$	$12\frac{1}{2}$	$32\frac{1}{4}$
$6\frac{1}{4}$	$56\frac{2}{3}$	$67\frac{2}{3}$	$17\frac{1}{3}$	$15\frac{2}{3}$
$2\frac{1}{2}$	$12\frac{1}{6}$	$15\frac{3}{8}$	$15\frac{5}{6}$	$43\frac{1}{2}$
$14\frac{17}{60}$	$172\frac{1}{12}$	$142\frac{7}{24}$	$89\frac{4}{15}$	$172\frac{7}{12}$

Subtract:

24.	25.	26.	27.	28.
$18\frac{1}{3}$	$21\frac{1}{2}$	$34\frac{1}{4}$	$43\frac{2}{3}$	$23\frac{3}{8}$
$12\frac{4}{5}$	$12\frac{5}{8}$	$25\frac{3}{8}$	$26\frac{7}{8}$	$14\frac{5}{6}$
$5\frac{8}{15}$	$8\frac{7}{8}$	$8\frac{7}{8}$	$16\frac{19}{24}$	$8\frac{13}{24}$

Multiply:

29.	30.	31.	32.	33.
$4\frac{1}{2}$	$12\frac{1}{3}$	$42\frac{1}{5}$	$126\frac{1}{3}$	$16\frac{1}{3}$
$3\frac{1}{3}$	$6\frac{1}{4}$	$25\frac{1}{6}$	$147\frac{1}{4}$	$12\frac{1}{4}$
15	$77\frac{1}{12}$	$1062\frac{1}{30}$	$18{,}602\frac{7}{12}$	$200\frac{1}{12}$

Divide:

34. $126\frac{1}{3} \div 4\frac{1}{2} = 28\frac{2}{27}$. 35. $147\frac{1}{4} \div 3\frac{1}{3} = 44\frac{7}{40}$. 36. $42\frac{1}{5} \div 6\frac{1}{4} = 6\frac{94}{125}$.

Reduce the following fractions to four-place decimals:

37. $\frac{7}{9} = .7778$. 38. $\frac{9}{13} = .6923$. 39. $\frac{7}{16} = .4375$. 40. $\frac{9}{17} = .5294$.

41. At $1.10 per thousand cubic feet of gas what is the bill for 4860 cubic feet? **$5.35**

42. At $56.50 per thousand feet of lumber what is the bill for 14680 feet. **$829.42**

43. At $8.60 per ton of coal what is the value of a carload weighing 78600 pounds? **$337.98**

44. A box is 18 inches deep, 27 inches wide and 42 inches long. How many cubic feet will it hold? **11.81 (cu. ft.)**

45. How many cubic yards are there in an excavation 50 feet long, 34 feet wide, and $4\frac{3}{4}$ deep? **299.07 (cu. yd.)**

46. At 57 pounds to the cubic foot what is the weight of a block of ice 1 foot thick, 18 inches wide, and 30 inches long? **$213\frac{3}{4}$ (lb.)**

47. A boy has the following grades in arithmetic: 79, 84, 88, 83, 91, and 92. Find the average of these grades to the nearest unit. **86**

48. At $160 an acre what is the value of a farm 210 rods by 320 rods? **$67,200.00**

Page 160

New Matter. Formal definition of percentage.

Review and Drill. Simple problems in percentage.

Remarks. The purpose, at this point, is to acquire a more intimate and direct acquaintance with fractions whose denominators are 100; that is, with per cents.

Page 161

New Matter. The practical importance of percentage. A discussion of the solution of problems involving percentage.

Remarks. An understanding of this page is as important as that of any page in the whole book, and no effort should be spared to achieve this purpose.

Supplementary Matter. Have the children find as many instances as they can where percentage is used.

Page 162

New Matter. Definitions of the terms "base," "rate," and "percentage."

Remarks. Note that the word percentage is used with two different meanings, one of which indicates the topic in arithmetic dealing with so-called "percentage," and the other the result found as indicated on this page. The word will be used mainly in the latter sense in the course in arithmetic, and no emphasis need be placed on the other. However, each of these meanings is in perfectly good standing, and we must finally know them both.

Review and Drill. Simple problems in finding percentage.

Page 163

New Matter. Formal work in finding the percentage.

Review and Drill. Simple problems in finding percentage.

Remarks. These problems are simple exercises in multiplication, and the child should very soon get the idea of percentage well enough in mind so that he can solve them with the same speed that he solves ordinary problems in multiplication of decimals.

Pages 164 and 165

Applications. Miscellaneous problems in finding percentages.

Remarks. Note the remarks at the top of this page. In solving a problem in percentage always decide first which number is the base, which the rate, and which the percentage. In reciting on problems such as are given on this page, the child should be asked to point out these three numbers in each problem until he can do it unhesitatingly and with accuracy.

Supplementary Matter. At the end of page 165, play game No. 3, page 30. For the first event, add III, IV A to E, and V, VI A to E, page 47. The sums, in order, are:

34342.77; 6538.962; 202.297; 17.492; 27.00; 11538.357; 7096.692; 10255.053; 29.982; 31.33.

For the second event, multiply A III, IV by D III, IV, page 47. The products, in order, are:

766.0064; 3.5148; 131.77469; 1913.536; 31859.076; 141.66449; 4747.7988; 6983.9195; 25181.6382; 4015.98384.

In the third event, divide A III, IV by D III, IV, page 47, getting each quotient to the nearest tenth.

The quotients, in order, are:

41414.7; 6102.1; 4724.9; 355.5; 1060.9; 855.2; 446.7; 1661.8; 2554.0; 15562.0.

Page 165

Solution of Problems

1. $2000 \times .32 = 640$ (pounds of copper).
   $640 \times \$0.19 = \$121.60$ (value of copper).
6. 15% of \$1800 = \$270 (total rent).
   $\$270 \div 12 = \$22.50$ (monthly rent).
7. 24% of \$1800 = \$432 (total for food).
   $\$432 \div 12 = \$36$ (monthly expense for food).
9. 4% of 10480 = 419.2 (lbs. butter fat).
   $419.2 \times \$0.38 = \$159.30$ (value to nearest cent).

Page 166

New Matter. Fractional per cents.

Remarks. Compare the list of fractions and their decimal and per cent equivalents on this page with the fractional and decimal equivalents given on page 130. It is not recommended that the child should actually commit these to memory, but nevertheless they should be so treated that after a short time he will know practically all of them. This table includes practically all of the fractional and decimal equivalents that the child needs to know.

Page 167

Review and Drill. Finding fractional per cents of numbers, and per cents equivalent to certain fractions.

Remarks. These problems should be solved by referring to the table on page 166.

Pages 168 and 169

New Matter. Practical uses of fractions of a per cent.

Supplementary Matter. If the school is in the country, get all the data that can be had of the kind given on page 169. If there is a creamery in the neighborhood, there will be children in the school from a family delivering milk to the creamery, and the neeessary data may be obtained from them.

Solution of Problems (page 169)

1. $.041 \times 9420 = 386.22$ (lbs. butter fat).
   $386.22 \times \$.42 = \$162.21$ (value to nearest cent).

Pages 170 and 171

New Matter. Finding the rate.

Remarks. Note the two solutions given on the top of page 171. If the first solution is adopted we have the following sequence of steps: (*a*) Expressing the rate as a fraction; (*b*) Reducing the fraction to per cent. The second method is obvious and needs no further remarks here. The teacher should select one of these solutions and use that exclusively.

Page 172

Applications. Miscellaneous problems in percentage.

Remarks. All problems on this page, except one, require the finding of the percentage. This arrangement is made with a definite purpose, inasmuch as it is best not to mix different operations in the first application of a comparatively new subject.

Solution of Problems

9. The total number in school is 748.
   $124 \div 748 = .166 = 16.6\%$.

The other per cents, in order, are: 15.0%, 14.2%, 13.1%, 12.3%, 11.5%, 9.6%, 7.8%. The sum of these is 100.1%. The difference is due to the approximate character of these per cents.

Page 173

Drill in Finding the Rate and the Percentage. Miscellaneous problems.

Remarks. In this book, the problem of finding the base when the rate and the percentage are given is not given as a separate case.

The finding of the rate is an important problem, which occurs frequently in practice.

Page 174

Applications. Miscellaneous problems in percentage.

Remarks. In connection with problems Nos. 1, 2, and 3, bring out that the facts used are such as occur in real life. Point out that a certain part of commercial accounts is always lost; that in most subscriptions some people fail to pay the subscription, and so on. In connection with problem 4, point out that collection agencies make all their income by getting a certain per cent of the debts they collect. If the school is in the country, enlarge upon problems No. 5 and 6.

In connection with problem No. 9, point out, without going farther into the subject of insurance, that property is usually insured for a certain per cent of its full value. Thus it may be that a house in a certain neighborhood which would sell for six thousand dollars is insured for forty-five hundred.

Supplementary Matter. Endeavor to get the children to bring in material for problems like these. There may be a child in the class whose father has debts owing him, of which he estimates that a certain per cent can be collected. Some may know about a church subscription where a certain per cent was finally paid in. If the school is in the country, the teacher may place on the board the following list of shrinkages of corn put into the crib: Nov., 5.2%; Dec., 6.9%; Jan., 7.5%; Feb., 7.8%; March, 9.7%; April, 12.8%; May, 14.7%; June, 16.3%; July, 17.3%; August, 17.8%; Sept., 18.2%; Oct., 18.2%.

Someone may be able to bring in a statement about the normal value of a house which is insured for a certain per cent of its value.

Page 175

Oral Review.

Remarks. If the children have difficulty in answering any of the questions on this page, lead up to the answers through simple concrete problems. Thus, if the child cannot answer the questions under No. 6, take a certain income, such as two thousand, and suppose that seventy-five per cent of it is spent. "How do you find out how much is spent, and how much is saved?" Ask questions of this kind to enable the children to answer the questions asked in problem 6 on this page. The accuracy and readiness with which such questions are answered afford a good test of how well the children have learned some of the really important things in arithmetic

Pages 176 and 177

New Matter. Summarizing the problems in percentage under the general idea of product and factors.

Remarks. Equation (P) on page 176 contains the fundamental idea of percentage. Unless the child knows just what this equation means, he is not qualified to solve properly any problem whatever involving percentage. The aim should be to have the child understand thoroughly that every problem in percentage is one of the three problems discussed on these two pages, and that in reality the theory is no more difficult than is the finding of any one of the three numbers in $3 \times 4 = 12$, when the other two numbers are given. Lead the child to ask questions about what he does not understand.

Page 178

New Matter. Discount; price; marking price.

Remarks. The only new thing which confronts the child on this page are the facts which underlie the problems. The effort should be to bring clearly and vitally before him the business practice in connection with commercial discount, the nature of list prices and marking prices, and how discount is figured. All

this is, strictly speaking, information about current business methods rather than arithmetic. The child must, however, know about these things before he can solve the problems intelligently, and when he once knows about them the solution of these problems is no more difficult than that of many others which he has already solved. The difficulty is one of current business, and not of arithmetic.

Supplementary Matter. To bring home to the children the facts required, it may be well to have them bring in advertisements cut from local papers; possibly a catalog of a wholesale house can be found, with discounts from list prices quoted. The work on this topic has not been well done until the child is thoroughly informed of the real nature of the various kinds of commercial discount, and if he is well informed on this point and knows the elements of the arithmetic he has studied thus far, he cannot possibly have trouble with this subject. Do not avoid the real difficulty by giving the child a rule of arithmetic and telling him to solve a certain number of problems by means of it. That leaves him as much in the dark as ever.

Page 179

New Matter. Short method for finding the selling price, the list, or marking price, and the rate of discount.

Remarks. It would be fatal to good teaching to give the child a rule for solving such problems. If difficulty arises, ask questions like the following:

In a class, fifty per cent of the children are boys. How many per cent of the class are girls?

A certain baseball team won sixty per cent of all the games they played. How many per cent of the games did they lose?

Continue with similar questions until the matter is made clear. Then, finally:

If goods are listed at a certain price and sold fifteen per cent below that price, how many per cent of the list price does the seller get?

Note that problems involving this same idea have been solved frequently before; as, for instance, problems 4 and 5 on page 155; problems 3 and 4 on page 174; problems 5, 6, and 10 on page 175. If thought best, these problems should be recalled, and it should be made absolutely clear that there is no new difficulty whatever on this page. By doing so, we shall give a valuable review of what has gone before and secure thoroughgoing understanding of the work at hand.

Solution of Problems

8.	Gross amount	\$155.00
	12% Discount	18.60
	Net	\$136.40
9.	Gross amount	\$347.00
	8% Discount	27.76
	Net	\$319.24

As alternative solutions we have: 88% of \$155.00 = \$136.40, and 92% of \$347.00 = \$319.24

Pages 180 and 181

New Matter. Several short methods for finding the selling price when the list price and the rate of discount are given.

Remarks. As shown on page 180, the shortest method for finding the selling price is not the same for all types of examples. This page affords opportunity for exercise of ingenuity on the part of the more wide-awake children. By referring to the fractions and their per cent equivalents given on page 166, material for other oral exercises of this sort may be suggested. The problems on page 181 may now be solved directly by means of the ideas given on page 180.

Pages 182 and 183

New Matter. Profit and loss.

Remarks. Make sure that the children get a clear idea of the only new element in this subject, namely, the meaning of the words "profit" and "loss." The oral exercises given on the two pages should serve to develop these ideas. Remember that at the outset the children have no clear idea as to what is meant by profit, by gain, and by loss, and how the profit is computed.

The teacher should have clearly in mind that "gain" or "profit" is computed in two different ways. A real estate dealer buying a business block for $100,000 and selling it for $125,000 would invariably say that he made 25% on this transaction. That is, he would compute the rate of profit on the purchase price as a base. On the other hand a retail dealer buying an article for $1.00 and selling it for $1.50 would in many cases speak of his profit as 33⅓%, and not as 50%. That is, he would compute the gain or profit as a rate on the selling price. The usage in this respect differs in different parts of the country and in different kinds of business. The expression "gross gain" may be used to avoid another clash with those who want to use "gain" to represent merely "net gain." In some cases the dealer adds his cost of selling to his buying price, and he computes the rate of profit on this sum. In this case "profit" is used to indicate "net profit" or "net gain."

Page 184

New Matter. Short method for finding the selling price when the cost and the rate of gain or loss are known.

Remarks. Note that this is precisely the same subject as that considered on page 179. Make sure that the children see this clearly, and thus lead them to find their own rule for solving the problems proposed here. To give children an independent rule by which to solve these problems would be the worst kind of teaching imaginable.

Page 185

New Matter. Finding the rate of gain or loss.

Remarks. Make sure the children understand that this is precisely the same problem as that of finding the rate in percentage which was considered on page 171.

Supplementary Matter. At this stage it should be possible to get the children to bring in concrete cases from their own experience where things have been bought for a certain sum and sold for a certain other sum. Houses, automobiles, and so on should furnish accessible material. The children will bring in the material and will take a real interest in finding the rate of gain or loss.

Page 186

New Matter. Commission. The kinds of business transacted on commission. The method of computing commission.

Remarks. Again, it should be made perfectly clear that no new principle of arithmetic is involved, and that the only thing that can possibly cause trouble is the fact of commission itself and how commission is computed. When the child understands these things he will need no rule for solving problems that come under this topic.

Page 187

Applications. Miscellaneous problems involving commission.

Supplementary Matter. Have the children find out what work is done on commission in the neighborhood. If necessary, help them to make a fairly complete list. By using the data thus obtained, have the children make problems for themselves, and solve them. In this way they will be solving real live problems.

Drill in Fundamentals. Play game No. 3, page 30. For the first event, add II, III A to E, and IV, V A to E, page 47. The sums, in order, are:

38366.74; 3463.212; 275.553; 13.627; 18.88; 29473.734; 10738.449; 58.886; 24.034; 20.74.

For the second event, use D I, II as the base, and E I, II, page 32, as the percentage. Find the rate to the nearest tenth of one per cent. The rates, in order, are:

8.3%; 31.2%; 11.0%; 37.2%; 4.5%; 7.4%; 33.6%; 20.6%; 16.8%; 11.2%.

Page 187

Solution of Problems

4. $50 \times \$.38 = \$19.00$ (selling price).
$\$19.00 - \$1.90 = \$17.10$ (sent to producer).
5. $180 \times \$.13\frac{1}{2} = \$24.30$ (selling price).
$12\frac{1}{2}\%$ or $\frac{1}{8}$ of $\$24.30 = \$3.04$ (commission).
$\$24.30 - \$3.04 = \$21.26$ (amount received by farmer).
6. 4% of $\$125000 = \$5000$.
Cost of building is $\$125000 + \$5000 = \$130{,}000$.
7. Commission is \$100.00.
Owner gets $\$1250 - \$100 = \$1150$.
8. Commission is \$82.50.
Total cost of boat is \$2832.50
9. Commission is \$375.00.
$\$15000 - \$375 = \$14625$ (seller's receipt).

Material for Second Examination, Sixth Grade.

Add:

1.	2.	3.	4.	5.
\$519.74	\$528.92	\$581.79	\$552.85	\$584.71
291.46	836.50	598.50	849.82	973.90
819.80	191.37	239.51	405.46	153.83
934.60	757.10	876.45	739.80	405.29
275.90	919.40	861.79	189.61	599.65
878.60	584.80	449.38	936.72	418.50
354.50	974.40	953.14	755.47	848.67
678.40	869.60	846.46	314.20	425.52
519.35	543.46	207.00	901.59	601.27
\$5272.35	\$6205.55	\$5614.02	\$5645.52	\$5011.34

Subtract:

6.	7.	8.	9.	10.
4791.50	52982.56	39190.86	6943.27	126782.48
2193.76	24636.90	12879.34	2873.91	72847.92
2597.74	28345.66	26311.52	4069.36	53934.56

Multiply:

11.	12.	13.	14.	15.
19745	963684	986013	32749	814267
9376	5642	3482	2391	4792
185,129,120	5,437,105,128	3,433,297,266	78,302,859	3,901,967,464

Find quotients to the nearest thousandth:

16. 12.97)814.62 — 62.870

17. 8.142)73.417 — 9.017

18. 91.42)714.370 — 7.814

Add:

19.	20.	21.	22.	23.
$57\frac{1}{8}$	$47\frac{3}{5}$	$74\frac{5}{8}$	$15\frac{7}{8}$	$71\frac{3}{4}$
$21\frac{3}{4}$	$21\frac{2}{3}$	$12\frac{3}{4}$	$12\frac{4}{5}$	$37\frac{1}{5}$
$27\frac{2}{3}$	$63\frac{1}{2}$	$36\frac{2}{3}$	$72\frac{3}{4}$	$17\frac{2}{3}$
$18\frac{1}{2}$	$42\frac{1}{4}$	$24\frac{1}{2}$	$81\frac{2}{3}$	$14\frac{1}{4}$
$125\frac{1}{24}$	$175\frac{1}{60}$	$148\frac{13}{24}$	$183\frac{11}{120}$	$140\frac{13}{15}$

Multiply:

24.	25.	26.	27.	28.
$1\frac{1}{5}$	$6\frac{1}{2}$	$212\frac{1}{5}$	$618\frac{1}{2}$	$53\frac{1}{4}$
$3\frac{7}{8}$	$4\frac{1}{4}$	$75\frac{1}{4}$	$412\frac{1}{3}$	$25\frac{1}{3}$
$4\frac{13}{20}$	$27\frac{5}{8}$	$15968\frac{1}{20}$	$255028\frac{1}{6}$	1349

Find the percentage in each of the following:

	Base	*Rate*	*Percentage*		*Base*	*Rate*	*Percentage*
29.	846	27%	228.42.	32.	89.12	16%	14.2592.
30.	493.7	41%	202.417.	33.	74200	21%	15582.0.
31.	129.86	55%	71.423.	34.	8760	53%	4642.8.

Find the rate to the nearest tenth of one per cent in the following:

	Base	*Rate*	*Percentage*		*Base*	*Rate*	*Percentage*
35.	49.60	44.0%	21.80	38.	4280	12.1%	520
36.	106.00	5.7%	6.00	39.	8400	7.0%	590
37.	89.12	20.9%	18.60	40.	211	4.7%	10

41. The ore from a certain mine contains 12% copper. How many pounds of copper are there in a carload of this ore weighing 94800 pounds? 11,376 (pounds)

42. A man puts 2840 bushels of potatoes into his cellar in the fall. If they shrink 18%, how many bushels will he have to sell in the spring? **2328.8 (bu.)**
43. An automobile costing $2750 decreased 28% in value. How much was it worth then? **$1980**
44. In a school having 635 pupils 587 were promoted. How many per cent of these pupils were promoted? **92%**
45. An ice-man started out with a load of 5800 pounds and sold 4950 pounds. The rest was wasted. How many per cent were wasted? **15%**
46. In a consignment of 900 dozen eggs $21\frac{1}{2}$ dozen were broken. How many per cent of the eggs were broken? **2.4%**
47. Find the net price of an article listed at $85.00 and sold at a discount of 35%. **$55.25**
48. Find the net price of an article listed at $17.50 and sold at a discount of 15%. **$14.88**
49. Find the net price of an article listed at $5.50 and sold at a discount of 40%. **$3.30**
50. Find the selling price of an article bought for $3.50 and sold at a gain of 35%, using the buying price as the base. **$4.73**
51. Find the selling price of an article bought for $85.00 and sold at a gain of 17%, using the buying price as the base. **$99.45**
52. Find the selling price of an article bought for $45.00 and sold at a gain of 40%, using selling price as the base. **$63.00**
53. Find the rate of gain on an article bought for $3.50 and sold for $5.00, using the selling price as the base. **30%**
54. Find the rate of gain on an article bought for 45 cents and sold for $1.00, using the selling price as the base. **55%**
55. Find the rate of gain on an article bought for $1.25 and sold for $2.00, using the selling price as the base. **$37\frac{1}{2}$%**
56. An agent sold a lot for $1700, charging 6% commission. What did the owner of the lot get? **$1598.00**
57. An agent sold an auto for $1150, receiving 15% commission. What was his commission? **$172.50**

Page 188

New Matter. Interest for one year.

Remarks. The fact that interest is paid will be known to many of the children from their experience with accounts in the savings bank. They will also know about getting interest on liberty bonds. It would not do, however, to begin the study of interest with these, because the savings bank uses compound interest, and the interest on liberty bonds is payable semiannually instead of annually. An interesting discussion may be developed on "Who borrow money?" Do the poor people only borrow, or do the well-to-do people also borrow? Bring out that business people borrow from time to time, even though they are very wealthy. Bring out that a farmer who borrows money to improve his land may really be better off than his neighbor who does not borrow and who does not improve his land. Similarly, a business man who borrows to put up a new store building may be making more money than his neighbor who continues to rent an old store.

Also bring out the reasons why interest is paid on money that is borrowed. Suppose money could be borrowed on good security without interest; suppose a man had $10,000 with which to build a home; suppose, then, that by giving a mortgage on his home as security he could borrow three thousand more without interest, and use it in beautifying the grounds around his house. Then he could go on for the rest of his life enjoying the increased beauty of his home without additional cost and without extra trouble except to borrow in the first place, and to borrow from someone else in case the one who first made the loan should want his money. Under such conditions everybody would want to borrow money and nobody would want to lend, and the upshot of the whole thing would be that no money would be loaned.

Note that the only new thing in connection with interest at this stage is the fact of interest itself, and that it is computed at a certain per cent of the amount loaned for one year. When the children understand this clearly, they will need no rules for solving these problems.

Page 189

Review and Drill. Exercises on interest.

Remarks. Make it clear that the first problem is precisely like those solved in finding a certain per cent of a number. Connect problem 2, where a fractional per cent is used with the work on pages 166–168. If there is difficulty, review those pages. There is nothing new in the way of arithmetic here.

Drill in Fundamentals. Play game No. 3, page 30. For the first event, subtract B I, II, III from A I, II, III, page 47. The remainders, in order, are:

2409.856; 1102.83; 5314.37; 6456.6; 6548.937; 8713.04; 141.535; 5551.21; 5290.36; 4113.008; 5217.16; 110.085; 71.02; 211.46; 5484.65.

For the second event, divide B III, IV by D III, IV, page 47. Find each quotient to the nearest thousandth. The quotients, in order, are:

3053.235; 1515.208; 4299.701; 264.371; 60.046; 775.479; 4.481; 105.205; 1003.975; 1434.134.

For the third event, use D I, II for the base and E I, II, page 47, for the percentage. Find the rates to the nearest tenth of one per cent. The per cents, in order, are:

148.8%; 19.4%; 179.1%; 212.9%; 118.6%; 108.0%; 5000%; 7.7%; 68.0%; 896.6%.

Page 190

Applications. Miscellaneous problems involving percentage and interest.

Supplementary Matter. The children may be induced to bring in material from their local experiences from which problems like these can be made. If the school is in the country, it should be possible to find out about farm loans which have been made in the locality, and about the rate of interest paid on them. It may be that the township or the county has borrowed money to build a road, a bridge, or a schoolhouse. Similar problems may be made in the city. It may be possible to find how much a school building cost and how much interest per year is required to carry this amount. In all localities there are automobiles that are depre-

ciating in value and for which interest is paid on the purchase price. It will be difficult to compute the actual yearly cost of running an automobile. Most people forget the interest on the investment and the depreciation of the machine. Never forget that this local material constitutes the very best basis for really live work in arithmetic. In after life this is the kind of material to which each person has to apply his arithmetic. If the arithmetic is learned only from books, and is applied only to problems stated in books, the application to practical problems will not be made readily.

Solution of Problems

7. Interest is $45150

$45150 ÷ 2400 = $18.81 (share of interest for each child).

8. Rent = 12 × $60. = $720. (cost of rented house).

6% interest on $8000 =	$480.00
Other expenses	260.00
Total expenses	$740.00

Rented house is $20 per year cheaper.

9.

Depreciation of machine	$450.00
7% of $1650 (interest)	115.50
Gasoline, etc.	470.00
Total cost	$1035.50

10. 240 × $110 = $26400

6% of $26400 =	$1584.00
Taxes	180.00
Repairs, etc.	250.00
Total yearly cost	$2014.00

Page 191

Review and Drill. Miscellaneous exercises involving simple applications of percentage.

Remarks. These exercises are intended to accustom the child to seeing the various applications of percentage placed together so that he may come to feel increasingly certain that all these are alike. The thing to be fought against is to have one rule for percentage, another for profit and loss, another for agent's commissions, and so on, thus tending to create the impression that these are separate and independent, even from the point of view of arithmetic, whereas as a matter of fact they are identical in this respect.

Pages 192 and 193

New Matter. Finding the rate of interest.

Remarks. Again, it must be made clear that the finding of the rate of interest, when the principal and the yearly rate of interest are given, is precisely the problem of finding the rate in percentage which was discussed on page 171. The steps for finding the rate should be: First, express the rate as a fraction; then reduce this to a decimal, and finally write this decimal as per cent. All of these ideas have been developed fully before, and now find effective review.

Supplementary Matter. Find local facts and make them the basis for exercises like those solved on these pages.

Page 194

Applications. Miscellaneous problems, involving the finding of the rate of interest.

Remarks. Problems of this kind are of very frequent occurrence and of great practical usefulness and should not be slighted. It may be noted that problems 5 and 10 are such as occur in the subject of investment. Thus it frequently happens that a bond yielding an income of $6.00 a year is bought at a sum something like $109.50, and that a share of stock yielding an income of $8.00 a year is bought for a sum something like $139.00. The method of solution is, of course, absolutely simple, and is the same as that developed on page 171.

Supplementary Matter. It should be easy to get the children to bring in problems similar to some of those given on this page. Problems 6 and 8 are such as may turn up in any thrifty family that has saved a little money and invested it, and the same is true of problem 11. By actually making up problems of this kind, the child not only gets a vital understanding of the meaning of the work, but he may also begin to develop the rudiments of the idea of saving and obtaining an income from investments. At this point it may be possible to take the current prices of Liberty Bonds and to thus find what rate of income an investment in such bonds will yield at the present time.

Solution of Problems

8. Rent = 12 × \$50 = ...	\$600
Taxes, etc.	175
Net income	\$425

\$425 is 5.6% of \$7600.

11. Yearly increase in income is \$12 × \$10 = \$120.00.
\$120 is 4.8% of \$2480.

Page 195

New Matter. The finding of interest when the time is a certain whole number of years.

Remarks. Be sure to bring out the simplicity of this case. This is best done by asking the youngster to solve the problems, without giving him a rule or making further remarks.

Page 196

New Matter. The finding of interest when the time is a whole number of months.

Remarks. The simplicity of this case is best brought out by suggesting that so many months is a certain fraction of a year, as indicated on this page.

Page 197

New Matter. Finding the interest when the time is given as a certain number of days, or as months and days.

Remarks. For the purpose of the first understanding of the subject, it is easiest to reduce the time to a fraction of a year and then to multiply, as is illustrated in the example solved on this page. Do not attempt at this time to teach any of the more specialized methods of computing interest. If the child leaves school at the end of the sixth grade he is sure to forget such special method, and if he continues through the seventh grade he will be taught whatever other method may be thought desirable.

Page 198

New Matter. Finding interest when a fractional rate of one per cent is used.

Remarks. This is a rather common problem. The most convenient form of solution is the one shown here, by which first one per cent is found, then the fractional per cent, and then the whole-number per cents, and these finally added. Do not over-explain. Just state the steps in order.

Page 199

Review and Drill. Computing interest.

Remarks. Note that on this page there is no case where time is given as a combination of months, years and days. There are examples involving months only, years only, and days only.

Page 200

New Matter. Finding interest when the first and last dates of the interest period are given.

Remarks. This involves the process of finding the length of time between two dates, which is done very informally at this stage. It is not thought best to introduce here the more complicated methods for finding the length of time.

Page 201

New Matter. The savings bank; the interest period; the interest rate.

Remarks. Many of the children will have had personal experience with the savings bank. Bring out that the savings bank computes interest on the smallest amount of an account during any one interest period. Note that at this stage the only computation of interest is simple interest; hence the idea of compound interest is not brought in. Note, also, the simple way for finding the interest for six months, three months, etc., namely, by first finding the interest for a whole year and then taking one-half or one-fourth of that amount.

Page 202

Review and Drill. Miscellaneous exercises involving percentage.

Standard Tests. Second Test, Grade Six (see page 308)

Test in Addition. Time, 6 minutes.

Half of the class should have 10 or more correct answers.

1.	2.	3.	4.	5.
$7.84	$5.74	$3.42	$7.48	$4.60
3.92	1.82	3.97	2.39	4.08
.67	2.46	3.16	5.67	.92
3.94	7.03	.49	3.09	6.19
2.01	.89	.24	1.78	2.40
7.68	.42	3.07	.51	3.17
.19	1.64	1.57	.79	.38
$26.25	$20.00	$15.92	$21.71	$20.74

6.	7.	8.	9.	10.
$1.76	$2.90	$7.02	$5.74	$3.68
5.78	3.54	4.63	2.93	4.09
.29	2.70	1.92	.56	1.69
1.69	6.38	.36	.63	4.00
.42	.23	2.94	7.19	.37
3.63	3.78	.47	5.17	.12
8.76	.79	1.75	3.46	6.23
$22.33	$20.32	$19.09	$25.68	$20.18

11. $3.78	12. $2.54	13. $5.78	14. $4.83	15. $3.68
2.69	1.73	4.36	5.62	5.06
1.94	2.75	.73	.65	.37
.36	3.23	2.48	.72	.54
.18	.89	.35	3.16	3.16
2.78	.72	4.69	1.37	9.61
3.96	6.05	7.38	2.07	1.98
$15.69	$17.91	$25.77	$18.42	$24.40

16. $ .87	17. $3.72	18. $4.78	19. $ .18	20. $7.38
2.05	9.61	2.46	3.45	1.79
1.97	.83	3.74	.75	2.64
2.36	.26	.28	4.67	.59
.95	4.73	1.46	1.75	3.72
8.70	3.49	.89	3.72	5.46
7.56	1.96	1.37	8.49	.38
$24.46	$24.60	$14.98	$23.01	$21.96

Test in Subtraction. Time, 6 minutes.

Half of the class should have 20 or more correct answers.

1. 627451	2. 630487	3. 567349	4. 217598	5. 342657
315329	210295	325724	195243	127346
312122	420192	241625	22355	215311
6. 237216	7. 867341	8. 636892	9. 498647	10. 451652
175113	352139	253751	364592	238421
62103	515202	383141	134055	213231
11. 704346	12. 716361	13. 350462	14. 170284	15. 375496
372132	209240	147231	160721	254963
332214	507121	203231	9563	120533
16. 237469	17. 298802	18. 358356	19. 625891	20. 428638
175357	157641	247831	214568	274516
62112	141161	110525	411323	154122

21. 710652	22. 514692	23. 945279	24. 957485	25. 862303
209341	372471	234726	536219	251079
501311	142221	710553	421266	611236
26. 546268	27. 316297	28. 947236	29. 690135	30. 646129
321745	254172	526074	379021	315072
224523	62125	421162	311114	331057
31. 753136	32. 962143	33. 718362	34. 238568	35. 789413
231027	541072	507841	197326	257309
522109	421071	210521	41242	532104
36. 529147	37. 172362	38. 436257	39. 470351	40. 671314
254042	157341	213625	256130	250172
275105	15021	222632	214221	421142

Test in Multiplication. Time, 6 minutes.

Half of the class should have 10 or more correct answers.

1. 321	2. 7325	3. 7462	4. 468	5. 2568
59	89	35	79	14
18939	651925	261170	36972	35952
6. 395	7. 3194	8. 764	9. 4687	10. 5674
47	28	35	91	12
18565	89432	26740	426517	68088
11. 572	12. 2891	13. 4657	14. 8749	15. 158
39	36	31	52	64
22308	104076	144367	454948	10112
16. 3129	17. 2319	18. 467	19. 869	20. 5614
57	28	35	47	82
178353	64932	16345	40843	460348

Test in Division. Time, 6 minutes.

Half of the class should have 5 or more correct answers.

1. $315\overline{)379864}$ 1205–289
2. $426\overline{)319247}$ 749–173
3. $237\overline{)357146}$ 1506–224
4. $423\overline{)190346}$ 449–419
5. $728\overline{)629170}$ 864–178
6. $189\overline{)375465}$ 1986–111
7. $319\overline{)154056}$ 482–298
8. $246\overline{)741620}$ 3014–176
9. $427\overline{)294628}$ 689–425
10. $243\overline{)375462}$ 1545–27

Page 203

Applications. Miscellaneous problems involving percentage and interest.

Remarks. It may well be that the children have been brought to a state of understanding of interest and percentage, so that there will be no need of solving the problems on these pages.

Solution of Problems

4. \$11,500 − \$10,000 = \$1,500 (gain).
   \$1,500 is 15% of \$10,000 (rate of gain).

7. 7% of \$7600 = \$532.00 (gain).
   \$7600 + \$532 = \$8132 (selling price).

11. The cost of the soap at retail is 25 × \$.30 = \$7.50.
    The saving is \$2.50, which is $33\frac{1}{3}$% of \$7.50.

Pages 204 and 205

Applications. Miscellaneous problems involving percentage and its application.

Remarks. Whenever possible, solve the problems without using pencil and paper. There are several problems on these pages which can be solved this way.

Material for the Third Examination, Grade Six.

If it is desired to include examples in the four fundamental operations, use some of those given on pages 373-7 and 385-6.

Find the percentage in the following:

	Base	Rate	Percentage		Base	Rate	Percentage
1.	41.60	2%	.832.	4.	9160	$8\frac{1}{2}$%	778.60.
2.	79.85	3%	2.395.	5.	3570	$15\frac{1}{3}$%	547.40.
3.	167.55	$5\frac{1}{2}$%	9.2153.	6.	275.60	27%	74.412.

Find the rates to the nearest hundredth of one per cent.

	Base	Rate	Percentage		Base	Rate	Percentage
7.	79.50	3.77%	3.00.	10.	275.50	3.67%	10.00.
8.	12560	6.21%	780.	11.	39.75	5.03%	2.
9.	93.75	5.33%	5.	12.	8760	14.27%	1250.

Find the interest for one year on the following:

	Principal	Rate	Interest		Principal	Rate	Interest
13.	$1250	$5\frac{1}{2}$%	$68.75.	16.	$1780	$6\frac{1}{2}$%	$115.70.
14.	$8960	7%	$627.20.	17.	$3970	7%	$277.90.
15.	$3250	8%	$260.00.	18.	$19250	$5\frac{1}{2}$%	$1058.75.

Find the interest on the following:

	Principal	Rate	Time	Interest		Principal	Rate	Time	Interest
19.	$3400	5%	40 days	$18.89.	22.	$5500	$5\frac{1}{2}$%	76 days	$63.86.
20.	$2100	6%	75 days	$26.25.	23.	$240	7%	120 days	$5.60.
21.	$4950	6%	93 days	$76.73.	24.	$4600	$6\frac{1}{2}$%	55 days	$45.68.

25. A man buys goods for $115 and sells them for $150. What is his gain per cent on the purchase price? 30.4%
26. A merchant buys a piece of furniture for $30 and sells it at a gain of 80% on the purchase price. What is the selling price? $54
27. A dealer buys shirts for $1.20 and sells them for $2.00. What is the gain per cent on the selling price? 40%
28. A man sells an automobile for $1200, which was 75% of the purchase price. Find the purchase price. $1600

29. A man owes \$3480, and pays 40% of this debt. How much does he still owe? **\$2088**

30. A merchant expects to lose 7% of his accounts. If he has \$18640 in accounts, how much does he expect to collect? **\$17335.20**

31. An agent sells 150 dozen eggs at 38 cents a dozen. What is his commission if the rate is 7%. **\$3.99**

32. An agent sells a used piano for \$285. How much does the owner get if the agent charges 12% commission? **\$250.80**

33. A man buys a building for \$14500 and gets \$1300 net yearly income from it. What rate per cent is this on the investment? (Find result to the nearest tenth of one per cent.) **9.0%**

34. A man has investments amounting to \$54,800 and his income from them is \$4300. Find the rate of income on these investments to the nearest tenth of one per cent. **7.8%**

35. A farm is bought for \$280 an acre. If the net yearly income on the investment is \$22.50 per acre, what is the rate of income on this investment? Find the rate to the nearest tenth of one per cent. **8.0%**

Page 206

New Matter. Formal study of the solution of problems.

Remarks. Pages 206 to 220 are all devoted to the study of the solution of problems. Three distinct elements in the solving of problems are brought out here. These are:

(*a*) Stating all the steps in a solution before performing any of the computations.

(*b*) The principle of products and factors; that is, if the product and one factor are given, the other factor may be found by dividing.

(*c*) The cancelling of the common factor in the numerator and denominator.

The teacher should have these three elements clearly in mind, and make it her purpose to develop them so that they will stand out clearly in the child's mind.

Page 207

Review and Drill. Use of the principle of products and factors.

Remarks. There should be reference, in connection with these problems, to very simple combinations. On this page the reference is to the combination $3 \times 4 = 12$. One really essential element in arithmetic is to get the children to grasp thoroughly that the ideas involved are very simple and very far-reaching.

If literal symbols (such as x) have been introduced, these should be used consistently on pages 206–219. The literal symbols may appropriately be introduced for the first time at this point.

Pages 208 and 209

New Matter. Area; and the idea of products and factors.

Applications. Miscellaneous problems involving area.

Remarks. In assigning the work on pages like these, first ask the children to read the problems and to make up their minds fully how each is to be solved. This may be developed in a class recitation. Then have them carry out the proposed solutions. Discuss thoroughly problem 10 on page 208. Such a discussion should develop a full statement of what the child really knows about the problem of finding areas and also about finding one of the dimensions when the area and the other dimensions are given.

Drill in Fundamentals. At the end of page 209, play game No. 3, page 30. For the first event, add VIII, IX A to E, and X, XI A to E, page 33. The sums, in order, are:

662,343; 497,931; 67,072; 6516; 561; 553,714; 567,982; 52316; 6740; 594.

For the second event, multiply B VIII, IX by D VIII, IX, page 33. The products, in order, are:

32,483,518; 13,952,169; 73,563,525; 55,265,056; 58,515,016; 30,742,410; 47,239,226; 14,859,711; 13,306,044; 8,771,862.

For the third event, divide A VII, VIII by D VII, VIII, page 33, carrying each quotient to the nearest hundredth. The quotients, in order, are:

189.34; 176.73; 63.72; 95.42; 59.88; 69.68; 113.04; 90.27; 112.73; 74.71.

Solution of Problems (page 208)

4. $18 \times 160 = 2880$ (area in sq. rd.).
$2880 \div 120 = 24$ (width in rods).

The solution may be put into this form:

$$\frac{18 \times 160}{120} = 24$$

9. $\dfrac{1250}{42}$ = length in feet of street paved in one day.

Paved in a week: $\dfrac{6 \times 1250}{42} = 178\frac{4}{7}$ (length in feet).

Solution of Problems (page 209)

1. $150 \times 25 = 3750$ (area in sq. ft.).
$\$7500 \div 3750 = \$2.00$ (price per sq. ft.).

$$\text{Or} \quad \frac{7500}{25 \times 150} = 2$$

2. $26.8 \times 94.3 = 2527.24$ (area in sq. ft.).
$2527.24 \times \$1.75 = \$4422.67$ (value).

4. $\dfrac{182 \times 290}{160}$ = number of acres.

And $\$10000 \times \dfrac{160}{182 \times 290} = \$30.31$ (price per acre)

5. $\dfrac{68 \times 120}{160}$ = number of acres.

$$860 \div \frac{68 \times 120}{160} = 860 \times \frac{160}{68 \times 120} = 16.9 \text{ (bu.)}.$$

$$\frac{260}{5\times350}= .149 \text{ (cost in dollars per sq. ft.).}$$

$$\frac{9\times260}{5\times350}=1.34 \text{ (cost in dollars per sq. yd.).}$$

9. $$\frac{35\times680}{9}\times1.80=4760 \text{ (cost in dollars).}$$

10. $$\frac{\overset{60}{\cancel{120}}\times\cancel{80}}{\underset{2}{\cancel{160}}}=60 \text{ (acres); } \$325.00\div60=\$5.42.$$

Pages 210 and 211

Review. Volumes; and the principle of product and factors.

Applications. Miscellaneous problems on volume.

Remarks. It may be best to make these two pages a class exercise. The problems on page 211 may be read, and the indicated solutions written on the board by the teacher, the pupils offering help and suggestions when they can. Do not have these indicated solutions copied by the children, but when the work is finished assign these pages again to see how well the children can do the work. The equation for volume should be discussed to find just what problems can be solved by using it, in the same manner that equation "A" was discussed on a preceding page.

Solution of Problems (page 211)

6. $$\frac{64\times76\times\overset{4}{\cancel{20}}}{\underset{45}{\cancel{225}}}=\frac{19456}{45}=432 \text{ (to nearest integer).}$$

Pages 212 and 213

Review and Drill. The three problems in fractions. These pages are really a review of pages 76 and 77.

Remarks. The point to be made clear is, that $\frac{1}{4}$ of a number and $\frac{1}{4}\times$ a number is the same. When that is once clear, the three problems in fractions given on page 212 are readily solved by the product-factors principle, and these pages, therefore, properly belong in this place. The direct ease and simplicity with which these problems are solved at this point shows the effectiveness of the method which is used here. If these two pages are mastered thoroughly the child should have no trouble with problems in fractions unless they are stated in such an involved form that there is trouble in comprehending the statement itself. If at all possible, get the children to make other problems like those on page 213. If this can be done, the children will have conquered the whole difficulty. The real trouble usually is, to get at the exact meaning of the problem.

Pages 214 and 215

Review. Percentage; interest; the principle of product and factors.

Remarks. The purpose of this work is apparent. It involves essentially the principle of product and factors. There is a large number of important problems which may be solved directly by its use. A full conception of the simplicity and effectiveness of this principle can be obtained only by persistently grouping around it many different problems, as is done on these pages.

The children should now be able to answer readily the questions on page 215. If they are not able to do so, it shows that they do not really know what is meant by the various terms used, such as "discount," "commission," "gain," "loss," etc.

Drill in Fundamentals. Play game No. 3, page 30. For the first event, add III, IV A to E, and V, VI A to E, page 47. The sums, in order, are:

34342.77; 6538.962; 202.297; 17.492; 27.00; 11538.357; 7096.692; 10255.053; 29.982; 31.33.

For the second event, solve problems 21 to 40, on page 167.

For the third event, solve problems 6 to 17, page 199.

Page 216

Review. The principle of factors and products in the solution of problems.

Remarks. By some means or other, the statement in the middle of this page should be brought home to the pupils (also to the teacher) in as thoroughgoing fashion as may be possible. This page brings out one of the most important ideas in the teaching of arithmetic, namely, the tying together of a great many apparently different things under one far-reaching principle. If this is done, all the many things which are considered on this page will be mastered with the utmost ease. The understanding of them consists, first of all, in thoroughly understanding the equations, and in the second place, in understanding the principle of products and factors, which really amounts only to an understanding of the definition of division: "Division is the process of finding one of two factors, when their product and the other factor are given." When these elements are understood, the solution of the problems is so simple that further explanation is unnecessary, and indeed impossible. For instance, since "speed × time = distance," we know that if distance and time are given, we find the speed by dividing the speed by the time, because *that is what division is*.

Pages 217 to 219

Applications. Miscellaneous problems.

Remarks. It may be well to go over these pages, having the children give the solutions at sight where they can. After the solutions have thus been indicated, the pages may be assigned for regular seat work.

Drill in Fundamentals. In connection with page 218, play game No. 4 on page 30, using the following examples:

$12\frac{1}{3}\times 7\frac{1}{2}=92\frac{1}{2}$; $47\frac{3}{4}\times 12\frac{7}{8}=614\frac{25}{32}$; $18\frac{3}{4}\times 7\frac{1}{2}=140\frac{5}{8}$; $37\frac{1}{4}\times 15\frac{3}{8}=572\frac{23}{32}$; $53\frac{1}{5}\times 12\frac{1}{4}=651\frac{7}{10}$; $12\frac{3}{4}\times 1\frac{1}{2}=19\frac{1}{8}$; $6\frac{7}{8}\times 62\frac{1}{4}=427\frac{31}{32}$; $79\frac{2}{5}\times 17\frac{1}{3}=1376\frac{4}{15}$; $18\frac{5}{6}\times 14\frac{2}{3}=276\frac{2}{9}$; $31\frac{1}{2}\times 4\frac{7}{8}=153\frac{9}{16}$; $67\frac{1}{2}\times 7\frac{2}{3}=517\frac{1}{2}$; $42\frac{5}{6}\times 8\frac{1}{4}=353\frac{3}{8}$; $79\frac{7}{8}\times 14\frac{1}{3}=1144\frac{7}{8}$; $93\frac{2}{3}\times 12\frac{1}{4}=1147\frac{5}{12}$; $67\frac{1}{2}\times 15\frac{1}{3}=1035$.

Solution of Problems (page 217)

3. $38\times\frac{17}{2}\times5\times\frac{57}{2000}=46.027$ (tons), or nearly 46 (tons).

4. Interest is 6% of \$215 = \$12.90

$$\begin{array}{r} \$12.90 \\ 1.40 \\ \hline \$14.30 \end{array}$$

5. Purchase price = $40\times86\times\$1.15=\$3956$.
$5\frac{1}{2}\%$ of \$3956 = \$217.58.

7. 25% of a number is $\frac{1}{4}$ of it. Hence 67.50 is $\frac{3}{4}$ of the list price.
And the list price is \$90.00.

8.

$12\times\$50$ =	\$600 (from rent)
Taxes	175
Net income	\$425

\$425 is 6.5% of \$6500. This is correct to the nearest tenth of one %.

10. $727\div6\frac{5}{6}=106.4$ (miles per hour).
Correct to the nearest tenth of a mile.

Solution of Problems (page 218)

1. $\dfrac{40\times1280}{3\quad\cancel{3}}\times\overset{.80}{\cancel{2.40}}=\dfrac{4096}{3}=1365.33$ (dollars).

2. $8\times6\times5\times\frac{4}{5}=192$ (bus.).

3. $140\times\$9.50=\$1330$ (buying price).
15% of \$1330 = \$199.50 (gain).
$1330\div199.50=\$1529.50$ (selling price).

4. $\dfrac{36}{19}\times53=\dfrac{1908}{19}=100.4$ (tons).

6. $\dfrac{\overset{165}{\cancel{2640}}}{\underset{2}{\cancel{32}}}\times.65=\dfrac{107.25}{2}=53.62$ (dollars).

Solution of Problems (page 219)

3. $196 \div \frac{3}{4} = 261\frac{1}{3}$ or 261 (number of loaves to the nearest integer).
$\$12.15 \div 261 = \$.047$.

10. $\dfrac{18\times 40}{160} = 4\frac{1}{2}$ acres; $670 \div 4\frac{1}{2} = 149$ (bu. per acre to nearest unit).

12. $170 \times \$95 = \$16150$ (value of farm).
$.0092 \times \$16150 = \$148.58$ (tax).

13. $$\frac{54\times \overset{11}{\cancel{22}}\times 8.1}{\underset{1000}{\cancel{2000}}} = \frac{4811.4}{1000} = 4.8114 \text{ (tons)}$$

Page 220

New Matter. Introduction to a systematic study of measurements. The importance of measurements in practical life.

Supplementary Matter. Get the children to bring in data about measurements used in their homes and their neighborhood. Try to make a fairly complete list of the kinds of measurements that are in practical use. Encourage the children to build up the list from their observation of what is going on around them.

Page 221

New Matter. Studying measurements; definition of denominate numbers.

Remarks. Paragraph 121 is addressed as much to the teacher, the principal, and the superintendent, as to the pupil. No matter how much we may talk about the desirability of making our education practical, very few parents would be willing that their children, when grown up, should be ignorant of things a knowledge of which distinguishes the utterly commonplace from the intelligent and well-informed. It is significant that as a race we have taken great pains and made great efforts to learn about matters which cannot by any imaginable possibility have a direct bearing on our so-called practical life; and it will be a sad day for our educa-

tional system if it becomes known that we are restricting it entirely to those matters which are supposed to have a direct bearing on economic success. The child is now old enough to feel that he does not want to be an ordinary ignoramus; he wants to know as much as other well-informed and intelligent people do.

The children should be made to understand that the art of measurements has been one of the greatest elements in raising our conveniences and comforts, our arts and industries, above those of the savage.

Supplementary Matter. For drills in fundamentals play game No. 3, page 30. For the first event solve Examples 1–12, page 141.

For the second event solve Examples 2–13, page 13.

For the third event solve Examples 29–40, page 173.

For the fourth event solve Examples 1–12, page 196.

Page 222

Review. The tables of liquid and of dry measure.

Remarks. It is not contemplated that the children should memorize the complete tables of measures as given on page 288. They should, however, know the pint, the quart, and the gallon of liquid measure, and the relations among them; also the quart, peck, and bushel of dry measure. If the children live in the country, they should know the weight of a gallon of milk and the weight of a bushel of such things as grains, and potatoes. They should also know the relation between a bushel of grain and a cubic foot, that is, that a cubic foot equals very nearly eight-tenths of a bushel.

Solution of Problems

6. Amount delivered during week is 2837.1 lb.
   $2837.1 \div 8.62 = 329.1$ (gallons).
   $329.1 \times \$0.18\frac{2}{3} = \$61.43$ (value of milk).

Page 223

Applications. Miscellaneous problems involving measures of capacity and weights of objects measured by dry or liquid measure.

Supplementary Matter. If the school is in the city, many problems similar to No. 1 on this page should be made up by the pupils, and solved. If the school is located in a poor part of a large city, where coal is sold by the bushel, problem No. 8 may be duplicated and studied in various ways. Thus the question as to how much is lost by buying coal by the bushel as over against buying it by the ton may be raised to good advantage. If the school is located in the country, problems like No. 2 may be made by having the children find how much a horse is fed while hard at work, how much he is fed when he is not working, etc. Problems like Nos. 3, 5, 6, and 7, may be made from data brought in by the children.

Solution of Problems

2. $$\frac{365\times\overset{3}{\cancel{12}}}{\underset{8}{\cancel{32}}}=\frac{1095}{8}=135\tfrac{5}{8}\text{ (bu.)}.$$

$135\frac{5}{8}\times\$.45=\$61.03$ (cost per year).

4. $4\times\$1.30=\$5.20$ (cost of 4 bu.).
$4\times32=128$ (qts. in 4 bu.).
$\$128\times\$.06=\$7.68$ (selling price).
$\$7.68-\$5.20=\$2.48$ (gain).

7. $\frac{5}{4}\times400=500$ (cu. ft. in 400 bu.).
$$\frac{500}{12\times8}=\frac{500}{96}=5\tfrac{5}{24}\text{ (depth in ft.)}.$$

8. $$\frac{2000}{76}=26.3\text{ (bu. to nearest tenth)}.$$

$26.3\times\$.25=\$6.58$ (cost of one ton).

Page 224

Applications. Miscellaneous problems involving weight.

Supplementary Matter. If the school is in the country, it should not be difficult to have the children bring in data like those given in problem No. 1. If the school is in the city, or any place where coal is bought and sold, data for making problems like No. 4 should be obtainable from almost any household in the course of a short time. The information in problem No. 5 will furnish an excellent check on the weighing done by the coalman. It has proved his undoing more than once.

Solution of Problems

1. Total weight of loads with wagon $=21000$ lbs.
   6×1480 lbs. $=8880$ lb. (total to be deducted).
   $21000-8880=12120$ (net weight in lbs.).
   $\frac{12120}{2000}=6.06$ (tons of hay).

3. $46\times2240=103040$ (pounds loaded at mine).
   $.97\times103040=99948.8$ (pounds sold).
   $\frac{99948.8}{2000}=49.97$ (short tons sold).

4. Total weight $=39420$ lbs. or 19.71 tons.
   $19.71\times\$7.80=\$153.74$ (total cost).

5. $8\frac{1}{2}\times42\times4\frac{1}{2}\div35=\frac{17}{2}\times42\times\frac{9}{2}\times\frac{1}{35}=45.9$ (tons).

Page 225

Applications. Problems involving the weight of live cattle and hogs, and the weights of the carcasses of the slaughtered animals.

Supplementary Matter. By noting local prices of cattle and hogs, and by finding these prices in the market quotations in the newspapers, it will be easy to find how much a pound of beef or a pound of pork actually costs before all the cost of butchering and distributing is added. If it is assumed that the by-products, such as hides, etc., are sufficient to pay for the cost of slaughtering

and placing the meat in the butcher shop, the actual cost per pound can be found fairly closely. In this way, interesting comparisons may be made as to what share of the price of meat paid by the consumer goes to the meat packer who butchers the animal, and to the wholesaler and retailer on the one hand, and to the farmer on the other hand. Such problems should, of course, be supplemented by a discussion as to what part of the animal sells at retail for each of several different prices.

Solution of Problems

1. Suppose the live animal weighs 1000 lbs. Then the cost is \$85.00. The meat weighs 62% of 1000, or 620 lbs. \$85 ÷ 620 = \$0.137, or 13.7 cents (a pound).
2. The cost of a 1000 lb. steer is \$72.50. The weight of meat is 550 lb. \$72.50 ÷ 550 = \$0.132, or 13.2 cents (a pound).
3. A 100 pound hog costs \$8.45. The meat weighs 75 pounds. \$8.45 ÷ 75 = \$0.113, or 11.3 cents (a pound).

Page 226

Review. Table of measures of length.

Remarks. If in the city, the child should be expected to remember permanently, the inch, the foot, the yard, and the mile, and some relations among them. If in the country, he should add to this the rod. Of course, he should know about the other units of measure, but he need not memorize their relations permanently.

Supplementary Matter. On two successive days, play game No. 2 on page 30. Let the teacher read the following, and have the pupil write down the results. For the first day read:

1 ft. is how many in.?	1 sq. ft. is how many sq. in.?
1 yd. is how many ft.?	1 sq. yd. is how many sq. ft.?
1 mile is how many ft.?	1 acre is how many sq. rd.?
1 qt. is how many pt.?	1 pk. is how many qt.?
1 gal. is how many qt.?	1 bu. is how many pk.?

For the second day read:

1 min, is how many sec.?
1 hr. is how many min.?
1 da. is how many hr.?
1 wk. is how many da.?
1 yr. is how many da.?
1 lb. is how many oz.?
1 ton is how many lb.?
1 doz. is how many units?
1 gross is how many doz.?
1 cu. yd. is how many cu. ft.?

In these games only those relations of denominate numbers which the child is expected to memorize permanently are required.

Solution of Problems

2. $22 \times 5280 \times \frac{2}{5} = 1056 \times 44 = 46{,}464$ (steps).

3. $5280 \div 12 = 440$, $440 \div 1 = 441$ (number of posts).

5. $5280 \div 2 = 2640$; $2640 \times \$.80 = \$2112.00$ (cost).

6. $$\frac{\overset{2640}{\cancel{5280}}}{\cancel{2}} \times \frac{\overset{35}{\cancel{105}}}{\underset{1000}{\cancel{2000}}} = 92.400 \text{ or } 92.4 \text{ (tons) for one rail.}$$

$2 \times 92.4 = 184.8$ (tons) for one mile of track.

$184.8 \times \$28.50 = \$5266.80$ (cost of rails).

Page 227

Application. Miscellaneous problems involving square measure.

Remarks. There is little need to memorize square measure, as the relations between any two such units may be obtained from the known relation between the corresponding linear units. However, the child should know that 144 square inches equal a square foot, and that nine square feet equal a square yard. If in the country, the child should also know that 160 square rods equal an acre, and that 640 acres equal one square mile.

The teacher should note, and possibly tell the children, that the data in problem No. 11 are only approximate. There are many side-tracks which are not counted, and, what is even more to the point, many double, and even quadruple, track railways which are here classed as single tracks.

Solution of Problems

11. First express width in terms of miles. $4\frac{1}{4} \div 320 = \frac{17}{1280}$.

$$\frac{17}{1280} \times 244180 = 3243\tfrac{1}{64} \text{ (sq. miles)}.$$

12. $\frac{69}{2} \times \frac{94}{9} \times 65 = 23422$ (cents, or \$234.22).

Material for the Fourth Examination, Grade Six.

If it is desired to include examples in the four fundamentals, percentage, and interest, use some of those given on pages 373, 385, 398.

Find the rate per cent loss or gain to the nearest tenth of one per cent, using the buying price as the base.

	Cost Price	*Selling Price*			*Cost Price*	*Selling Price*	
1.	\$.75	\$1.20	60%	4.	\$25.00	\$15.00	40%
2.	\$1.25	\$2.25	80%	5.	\$42.50	\$65.00	52.9%
3.	\$5.00	\$8.50	70%	6.	\$35.00	\$28.50	18.6%

Find the selling price.

	List Price	*Rate of Discount*			*List Price*	*Rate of Discount*	
7.	\$6.50	35%	\$4.23	10.	\$40.00	55%	\$18.00
8.	\$12.75	42%	\$7.40	11.	\$1550	17%	\$1286.50
9.	\$3.50	65%	\$1.23	12.	\$160.00	22%	\$124.80

13. If 1760 bushels of wheat is harvested from a field containing 94 acres, what is the average yield per acre? 18.7 (bu. per acre)
14. 42 horses are bought for \$8500, and 35 horses for \$6800. What is the average cost per head of these horses? \$198.70
15. At \$4.25 a square yard, how much will it cost to pave a street 40 feet wide and 380 feet long? \$7177.78
16. A farmer sold a field 36 rods wide and 90 rods long for \$3800. What was the price per acre? \$187.65
17. A farm 175 rods by 260 rods was sold at \$185 per acre. What was the selling price? \$52609.38
18. If it cost \$460 to cultivate a field 110 rods long and 80 rods wide, what was the cost per acre? \$88.36

19. At 32 cents a square foot, what is the cost of laying a hardwood floor in a hall 45 feet wide and 70 feet long? **$1008**

20. A room 28 feet wide, 32 feet long and 12 feet high is occupied by 35 persons. How many cubic feet per person does the room contain? **306.2 (cu. ft. per person)**

21. A piece of furniture sold 45% below the list price is sold for $44.00. What was the list price? **$80.00**

22. An aeroplane flew 680 miles in 4 hours 35 minutes. What was the speed per hour? **$148\frac{4}{11}$ (miles per hour)**

23. If 47 tons of hay is sufficient to winter 24 cows, how many tons will it take to winter 37 cows? **72.46 (tons)**

24. At 57 cents a bushel, find the value of a load of oats weighing 3760 pounds (1 bu. oats = 32 lb.). **$66.98**

25. If 84% of a certain grade of butter is butter fat, how many pounds of butter can be made from 795 pounds of butter fat? **946.43 (lb.)**

26. One cubic foot will hold $\frac{4}{5}$ of one bushel of grain. How many bushels of grain are there in a bin 14 feet long and 12 feet wide if the grain is $6\frac{1}{2}$ feet deep? **873.6 (bu.)**

27. If one ton of coal occupies 35 cubic feet, how many tons are there in a bin 18 feet long and 10 feet wide if the coal is $5\frac{1}{4}$ feet deep? **27 (tons)**

28. At $2.75 per square yard, what is the cost of paving a school yard containing 890 square feet? **$271.94**

Page 228

New Matter. The area of a triangle.

Remarks. The child should be made to see by means of demonstration on the board carried out by the teacher, that it is very probable that the area of a triangle equals one-half the base times the altitude. The purpose here is not to demonstrate this truth in the mathematical sense, but to give it a reasonable basis upon which the child feels he can depend. A rigorous deductive proof is far beyond the average child at this age.

Page 229

New Matter. Definition of a circle, diameter, radius.

Remarks. Again, there should be no attempt to give a formal deductive proof. The figure shown will be sufficient to make the child feel that the circumference is something more than three times the diameter. If the teacher thinks best, she may, of course, tell the children that 3.1416 is a better multiplier than $3\frac{1}{7}$.

Page 230

New Matter. The area of a circle.

Remarks. The argument given on this page will be sufficient to make the child believe that the circumference multiplied by one-half the radius gives the area. No attempt should be made at this time to develop the *formula* for finding the area of the circle.

Supplementary Matter. Circles may be drawn on the board, as suggested in problem No. 8, and their areas computed. That will be a source of real interest. If the children live in the country they will like to measure the circumference of a tree, and then to compute the diameter and cross-section area.

Solution of Problems

3. $3\frac{1}{7} \times 36 = 113\frac{1}{7}$ (feet) circumference.
   $9 \times 113\frac{1}{7} = 1018\frac{2}{7}$ (area in sq. ft.).
7. Circumference $= 3\frac{1}{7}$ feet.
   Area $= \frac{1}{4} \times 3\frac{1}{7} = \frac{11}{14}$ (sq. ft.), or $113\frac{1}{7}$ sq. in.
8. Circumference $= 3\frac{1}{7} \times 16 = 50\frac{2}{7}$ (in.).
   $4 \times 50\frac{2}{7} = 201\frac{1}{7} =$ area in sq. in.

Page 231

Applications. Miscellaneous problems.

Remarks. The idea of the volume of a cylinder may be developed very informally at this point. Simply multiply the area of its base by the length, and let it go at that.

The problems given on this page should be of very considerable interest to the child. The fact that the hardheaded grown-ups may not have use for this kind of problem will prove no argument against them to the children.

Supplementary Matter. For drill in fundamentals, play game No. 1 on page 30. Add VIII, IX A to E, and X, XI, A to E, page 33. The sums, in order, are:

662,343; 497,931; 67072; 6516; 561; 553,714; 567,982; 52,316; 6740; 594.

Solution of Problems

2. $180\times3\frac{1}{7}=565\frac{5}{7}$ (circumference in feet).
$45\times565\frac{5}{7}=25457\frac{1}{7}$ (area in sq. ft.).

3. $\dfrac{\overset{25}{\cancel{250}}\times\overset{480}{\cancel{5280}}\times12\times7}{\underset{7}{\cancel{35}}\times\underset{11}{\cancel{22}}}=144{,}000$ (revolutions).

4. The diameter is 3 feet.

Then $\dfrac{1120\times5280}{3\times\frac{22}{7}}=\dfrac{1120\times\overset{880\quad 80}{\overset{\cancel{1760}}{\cancel{5280}}}\times\cancel{7}}{\cancel{3}\times\underset{11}{\cancel{22}}}=1120\times807=627{,}200$ (revolutions).

5. $\dfrac{8480\times5280\times12}{34\times\frac{22}{7}}=\dfrac{8480\times\overset{\cancel{240}\quad 120}{5280}\times12\times7}{\underset{17}{\cancel{34}}\times\cancel{22}}=\dfrac{85{,}478{,}400}{17}=5{,}028{,}141$

(number of revolutions, to nearest unit).

Page 232

New Matter. Miscellaneous problems involving cubic measure.

Remarks. Go through this page with the children and have them answer all the questions that they can orally. This will form a review of the idea of cubic contents.

Solution of Problems

8. $\frac{40\times25\times5}{27}=\frac{5000}{27}=185\frac{5}{27}$ (cu. yd.).

Page 233

Applications. Miscellaneous problems involving cubic measure.

Remarks. The purpose of the first two problems is to find that .8 of a bushel is equal to 1 cubic foot. There is no suggestion, for instance, that the child should remember the number of cubic inches in a bushel, but it is very healthy for him to find out for himself the important results obtained from problem No. 2. If in the country, this result should be memorized permanently.

Supplementary Matter. Wherever practicable, data should be brought in from local affairs to form the basis for other problems of this sort.

Solution of Problems

4. $16\times14\times48=10752$ (cu. ft.).
   $10752\times.8036=8640.3$ (bu. correct to nearest tenth).

6. $\overset{3}{\cancel{12}}\times\frac{7}{\cancel{2}}\times\frac{3}{\cancel{2}}\times62.5=63\times62.5=3937.5$ (lbs.). 3937.5 lbs. = 1.96875 or 1.97 tons.

Pages 234 and 235

New Matter. Board measure.

Remarks. If the school is located in a place where lumber is used to a considerable extent (not in the center of a big city like New York or Chicago) the children should be made fairly familiar with the idea of a board-foot, and of the rule for finding board-feet given on page 234. The figures given in exercise 3 on page 235 are taken from an actual bill of lumber that was bought for the building of a small frame house.

Page 236

New Matter. Computing area; finding the cost of plastering and painting.

Remarks. A page of this kind may be regarded as giving practical problems without any special rules. The information needed to solve them is contained in the paragraph at the top of the page.

Solution of Problems

3. The room is 5 yards by 4 yards and 3 yards high. The area of walls is $2\times5\times3+2\times4\times3=54$ sq. yd.
   Area of ceiling $5\times4=$ 20 sq. yd.

   Total area................74 sq. yd.

4. $\dfrac{16\times14}{9}=\dfrac{226}{9}=24\frac{8}{9}$ (sq. yd.).

6. The dimensions of room in yards are $\frac{22}{3}$, 6, 3 yards.
   Area of walls $=2\times\frac{22}{3}\times3+2\times6\times3=44+36=$ 80 sq. yd.
   Area of ceiling.............................. 44 sq. yd.

   Total area ....124 sq. yd.

   $124\times\$.15=\$18.60$ (cost).

7. The total length of wall is 52 ft.+66 ft.+48 ft.+56 ft.+40 ft.= 262 ft, or $87\frac{1}{3}$ yards.
   Hence area of walls is $3\times87\frac{1}{3}=262$ (sq. yd.).
   Areas of ceiling are $18\frac{2}{3}+30+15\frac{5}{9}+21\frac{1}{3}+10\frac{2}{3}=96\frac{2}{9}$ (sq. yd.).
   Total area is $358\frac{2}{9}$ (sq. yd.). $12\times358\frac{2}{9}=4299$ (cost in cents), or \$42.99.

Page 237

New Matter. Estimating wall paper.

Remarks. The topic of this page is of interest to all persons who do not live in a rented apartment in a big city. The page should present no serious difficulties. If some of the children are having or have had papering done lately in their homes, they

should be led to measure the rooms and bring in the data so that problems can be made in the class. Problems made up by the children from local data are infinitely superior in several respects to those obtained from the pages of a book, no matter how well these may be chosen.

Page 238

New Matter. Estimating the length of a carpet needed for a room of a given size.

Remarks. When the dimensions of the room to be carpeted are given, a picture like the one on this page may be drawn on the board to show just how many strips will be needed for each way of laying the carpet.

Solution of Problems

1. Each strip is $17\frac{1}{2}$ feet long.
   The total length is $5 \times 17\frac{1}{2} = 87\frac{1}{2}$ feet, or $29\frac{1}{6}$ yards.
2. There will be $48 \div 3 = 16$ strips.
   The total length is $16 \times 64 = 1024$ feet, or $341\frac{1}{3}$ yards.
3. There will be 22 strips, and the total length will be $22 \times 48 = 1056$ feet, or 352 yards.
4. Each strip is $21\frac{2}{3}$ feet long, and there are $6 \div \frac{3}{4} = 8$ strips. The total length is $8 \times 21\frac{2}{3} = 173\frac{1}{3}$ ft., or $57\frac{7}{9}$ yd.

Page 239

New Matter. Estimating the lumber for floors, and the number of shingles for a roof.

Remarks. The information given on these pages should not be memorized. It is such information as every carpenter will have. The problems themselves are simple. They should be regarded not as separate topics to be developed fully, but simply as ordinary problems to be solved by means of common sense and the information given in the text.

Solution of Problems

1. $45 \times 32 = 1440$ (board-feet). $1440 + \frac{1}{6}$ of $1440 = 1680$.
   $1.68 \times \$85.00 = \$142.80$ (value of lumber).
2. $2 \times 42 \times 48 = 4032$ (board-feet).
   $4032 + \frac{1}{6}$ of $4032 = 4704$
   $4.704 \times \$78.00 = \$366.92$ (cost).
3. $2 \times 24 \times 46 = 2208$ (sq. ft.), or 22.08 squares.
   $935 \times 22.08 = 20{,}644.8$, or 20,645 (shingles).
4. $24\frac{1}{2} \times 950 = 23{,}275$ shingles, or 23.275 thousands.
   $23.275 \times \$3.25 = \$75.64$ (cost).

Page 240

New Matter. The number of bricks in a wall.

Remarks. The remarks made about the preceding few pages apply to this page also.

Supplementary Matter. It should be easy to find cases of the use of brick in the neighborhood, and to get the children to obtain the dimensions and bring them in to make problems.

Page 241

Review. The table of time.

Remarks. Note the spirit of reviews like those given on this page. The old material is gone over again and brushed up, and at the same time some new material is put in.

Pages 242 and 243

New Matter. Temperature and heat.

Remarks. This is, of course, partly a review of what is already known, but much of the material is new. The idea of a unit of heat and the number of heat units required to thaw a pound of ice and to boil away a pound of boiling water should be of real interest to the child. Suggest that a pound of ice be dropped into a pound (about 1 pint) of boiling water. How warm will the water be when the ice is thawed?

Do not measure the interest of the class by that of the dullest child. Once in a while introduce material which is of real interest to the brightest ones. They too are entitled to some attention at odd times. A really bright boy of eleven has more varied interests than many fairly intelligent grown persons. Do not stunt that boy's growth by failing to make use of his interests.

Solution of Problems (page 243)

1. $158 \times 12 = 1896$ (heat units).
2. $12 \times 966 = 11592$ (heat units).
3. $16 \times 144 = 2304$ (heat units).
4. $20 \times 144 = 2880$ (heat units).
   $15 \times 182 = \underline{2700}$ (heat units).
   Difference $= 181$ (heat units).
5. $20 \times 180 = 3600$ (heat units).
   $4 \times 966 = \underline{3864}$ (heat units).
   Difference $= 264$ (heat units).
6. $65 \times 144 = 9360$ (heat units).
   142 heat units will bring 1 lb. of water from 70° to 212°.
   Hence $\frac{9360}{142} = 65.9$ (lbs. water) is the required result.

Pages 244 and 245

New Matter. Reduction of denominate numbers.

Remarks. Do not try to develop this subject formally. Get the children to solve the problems by direct use of their common sense. The information given on these pages need not be committed to memory. Whenever land is measured by the hundred-foot chain, the dimensions are obtained in feet, and the area in square feet, but the interesting thing to know is how many acres there are in the area; hence the reduction of square feet to acres arises.

Pages 246 and 247

New Matter. The four fundamental operations on compound denominate numbers.

Remarks. The work on these pages need not be stressed. Note that both methods of subtraction work equally well for denominate numbers also.

Supplementary Matter. Play game No. 4, on page 30, on three successive days. For the first game find the sums required on page 5.

For the second game, find the products in Examples 1–13, page 13.

For the third game find the quotients asked for in 3–12, page 19.

Pages 248 and 249

New Matter. Making out bills; extending bills and footing them.

Supplementary Matter. Have the children bring in bills from actual local business transactions, and have them extend the items and foot the columns in the usual manner. Also have the children make out receipts for money paid. Tell the children that a formal receipt is given only when not all of the bill is paid. If the bill is paid in full it is receipted and returned, and then forms a perfectly good receipt in full to date.

Solution of Problems (page 249)

1.

10 lb. at $7\frac{1}{2}$¢	\$0.75
1 sack	2.20
$1\frac{1}{2}$ bu. at \$1.25	1.88
2 bags salt at 6¢	.12
2 lb. at 16¢	.32
2 loaves at 5¢	.10
$2\frac{1}{4}$ doz. at 45¢	1.01
2 lb. at 40¢	.80
$1\frac{3}{4}$ doz. at 35¢	.61
2 cans at 20¢	.40
	\$8.19

2.

1400 bu. at 72¢	\$1008.00
580 boxes at \$3.14	1821.20
15 bunches at \$3.98	59.70
	\$2888.90

3.

20 bu. at $11.25.....	$225.00
18 bu. at $11.75.....	211.50
8 bu. at $4.30	34.40
	$470.90

4.

860 yd. at $6\frac{1}{4}$¢.....	$53.75
1450 yd. at 62¢.....	899.00
580 yd. at $1.15...	667.00
1840 yd. at $12\frac{1}{2}$¢....	230.00
	$1849.75

Page 250

New Matter. The cash account.

Remarks. It should not prove difficult to explain to the child how the cash account is kept, namely, that all cash received or on hand at the beginning of the period is entered on the left, or debit side, while all cash actually paid out during the period is entered on the right or credit side. At this stage no effort should be made to develop the reasons for "debiting" and "crediting" cash. Simply state the rule, and make sure that children are able to follow it in practice.

Supplementary Matter. If possible, get some of the children to bring in data for their own cash account for a month or for some other convenient period. The children will also be willing and able to fix up fictitious data for cash accounts. If they do, the teacher should see to it that the amounts are reasonable.

Page 251

New Matter. An account with real estate.

Remarks. This is taken from an actual case, the figures themselves being those which arose in real practice. Point out that the account shows how much was gained by the transaction. Show that the debit side is used for all expenditures in connection with this real estate, while the credit side is used for all incoming amounts in connection with it.

Supplementary Matter. Play game No. 3, page 30. For the first event solve Examples 1–28, page 173.

For the second event, solve Examples 41–52, page 173.

For the third event, solve Examples 4–17, page 184.

Pages 252 and 253

New Matter. Account with selling papers. As in the preceding account, show that the debit side is used for all expenditures, and the credit side for all income; and that the difference between the two sides shows the actual gain.

Supplementary Matter. If some of the boys in the class are selling papers, have them bring in data from their own experience, and use this as material for other accounts of this kind. Encourage the boys to keep accounts of this sort in their paper business.

Pages 254 and 255

New Matter. Accounts with a vegetable garden and with raising Belgian hares.

Remarks. These data are taken from actual cases. Show, as before, the function of the credit and debit sides of the account.

Supplementary Matter. Have as many original data brought in as possible, for the purpose of making up similar accounts. For a drill in fundamentals, play game No. 4, page 30. Add II, III A to E, and IV, V A to E, page 47.

The sums, in order, are:

38,366.74; 3463.212; 275.553; 13.627; 18.88; 29,473.734; 10,738.449; 358.886; 24.034; 20.74.

DR. Acct. with Belgian Hares. CR.

12 bales at 90¢	$10.80	April	37 lb. at 38¢	$14.06
650 bbl. bran at $1.10 cwt.	7.15	May	34 lb. at 26¢	8.84
500 lb. at $1.60 cwt.	8.00	June	41 lb. at 25¢	11.75
183 days' work at 10¢	18.30	July	49 lb. at 35¢	17.15
	$44.25	August	34 lb. at 35¢	11.90
Balance	31.35	September	34 lb. at 35¢	11.90
	$75.60			75.60
		Gain		$31.35

Material for Fifth Examination, Grade Six.

If it is desired to include examples in the four fundamentals, percentage, interest, or profit and loss, use some of those on pages 373, 385, 398, 412.

1. Find the area of a triangle whose base is 12 feet and altitude 8 feet. **48 (sq. ft.)**
2. Find the area in square feet of a triangle whose base is 22 inches and altitude is 28 inches. **2.14 (sq. ft.)**
3. Find the area in square yards of a triangle whose base is 18 feet and altitude is 16 feet. **16 (sq. yd.)**
4. Find the circumference of an auto tire whose diameter is 32 inches. **100$\frac{4}{7}$ (inches)**
5. Find the circumference of an auto tire whose diameter is 37 inches. **116$\frac{2}{7}$ (inches)**
6. Find the number of board-feet in 36 boards each 10′ x 10″ x 1″. **(300 board-ft.)**
7. Find the number of board-feet in 140 pieces of lumber each 14′ x 6″ x 2″. **1960 (board-ft.)**
8. Find the number of board-feet in 260 pieces of lumber each 16′ x 4″ x 2″. **2773$\frac{1}{3}$ (board-ft.)**
9. Reduce 2680 square feet to square yards. **297$\frac{7}{9}$ (sq. yd.)**
10. Reduce 864 square inches to square feet. **6 (sq. ft.)**
11. Reduce 960 feet to rods. (16½ feet = 1 rod.) **58.18 (rd.)**
12. Reduce 1260 cubic feet to cubic yards. **46.67 (cu. yd.)**
13. 1480 pounds of milk are how many gallons? (Use 8.6 lbs. as the weight of 1 gal.) **172.09 (gal.)**
14. A bin contains 1180 cubic feet. How many bushels does it hold? **944 (bu.)**
15. Make out a bill for the following items. Extend each item and foot the bill. Use the date of the examination.

J. W. Hansom & Co. sold to Mr. L. W. Ward, both of Madison, Wisconsin, 50 lb. of sugar at 9½¢ a lb., 98 lb. flour for $5.60, 1 box of soap for $7.25, 3 lb. of butter at 55¢, 6 cans of corn at 18¢, and 12 cans of peas at 27¢. **Total of bill, $23.57**

16. Make out bill as in example 15:

A. L. Stevenson sold to Mr. B. F. Anderson, both of Cleveland, Ohio, 6 doors at $9.50, 12 window casings at $4.75, 4 locks at $2.80, 2 locks at $3.50, 6 pair of door hinges at $1.40, 5 windows at $12.50, and 7 windows at $14.25. **Total of bill, $302.85**

17. Make out bill as above:

Anderson's Greenhouse sold to J. P. Larsen, both of Anaconda, 3 doz. gladiolas at $1.75, 5 doz. red roses at $2.25, 2 doz. asters at $1.00, 1 spray, $2.50, 2 bouquets at $1.25, 9 rose buds, $1.50, 3 doz. pinks at 75¢. **Total of bill, $39.25**

Page 256

New Matter. Classes of mail matter.

Supplementary Matter. Obtain from the postoffice the printed postoffice regulations. These will describe the various classes of mail matter. Have the children read these regulations through, to see whether they can understand them. Let this lead to a class discussion as to what kinds of material may be sent in the various classes of mail.

Page 257

New Matter. Postal money orders and express money orders.

Supplementary Matter. Obtain from an express company a statement giving information about the issuance of express money orders. Raise such questions as these:

At what postoffice may a postal money order be cashed?

At what express office may an express money order be cashed?

Compare the advantages of these two arrangements.

Pages 258 and 259

New Matter Fuller description of fourth-class mail matter. Insurance of mail packages.

Supplementary Matter. From an express company, get a list of rates and compare these rates with the postal rates.

Pages 260 to 263 (inc.)

Review. Drawing to scale, and reading the scale on drawings.

Remarks. Simple drawings of this kind should now be quite easy to make. Reading the scale on drawings and deciding the distances between points is as important and sometimes as difficult as the making of the drawings themselves.

Supplementary Matter. On the board draw to scale figures representing a football field, a tennis court; and measure to find the distances between interesting points.

For drill in fundamentals at the end of page 261, play game No. 3, page 30. For the first event, add III, IV A to E, and V, VI A to E, page 32. The sums, in order, are:

617,695; 443,210; 45,235; 5971; 639; 731,555; 591,597; 57,036; 5412; 645.

For the second event, subtract B II, III, IV from A II, III, IV, page 32. The differences, in order, are:

1676; 8106; 6280; 12738, 4978; 4442; 14761; 802; 15418; 7710; 18903; 46176; 31918; 20862; 13493.

For the third event, multiply B IV, V by C IV, V, page 32. The products, in order, are:

106,475,088; 83,233,320; 235,554,052; 189,843,680; 715,705,616; 253,600,203, 220,771,172; 306,591,917; 685,609,794; 444,317,352.

For the fourth event, divide A V, VI by D V, VI, page 32, obtaining quotients to the nearest hundreths. The quotients, in order, are:

65.75; 127.48; 292.29; 134.32; 196.14; 66.76; 142.37; 109.16; 330.00; 167.42.

Solution of Problems (page 261)

1' A B $=3\frac{3}{4}''$, B C $=2''$.
$80 \times 3\frac{3}{4}=300$. $80 \times 2=160$.
Hence the field is 160 feet by 300 feet.

3' If E F $=18\frac{1}{2}$ feet and A D $=160$ feet,
then A E + F D $=160-18\frac{1}{2}$ (feet) $=141\frac{1}{2}$ (feet).
But A E and F D are equal. Hence A E is half $141\frac{1}{2}$ or $70\frac{3}{4}$ feet.

Pages 264 to 267

New Matter. Graphical representations.

Remarks. Study page 265 to gain from the graph the actual information contained in it. This page is a copy of a graph given in the report of the Surgeon-General of the United States army.

In connection with pages 266 and 267, bring out the approximate character of the numbers used. Show why the exact number cannot be obtained, and further, that the approximate numbers answer the purpose just as well as exact numbers would in case they could be obtained.

Do not treat these pages lightly. Graphic representation is now in very general use. A child in the public schools has a right to be instructed so that he can understand simple graphs.

Supplementary Matter. Find the population of your city and of your state for several decades past, and make graphs like these to show the growth of population. Bring in graphs which are found in magazines, in advertisements, etc., and study them.

Pages 268 to 275

Applications. Miscellaneous problems involving the whole field of arithmetic thus far studied.

Remarks. These problems have been made simple, so that it may reasonably be expected that the average child who reaches this point in arithmetic can solve them without much trouble. It will be excellent practice to have the children make problems like these and solve them. We must not forget that our purpose is, not merely to teach the child to read problems in a book and solve them, but to handle effectively and with a definite purpose data which occur in daily life. If a child can be led to make a problem of his own, stating the conditions in his own language, and, if possible, using data which have actually occurred in his experience, we shall be very much more certain that he understands what he is doing than when he simply solves problems given on the printed page.

Naturally, the kind of problems to be stressed will depend upon the character of the school. In a country school, the farm problems should be stressed, while in a city school the data involved in them will often be unintelligible. Similarly, some problems involving data that arise only in the city will be unintelligible to the country boy and girl.

Frequently problems of this sort contain information which is of interest to all well informed people. Thus, the amount of butter-fat in butter, and the percentage of butter-fat in milk, are interesting facts to any wide-awake person, no matter what his occupation may be. Similarly, the data about dead-weight and live-weight of a freight train are interesting to all intelligent people. Such problems as No. 2 on page 270 will enable the practical housekeeper to determine whether or not the ice-man is giving her full weight.

Supplementary Matter. Drill in fundamentals. Somewhere during the progress of these problems play game No. 3, page 30. For the first event, add I, II A to E, and III, IV A to E, page 47. The sums, in order, are:

48253.54; 2611.794; 440.751; 14.060; 16.67; 34342.77; 6538.962; 202.297; 17.492; 27.00.

For the second event, multiply A V, VI by D V, VI, page 47. The products, in order, are:

139.10784; 9533.17; 132.80904; 4328.559; 230.02272; 4169.0993; 1199.84568; 23.14338; 75.233588; 6710.64057.

For the third event, solve examples 41–52, page 173.

For the fourth event, solve examples 6–12, page 199. Note that there are two examples under each number.

Solution of Problems (page 268)

1. $10\frac{1}{2}\times 3\frac{1}{2}\times 2\frac{1}{6}\times .8=\dfrac{\overset{7}{\cancel{21}}\times 7\times 13}{\cancel{2}\times\cancel{2}\times \underset{\cancel{3}}{\cancel{6}}}\times \overset{.1}{\cancel{.8}}=63.7$ (bu.).

2. $1850+$245=$2095 (cost).
$2095−$1926=$169 (loss).
$169 is 9.1% of $1850.

4. $$\frac{85\times5280}{6\times\frac{22}{7}}=\frac{85\times\overset{40}{\cancel{\overset{240}{\cancel{5280}}}}\times7}{\cancel{6}\times\cancel{22}}=23{,}800 \text{ (revolutions).}$$

5. 175% of $25=$43.75 (marking price).
75% of $43.75=$32.81 (selling price).

7. $9\frac{1}{2}\times11\frac{2}{3}=110\frac{5}{6}$ (square feet in a rug).
$110\frac{5}{6}\times\$2.15=\$238.29$ (value of rug).

8. $$\frac{160\times48.5}{120}=\frac{194.0}{3}=64\frac{2}{3} \text{ (width in rods).}$$

10. 92% of 2360=2171.2 (number of bu. sold).
2171.2×$.71=$1541.55 (amount received).

Solution of Problems (page 269)

4. $5\frac{1}{2}$% of $8000=$440

Taxes, etc.	180
Total	$620

Hence it costs $20 a year more to buy.

6. $323=95% of selling price.
Selling price=$323÷.95=$340.

7. $$\frac{1450}{25\times95}=\text{value per square foot.}$$

$$\frac{\overset{58}{\cancel{1450}}}{\underset{5}{\cancel{25}}\times\underset{19}{\cancel{95}}}\times\overset{9}{\cancel{45}}\times85=\frac{58\times9\times85}{19}=\frac{44370}{19}=2335.26$$

(value in dollars).

8. $402.50 = 115% of buying price.
$402.50 ÷ $1.15 = $350 (buying price).

10. $14\times\frac{21}{2}\times\frac{11}{2}\times\frac{1}{36}=\frac{539}{24}=22.46$ (tons).

Solution of Problems (page 270)

2. $\frac{17}{12}\times\frac{13}{12}\times\frac{11}{12}=\frac{2431}{1728}=1.4$ (cu. ft.).
$57\times1.4=79.8$ (lb. of ice or 80 lb.).
$.80\times\$.35=\$.28$ (cost of ice).

3. First reduce dimensions to rods.
$320\times29=9280$ (length in rods).
$66\div16\frac{1}{2}=4$ (width in rods).

$$\frac{\cancel{4}\times\overset{232}{\cancel{9280}}}{\underset{4}{\cancel{160}}}=232 \text{ (area in acres).}$$

$56\times232=12992$ (bushels of corn).

4. $78\times1173\times\$.11\frac{1}{4}=\$10,293.08$ (buying price).
$90844\times\$.12\frac{3}{4}=\$11,582.61$ (selling price).
$\$11,582.61-\$10,293.08=\$1289.53$
$\$1289.53-\$3850=\$1251.03$ (gain).

6. 16 inches $=1\frac{1}{3}$ feet. 1 acre $=160\times16.5\times16.5=43,560$ sq. ft.
$1\frac{1}{3}\times43,560=58,080$ (cubic feet). $58,080-10\%=52,272$ cu. ft.
$\frac{52272\times57}{2000}=\frac{26136\times57}{1000}=1489.752$ or $1489\frac{3}{4}$ (tons net).

Solution of Problems (page 271)

2. $$\frac{1180\times\overset{14}{\cancel{56}}}{\underset{2579}{\cancel{10316}}}=\frac{16520}{2579}=6.4 \text{ (lb. of grain per lb. of hay).}$$

3. 4.2% of milk = 75.
$75 \div .042 = 1785.7$ (lb. of milk).

5. 5% of \$5600 = \$280 (commission).
\$5600 − \$280 = \$5320 (owner's share).
\$280 − \$84.75 = \$195.25 (agents net profit).

6. \$2300 + \$12,500 = \$14,800 (total value).
1.952% of \$14,800 = \$288.90 (tax).

7. $16 \times 3\frac{1}{7} = 48\frac{16}{7} = 50\frac{2}{7}$ (circumference in inches).
$4 \times 50\frac{2}{7} = 201\frac{1}{7}$ (area in sq. in.).
$201\frac{1}{7} - 196 = 5\frac{1}{7}$ (difference in area in sq. in.).

11. $\frac{45 \times \overset{\cancel{4}\,2}{\cancel{72}} \times \frac{9}{2}}{\cancel{18}} = 810$ (tons of earth).

12. $\frac{11}{\underset{\cancel{2}}{\cancel{16}}} \times \overset{125}{\cancel{1000}} = 687.5$ (lb. of flour).

Solution of Problems (page 272)

1. $\frac{4\frac{1}{4} \times 2000}{4\frac{1}{2}} = \frac{17 \times \overset{1000}{\cancel{2000}} \times \cancel{2}}{9 \times \cancel{4}} = 1888.8$ or 1889 (bricks).

2. $560 \times 10 \times \frac{1}{3} \times 1 = 1866\frac{2}{3}$ (board-feet).
$1.866\frac{2}{3} \times \$65 = \$121.33$ (cost)

3. $7\frac{1}{2} \times 3\frac{1}{3} \times 1\frac{5}{6} = \frac{\overset{5}{\cancel{15}} \times \overset{5}{\cancel{10}} \times 11}{\cancel{2} \times \cancel{3} \times 6} = \frac{275}{6} = 45\frac{5}{6}$ (cu. ft.)

$45\frac{5}{6} \times \frac{1728}{231} = \frac{275 \times \overset{\overset{96}{\cancel{288}}}{\cancel{1728}}}{\cancel{6} \times \underset{77}{\cancel{231}}} = 342.9$ (gallons).

4. $\overset{5}{\cancel{60}}\times\overset{7}{\cancel{14}}\times\frac{10}{\cancel{12}}\times\frac{5}{\cancel{2}}=1750$ (board-feet).

5. $\frac{1}{3}$ in. $=\frac{1}{36}$ foot $=\frac{1}{36\times30}$ of the rail.
$1\div1080=.000926=.0926\%$.

6 The diameter of the circle is 12 inches.
Circumference is $12\times3\frac{1}{7}=37\frac{5}{7}$ (inches)
Area $=3\times37\frac{5}{7}=113\frac{1}{7}$ (sq. in.).
Area of square $=12\times12=144$ (sq. in.).

7. $\frac{8940\times18\frac{1}{2}}{\cancel{8.6}}=\frac{\overset{2235}{\cancel{8940}}\times37}{\underset{4.3}{\cancel{8.6}}\times\cancel{2}}=19231$ (cents), or \$192.31.

8. $\frac{80\times16\frac{1}{2}\times2\times3\frac{1}{2}}{320}=\frac{\cancel{80}\times33\times\cancel{2}\times7}{\underset{4}{\cancel{320}}\times2\times2}=\frac{231}{8}=28\frac{7}{8}$ (days).

9. $\frac{960}{40\times480}=$ cost in dollars per sq. ft.

$\frac{\overset{240}{\cancel{960}}\times45\times\overset{90}{\cancel{1800}}}{\underset{10}{\cancel{40}}\times\underset{2}{\cancel{480}}}=4050$ (cost in dollars).

10. A 17-inch wall required 28 bricks per sq. ft.
$160\times7\times28=31,360$ (number of bricks).

11. Total coal is 14.015 tons.
$14.015\times\$7.40=\$103.71$ (cost of coal).
$14.015\times\$.30=\$4.20$ (saving by buying early).

Solution of Problems (page 273)

3. $4\frac{1}{3}\times3\frac{1}{7}-13\frac{13}{21}$ (circumference in feet).
$5280\div13\frac{13}{21}=387.7$ (revolutions).

5. \$1755 = 65% of original cost.
Hence \$1755 ÷ .65 = \$2700 (original cost).

6. 3.8% of 11860 = 450.68 (lb. butter fat).
450.68 = 85% of butter.
450.68 ÷ .85 = 530.2 (lb. butter).

7. $15\times144+15\times48=2160+720=2880$.

8. $35\times144=5040$, $30\times152=4560$.
It requires $5040-4560=480$ heat units more to heat the 35 lb.

10. $\frac{56}{16\times18}$ = cost in dollars per sq. ft. $\frac{56\times25\times36}{16\times18}=175$ (cost in dollars).

Solution of Problems (page 274)

1. $$\frac{650\times\overset{2}{\cancel{2000}}\times2}{\underset{\cancel{2}}{\cancel{8}}\times\cancel{1000}}\times\overset{\overset{.75}{\cancel{2.25}}}{\cancel{6.75}}=1950$$ (value in dollars).

2. $640\times56{,}650=36{,}256{,}000$ (area in acres).
$16536500\div36256000=.459=45.9\%$.

3. $4\times3\frac{1}{7}=12\frac{4}{7}$ (circumference in feet).
$1\times12\frac{4}{7}=12\frac{4}{7}$ (area in sq. ft.).
$528000\div12\frac{4}{7}=42{,}000$ (number of revolutions).

4. $100\div2\frac{3}{4}=36\frac{4}{11}$ (miles per hour).
$420\div36\frac{4}{11}=420\times\frac{11}{400}=\frac{231}{20}=11.55$ (hours), or 11 hrs. 33 min.

5. Teacher's salary \$450
Other expenses 245
Interest, $5\frac{1}{2}\%$ of \$3500 192.50
\$887.50

\$887.50 ÷ 28 = 31.70 (cost per child).

6. $\dfrac{160\times21}{4}$ = sq. rd. per day.

$$160\times54+\frac{160\times21}{4\times2}=\frac{\cancel{160}\times\overset{18}{\cancel{54}}\times4\times2}{\cancel{160}\times\underset{7}{\cancel{28}}}=\frac{144}{7}=20\tfrac{4}{7}\text{ (days).}$$

8. $20\times36=720$ (cu. ft. in bin).
$10\times8\times9=720$, $20\times9\times4=720$, $5\times16\times9=720$ (etc.).

10. 104% of \$150,000 = \$156,000 (value at end of first year).
104% of \$156,000 = \$162,240 (value at end of second year).
104% of \$162,240 = \$168,729.60 (value at end of third year).

Solution of Problems (page 275)

1. 65% of \$1560 = \$1014 (value at end of first year).
75% of \$1014 = \$760.50 (value at end of second year).

2. $\dfrac{10435}{8.6}\times18\tfrac{1}{4}=22144$ (cents), or \$221.44.

3. 4.3% of 7830 lbs. = 336.69 (lb. butter fat).
$336.69 + .85 = 396.1$ (lb. of butter).

4. Dimensions in yards are 6, $3\tfrac{1}{6}$, 3.
Area of walls $=2\times6\times3+2\times3\tfrac{1}{6}\times3=36+31=67$ (sq. yd.).
Area of ceiling $=6\times3\tfrac{1}{6}=$ 31 (sq. yd.).
Total area = 98 (sq. yd.).
$98\times\$.35=\$34.30$ (total cost).

5. 3.6% of 11,895 lbs. = 428.22 (lb. butter fat).
$428.22 + .85 = 503.8$ (lb. butter).

6. $$\frac{\overset{\cancel{615}\ 205}{\cancel{2460}}\times5}{\underset{\cancel{6}\ 2}{\cancel{24}}}=\frac{1025}{2}=512.5\text{ (board-feet).}$$

$.5125\times\$84=\$43.05$ (cost).

7. $45000 \times \$.37 = \$16650.00$ (buying price).
   2% of 45000 bu. = 900 bushels (shrinkage).
   $44100 \times \$.41 = 18081.00$.
   $\$18081 - \$16650 - \$340 = \$1091$ (gain).

Material for Sixth Examination, Grade Six.

If it is desired to include examples in the four fundamental operations, percentage, interest, commission, discount, or profit and loss, use some of those on pages 373, 385, 398, 412, 424.

1. A field is 40 rods wide and 65 rods long. Draw a figure representing it, letting one inch represent 10 rods.
2. A table is 30 inches wide and 42 inches long. Draw a figure representing it, letting one inch represent a half foot.
3. Using the scale 1 : 24, draw a figure representing a room 10 feet wide and 12 feet long.
4. Make a graph representing the numbers 24, 30, 40, 56.
5. At a ball game there were 1500 men, 900 women and 700 children. Make a graph representing these numbers.
6. The attendance at a school for the last five years has been 260, 290, 340, 370, and 410. Make a graph representing these numbers.
7. How many revolutions will the wheels of an automobile make in going 100 miles if their diameter is 3 feet? **56,000**
8. The diameter of the drivers on a locomotive is 5 feet. How many revolutions will they make in going 350 miles? **117,600**
9. A garden contains $1\frac{3}{4}$ acres. How long is it, if it is 12 rods wide? **$23\frac{1}{3}$ (rods)**
10. An auctioneer sells goods for \$5860, and charges $3\frac{1}{2}$% commission. How much does his commission amount to? **\$205.10**
11. An agent gets a monthly salary of \$225.00 and 5% commission on all sales over \$40,000 for one year. One year his sales were \$120,000. What was his yearly income? **\$6700**
12. A fast train ran 176 miles in 3 hours and 15 minutes. What was the average speed in miles per hour? **54.15 (miles per hour)**

13. Find the number of board-feet in 18 planks each 16 feet long, 10 inches wide, and $2\frac{1}{2}$ inches thick. 600 (board-feet)

14. Using a tape measure, a boy finds the circumference of a tree to be 6 feet 8 inches. What is the diameter of the tree? 25.45 (inches)

15. If it takes 6 days to plow a field 120 rods long and 30 rods wide, how long will it take to plow a field 180 rods long and 150 rods wide? 45 (days)

16. At 42 cents a square yard, how much does it cost to plaster the walls and ceiling of a schoolroom 30 feet wide, 36 feet long, and 11 feet high, making no allowance for doors or windows? $118.16

Standard Tests

See notes to page 308

Test in Addition. (Third test, grade six). Time, 6 minutes. Half of the class should have 10 or more correct answers.

1.	2.	3.	4.	5.
$5.64	$2.19	$2.18	$7.92	$7.48
2.79	6.37	7.63	3.08	.70
.39	3.29	3.92	2.36	1.29
.80	1.67	9.36	.47	3.56
2.19	.49	2.61	.52	2.61
1.37	.67	.47	7.49	2.37
6.70	1.48	.36	1.42	.56
7.89	3.46	4.18	1.56	5.17
$27.77	$19.62	$30.71	$24.82	$23.74

6.	7.	8.	9.	10.
$5.46	$4.25	$3.72	$3.26	$1.47
7.06	.14	3.19	2.45	.65
3.82	3.72	.57	1.16	2.34
.12	.83	4.63	.75	1.45
.61	1.64	2.31	.10	2.46
2.57	2.31	.65	1.46	1.73
3.86	1.78	1.43	2.59	2.38
1.15	3.07	1.29	3.75	.29
$24.65	$17.74	$17.79	$15.52	$12.77

11.	12.	13.	14.	15.
$8.58	$4.79	$3.82	$ .24	$1.43
.37	.36	.96	.68	6.27
2.60	2.34	.37	5.46	.54
3.76	.61	2.39	1.72	3.60
6.08	2.68	1.56	2.16	.67
1.69	3.92	3.04	3.34	1.20
.45	3.11	1.70	1.07	3.08
3.72	1.56	1.15	6.00	2.75
$27.25	$19.37	$14.99	$20.67	$19.54

16.	17.	18.	19.	20.
$6.98	$1.65	$1.98	$1.39	$5.68
3.40	2.86	3.40	4.61	2.34
6.05	1.75	2.05	3.97	.72
.35	3.46	5.36	3.16	.56
.69	.47	.57	.56	3.44
7.08	9.05	.41	2.79	9.18
5.19	.69	6.92	2.50	6.16
2.47	1.37	3.60	.12	1.29
$32.21	$21.30	$24.29	$19.10	$29.37

Test in Subtraction. Time, 6 minutes.

Half of the class should have 20 or more correct answers.

1.	2.	3.	4.	5.
341658	840157	504379	928126	154776
179372	259024	272183	564912	127482
162286	581133	232196	363214	27294

6.	7.	8.	9.	10.
834897	272820	387914	546176	900794
317932	168470	253275	252734	378241
516965	104350	134639	293442	522553

11.	12.	13.	14.	15.
531263	835489	649291	743456	904157
383151	372534	251420	315834	293426
148112	462955	397871	427622	610731

16. 687452	17. 146780	18. 876406	19. 546167	20. 254386
253761	105719	367903	234781	175836
433691	41061	508503	311386	78550
21. 973318	22. 341374	23. 469689	24. 123456	25 708613
364602	192137	253792	102268	352531
608716	149237	215897	21188	356208
26. 746354	27. 926327	28. 544470	29. 124036	30. 340927
273149	734062	251560	105621	254316
473205	192265	292910	18415	86611
31. 819235	32. 926507	33. 502613	34. 243175	35. 128569
352721	367204	348212	192724	104783
466514	559303	154401	50451	23786
36. 993126	37. 763214	38. 431256	39. 487296	40. 607241
574721	361058	250732	273527	362139
418405	402156	180524	213769	245102

Test in Multiplication. Time, 6 minutes.

Half of the class should have 10 or more correct answers.

1. 6846	2. 7826	3. 329	4. 4162	5. 7491
43	19	456	58	68
294378	148694	150024	241396	509388
6. 3456	7. 9183	8. 593	9. 6581	10. 5349
73	27	326	79	48
252288	247941	193318	519899	256608
11. 6915	12. 3129	13. 231	14. 4765	15. 9748
68	57	892	36	51
470220	178353	206052	171540	497148
16. 6481	17. 5795	18. 289	19. 5638	20. 9518
23	13	765	45	46
149063	75335	221085	253710	437828

Test in Division. Time, $8\frac{1}{2}$ minutes.

Half of the class should have 5 or more correct answers.

1211–273	1169–393	1175–160
1. 519)628782	2. 491)574372	3. 399)468985
1526–204	1265–237	1326–236
4. 476)726580	5. 690)873087	6. 437)579698
1221–286	1117–331	1157–432
7. 614)749980	8. 391)437078	9. 536)620584
970–444		
10. 592)574684		

Pages 276 and 277

New Matter. Discount series.

Remarks. Make clear to the child why discount series are used. In the first place, the child should be made to understand that when price lists are sent out by wholesale dealers the price is purposely placed high so that for a considerable length of time, under normal conditions, the prices will not go above the list price. The wholesale dealer expects, under normal circumstances, to sell lower than the list price. This he does by quoting the list price with a certain discount. Suppose a bill of goods listed at \$100 is sold at a discount of 30%, with three months credit. Under certain conditions the wholesale dealer may be willing to give an extra discount of 10%. This means that he will deduct 10% of the \$70 that is due in three months, and not of the \$100 which is the list price. The wholesale dealer may be willing to give still another discount of 5% for immediate cash payment. Hence the following condition may arise. The goods may be sold normally at a discount of 30%; then an additional 10% may be given to obtain a particularly valuable sale, and another 5% for cash. This will then be computed as follows:

30% is first deducted from the \$100.00, leaving \$70.00; then 10% of \$70 is deducted leaving \$63.00; and finally 5% of \$63.00 is deducted, leaving \$59.85.

It should be borne in mind that the only difficulty here is to learn how the discounts are taken. The arithmetic proper is the same old story, and will cause no trouble. Therefore, whenever this subject is to be taught, the attention must be directed specifically to the one new thing in hand, namely, the commercial usage which gives rise to the problem.

Supplementary Matter. If possible, the teacher should collect real bills, discounted in series.

For the drill in fundamentals, play game No. 2, page 30. Solve examples 13 to 24, page 277.

Pages 278 and 279

New Matter. Taxes.

Remarks. The actual arithmetic involved in taxation is very simple. The only difficulty is to understand how taxes are levied, and the scheme according to which they are computed.

Supplementary Matter. If possible, collect data from a real tax spread. A part of such a spread will serve, and the children will then see exactly what is required. Bring out clearly that there are two distinct problems so far as the arithmetic is concerned. One is to find the rate of taxation when the assessed valuation and the amount of taxes to be raised are given, and the other is to find how much a particular person having a certain valuation will have to pay. Very interesting discussions may develop by finding for what purposes the taxes are used, and by raising questions as to whether they are wisely expended. The main idea involved, from a social point of view, is that a share of a person's income is taken by the group (the community, the state, or the nation) and expended collectively. The following question may be raised: Is it just that one-tenth, for instance, of a person's income should be taken away from him and spent in ways about which he has very little to say?

Discuss also whether, for instance, income from real estate and income in the form of wages are equally taxed.

Solution of Problems (page 278)

2. $140 \times \$65.00 = \$9100.00$.
$.005 \times \$12500 = \$62.50$ (tax).

4. $50 \times 120 = 6000$ (sq. ft.).
$6000 \times \$.45 = \$2700.00$ (value of lot).
$11200.00 value of house and lot.
$.00875 \times \$11,200 = \$98.00$ (tax).

5. $150 \times 140 = 21000$ (area of lot in sq. ft.).
$21000 \times \$.25 = \$5250.00$ (value of lot).
$\frac{4}{5}\% = .8\%$. $.008 \times \$5250 = \$42$ (tax).

Solution of Problems (page 279)

3. $2 \times 25 \times 120 = 6000$ (area of lots in sq. ft.).
$6000 \times \$.55 = \$3300.00$ (value of lots).
7000.00
$\$10300.00$ (value of lot and building).
$.00734 \times 10300 = \$75.60$ (tax).

4. $350 \times \$85 = \$29750$ (value of land).
6400
$\$36150$ (value of land and buildings).
$.00825 \times \$36150 = \$298.24$ (tax).

5. $.0054 \times \$12400 = \$66.96$ (tax on personal property).
$.0075 \times \$10000 = 75.00$ (tax on real estate).
$\$141.96$ (total tax).

6. $.00925 \times \$8600 = \$79.55$ (tax on personal property).
$.0125 \times \$14000 = 175.00$ (tax on real estate).
$\$254.55$ (total tax).

Pages 280 and 281

New Matter. Fire insurance.

Remarks. The only difficulty here is to understand what fire insurance is, and how the premium is computed.

Supplementary Matter. Find the fire insurance rate on various kinds of buildings and goods, and have problems made based on these rates. The value of certain buildings should be found, and the amount of insurance carried on them. In finding the total loss in case of fire deduct the premium from the face of the insurance policy. Make some study of comparative rates of insurance on various kinds of property, and of comparative rates of insurance in different localities. Discuss the reasons for such differences.

Solution of Problems (page 280)

5. $.004 \times \$8000 = \$32.00$ (premium on house).
$.005 \times \$5000 = \underline{\$25.00}$ (premium on personal property).
$\$57.00$ (premium on total).

6. $.0045 - .00275 = .00175$ (saving in rate).
$.00175 \times \$225{,}000 = \$393.75$ (amount saved).

Solution of Problems (page 281)

1. The three-year rate is $1\frac{1}{4}\%$.
$1\frac{1}{4}\%$ of $\$4000 = \$50.00$ (premium).

2. The five-year premium is $4 \times .45\% = 1.8\%$.
1.8% of $\$8500 = \$153.00$ (premium).

3. The premium rate for 90 days is 40% of $.56\% = .224\%$.
$.224\%$ of $2500 = \$5.60$ (premium).

4. 80% of $\$16{,}000 = \$12800$.
$.375\%$ of $\$12800 = \$48.00$ (premium).

5. 70% of $.6\% = .42\%$.
$.42\%$ of $\$3400 = \$14.28$ (premium).

6. The five-year rate is $4 \times .35\% = 1.4\%$.
1.4% of $\$25{,}000 = \$350.00$ (premium).

7. The five-year rate is $4 \times .45\% = 1.8\%$.
1.8% of $\$12000 = \$216$ (five-year premium).
$4 \times \$216 = \$864$ (total premium).

Pages 282 and 283

New Matter. Life insurance.

Remarks. The arithmetic involved in these problems consists almost entirely in using a table of rates and simple multiplication.

Supplementary Matter. It is easy to obtain a book of life insurance rates from an insurance agent. Make up problems from such a book, using the ages of people who are known to the children. It may be remarked here that when the children are a good deal better informed than they are at present, they may be able to solve problems such as the rate of interest which is obtained on an investment in life insurance policies. It doesn't do any harm to learn that there are things about which we do not yet know, and which future activity may place within our reach.

Solution of Problems (page 283)

1. $2.5 \times \$24.38 = \$60.95$ (yearly premium).

4. $5 \times \$28.11 = \$140.55$ (yearly premium).
$17 \times \$140.55 = \$2389.35$ (sum of all premiums).
$\$5000 - \$2389.35 = \$2610.65$ (excess of policy on total premiums).

5. $4 \times \$54.06 = \$216.24$ (yearly premium).
$20 \times \$216.24 = \$4324.80$ (total premiums).
$\$4324.80 - \$4000 = \$324.80$ (excess of total premiums over policy).

6. $28 \times \$395.50 = \$11{,}074.00$ (total premium).
$\$11{,}074.00 - \$10{,}000 = \$1{,}074.00$ (excess of total premium over policy).

Page 284

New Matter. Discounting a promissory note.

Remarks. Note in particular that the bank discount is deducted at the beginning of the period for which the note is to run after it is discounted.

Page 285

New Matter. Bills.

Remarks. The discounting of a bill is very much the same kind of a transaction as the discount made by the bank; that is, the discount is computed on the face of the bill. Of course the element of time does not enter into problems of commercial discount as it does into problems of bank discount.

Page 286

New Matter. Standard time.

Remarks. This topic should not be confused with longitude and time. All that is intended here is to make clear that there are so-called time belts in the United States, and that the time differs by one hour in any two consecutive belts. Nothing is said here about longitude, and the topic is entirely within reach of the child. Moreover, it is important that the child should be taught something about this, for sooner or later he will have to set his watch as he crosses a boundary between two time belts.

Page 287

Review. Problems without numbers. This page serves to summarize the supplementary matter given in this book. If the children are able to answer these questions promptly and understandingly, the topics themselves should give no trouble.

Pages 288 and 289

Review. Summary of all the tables that have been studied. These tables need not be studied separately. Reference is made to them throughout the book, but whatever part of each table is to be memorized would be memorized at the time the work is taken up in the text itself. However, the tables are given here for convenient reference both in the schoolroom and in later life.

Lennes, N. J. 613349

Applied arithmetic.

NAME	DATE	NAME	DATE

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